Assignment - 1

1) $X \sim Poi(\theta)$ and θ is large, then approx

$$\frac{X-\theta}{\sqrt{\theta}} \sim \text{approx. } N(0,1)$$

$X = 200$ $\quad \therefore \quad \theta = X \pm 1.96 \sqrt{\theta/n}$

Q2) $x_i : i = 1, 2, 3 \ldots n$
mean μ, variance σ^2

$\bar{x} = \sum_1^n x_i$ (sample mean)

$s^2 = \frac{1}{n-1} \sum_i^n (x_i - \bar{X})^2$ (sample variance)

$X_i \sim N(\mu, \sigma^2)$
$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

$E(\bar{X}) = E\left(\frac{\sum x_i}{n}\right) = \frac{n\mu}{n} = \mu.$

$var(\bar{X}) = var\left(\frac{\sum x_i}{n}\right) = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$

iii) $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$$E(s^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2$$

$$var(s^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{(n-1)}$$

iv) $P(50\% < s^2 < 150)$

$$= P\left(\frac{9\times 50}{100} < \chi^2_9 < \frac{9\times 150}{100}\right)$$

$$= 4.5,\ 13.5 \text{ Ans.}$$

3) MLE of p -

$$l(p) = \left(((1-p)^7\right)^{86} \left(4p(1-p)^3\right)^{75} \left(6p^2(1-p)^2\right)^{16} \left(4p^3(1-p)\right)^2 (p^4)$$

$$= (1-p)^{344}\ p^{75} (1-p)^{225}\ p^{32} (1-p)^{32}\ p^6 (1-p)^2 (p^4)$$

$$= (1-p)^{603}\ p^{117} \times \text{Const.}$$

$$\ln l(p) = 603 \ln(1-p) + 117 \ln p + \text{const.}$$

$$\frac{d}{dp} \ln l(p) = \frac{-603}{1-p} + \frac{117}{p} = 0$$

$$117(1-p) - 603p = 0$$

$$p = 0.1625$$

iii) $P(X=0) = (1-p)^4 = 0.492$

$P(X=1) = 4p(1-p)^3 = 0.3818$

$P(X=2) = 6p^2(1-p)^2 = 0.111$

$P(X=3) = 4p^3(1-p) = 0.0144$

$P(X=4) = 0.0007$

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.59	0.126.

By clubbing,

	0	1	2+
O_i	86	75	19
E_i	88.56	68.72	22.71
C_i	0.0757	0.5739	0.6061

$\Sigma C_i = 1.2557 \sim \chi^2_1$

$\therefore 1.2557 < 3.841$

Do not reject H_0.

5) $U(a,b)$ $\quad E(X) = \frac{a+b}{2}$ $\quad V(X) = \frac{(b-a)^2}{12}$

show this,

$a = \bar{x} - \sqrt{3}s$

$a = 2\bar{x} - b$

$a = b - 2\sqrt{3}s$

$2\bar{x} - b = b - 2\sqrt{3}s$

$b = \bar{x} + \sqrt{3}s$ $\quad a = \bar{x} - \sqrt{3}s$

Date:

6) $H_0 : p = 0.1 \qquad H_1 : p > 0.1$

i) Type 1 error.

P(reject H_0 when it was true) = $P(X > 5$ when

7) i) $\bar{X}_{public} = \frac{270}{9} = 30$

$\bar{X}_{private} = \frac{315.5}{9} = 35.05$

$S^2_{public} = \frac{1}{8} \times \left[\sum x_i^2 - n\bar{X}^2_{public}\right]$

$= \frac{1}{8} \times \left[8614.5 - 9 \times 30^2\right]$

$= 64.312$ Ans

$S^2_{private} = \frac{1}{8} \times \left[11599.25 - 9 \times 35.05^2\right]$

$= 67.84093$

$H_0 : \mu_1 = \mu_2 \qquad H_1 : \mu_1 \neq \mu_2$

ii) $Sp^2 = \frac{(n-1)S_1^2 + (n-1)S_2^2}{n_1 + n_2 - 2}$

$= \frac{8 \times 64.312 + 8 \times 67.84093}{16}$

$= 66.076465$

Test : $\frac{\bar{X}_1 - \bar{X}_2 - \delta}{Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{30 - 35.05 - 0}{8.1287\sqrt{\frac{2}{9}}}$

$= -1.31788$

Do not reject H_0.

~~$t_{18} \sim (\pm 2.101)$~~

iii) Private hospitals do not result higher claim size at 95%.

9) $t_k = \dfrac{N(0,1)}{\sqrt{\chi^2_k/k}}$ where $N(0,1)$

and χ^2_k random variables are independent

ii) $E(t_k) = 0$

$var(t_k) = \dfrac{k}{k-2}$

iii) a) $\bar{X} \pm t_{0.0001, n-1} \dfrac{s}{\sqrt{n}}$

$50 \pm t_{0.005, 9} \times \sqrt{\dfrac{48.667}{10}}$

$= 50 \pm 3.25 \times \sqrt{\dfrac{48.667}{10}}$

$= \cancel{(56.223, 43.7767)}$

$= (42.83, 57.17)$

(b) ~~$E(t_k) = 0$~~

~~$Var(t_k) = \dfrac{9}{7}$~~

$N_{0.005, 9} = 50 \pm 2.5758 \sqrt{\dfrac{48.667}{10}}$

$= (44.32, 55.68)$

Page No.:
Date:

Q10) i) $n = 1000$ $Poi(3)$

$$P(X > 3100 \text{ when } X \sim Poi(3))$$
$$X \sim N(3000, 3000)$$

$$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$$

$$= P(Z > 1.826)$$

$$= 3.39\%$$

ii) When H_0 should be rejected but is accepted

iii) The power is the probability of rejecting H_0 when true value of parameter μ is some value other than $\mu = 3000$.

In terms of μ

$$P(X > 3000 \mid X \sim Poi(\lambda))$$

$$= 1 - \frac{e^{-\mu} \times \mu^{3000}}{3000!}$$

Q8) Unbiased Estimator: $E(\hat{\theta}) = \theta$

Bias: $E(\hat{\theta}) \neq \theta$

MSE is when $\theta = (E(\theta)^2 - \theta)^2$ Ans.

11) $n = 12$ $\quad \Sigma x = 213200$

$\Sigma x^2 = 4919860000$

$\bar{X} = 17767$ $\quad S = 10144.$

$X_1 = 11000$ $\quad X_2 = 48000$

$X_1' = 27500$ $\quad X_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 48000$

$A = 154200$

$\Sigma x' = A + x_1' + x_2' = 213200$

$\bar{X}' = 17767$

$\cancel{\Sigma x^2 = A + x_1' + x_2' = 213200}$

$\Sigma x^2 = 4919860000 = B + 11000^2 + 48000^2$

$B = 4243360000$

$$S = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$$

$= 6434.03$ Ans