Numerical Methods And Algebra

Date
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Assignment - 1

Q-1 To find : $\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

Let $x = \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$

Squaring both sides

$$x^2 = 7+\sqrt{7+\sqrt{7+\sqrt{7+\ldots}}}$$

$$x^2 = 7 + x$$

$$x^2 - x - 7 = 0$$

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$x = 3.1926, \quad -2.1726$

Rejected

$\therefore \sqrt{7+\sqrt{7+\sqrt{7+\ldots}}} = 3.1926$

Q-2 $7x^2 - 6y^2 = -11x^2 - 4y^2$

$$18x^2 = 2y^2$$

$$9x^2 = y^2$$

$$\frac{x^2}{y^2} = \frac{1}{9}$$

$$\boxed{\frac{x}{y} = \frac{1}{3}}$$

Q-3 $x + 3y = 4$ $\quad x = 4 - 3y$ (i)

$x^2 + y^2 - 4y = 12$

$x^2 + y^2 - 4y = 12$

$(4-3y)^2 + y^2 - 4y = 12$

$10y^2 - 28y + 4 = 0$

$5y^2 - 14y + 2 = 0$

$y = 2.648996$ or 0.151004

$\therefore$ ↓ ↓

$x = -3.946998$ or 3.546998

Q-4 $d = 2r^2 - 16r + 34$

a) At $r = 3.5$

$d = 2(3.5)^2 - 16(3.5) + 34$

$= \underline{2.5}$

b) $\frac{d}{dr} = 4r - 16$

$r = 4$

$\frac{d^2}{dr^2} = 4 > 0$

Minimum at $\underline{r = 4}$

c) $d = 100$

$100 = 2r^2 - 16r + 34$

$0 = r^2 - 8r - 33$

$0 = r^2 - 11r + 3r - 33$

$0 = r(r-11) + 3(r-11)$

$0 = (r+3)(r-11)$

$r = 11, -3$

Rejected

∴ Speed of engine is 11

Q-5

Company 1	Company 2
1500 + 25/km	2100 + 22/km

a) $1500 + 25n$

b) $2100 + 22n$

c) $2100 + 22n < 1500 + 25n$

$600 < 3n$

$\underline{200 < n}$

Q-6 $f(x) = x^3 - 7x^2 + 14x - 6 = 0$

a) Let $a = 0$, $b = 1$

$f(a) = f(0) = -6$

$f(b) = f(1) = 2$

Since $f(a)$ and $f(b)$ have opposite signs the interval is appropriate

$x = 0.5811$

b) Considering $a = 1$, $b = 3.2$

$f(a) = f(1) = 2$

$f(b) = f(3.2) = -0.112$

The interval is appropriate as $f(a)$ and $f(b)$ have opposite signs.

$x = 3.0011$

c) Let $a = 3.2$, $b = 4$

$f(a) = f(3.2) = -0.112$

$f(b) = f(4) = 2$

$\therefore x = 3.42$

Q-7 $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

Since $f(n)$ has real zero in $(-1, 0)$ we consider the midpoint -0.5 as x_0.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x)} \Rightarrow -0.5 - \frac{f(0.5)}{f'(-0.5)} = -0.15045$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.0418168$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.040659343S$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.040659288832$$

Since $f(x)$ has a real zero in $(0,1)$ we consider the midpoint 0.5 as x_0

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -0.70508920$$

Similarly we calculate

$x_2 = -0.323791142$

$x_3 = -0.064603131O3$

$x_4 = -0.040688615115$

$x_5 = -0.040659288835$

Q-8 70, 86, 81, 83

a) Converse Average $= \frac{70+81+86+83+x}{5} = \frac{320+x}{5}$

b) $80 \leq \frac{320+x}{5} < 90$

c) $400 \leq 320+x < 450$

$80 \leq x < 130$

Since marks are out of 100

$80 \leq x \leq 100$

Q-9

Discount Rate	4%	?	20%
NPV	8.54	0	-11.81

$$y - 0.04 = \frac{0.20 - 0.04}{-11.81 - 8.54}(0 - 8.54)$$

$$y = 10.7144963\%$$

Discount Rate = 10.71480%

Q-10 $R(x) = 32x - 0.21x^2$ $\quad C(x) = 195 + 12x$

$$32x - 0.21x^2 > 195 + 12x$$

$$0.21x^2 - 20x + 195 < 0$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= 84.2114253, \quad 11.0266771$$

$$11.0266771 < x < 84.2114253$$

$$12 < x < 84$$

No. of orders to remain profitable should be greater than 12 and less than 84.