Assignment - 11

Ques ①

Let x_i be claim size ~~of~~ on home insurance.

y_i " " " " car insurance.

So, ATQ..

$X \sim N(800, 100^2)$; P ~~$(x_1+x_2+x_3+x_4)$~~

$Y \sim N(1200, 300^2)$

~~$P(\ldots 800)$~~ $P(y_1+y_2+y_3 > x_1+x_2+x_3+x_4+800)$

$P(y_1+y_2+y_3 - (x_1+x_2+x_3+x_4) > 800)$

$(y_1+y_2+y_3) - (x_1+x_2+x_3+x_4) \sim N(3\times1200 - 4(800) ; 300^2(3)+100^2(4))$

$\sim N(400, 310000)$

~~$P(\ldots 800)$~~ $P\left(z > \frac{800-400}{\sqrt{310000}}\right)$

$P(z > 0.71842) \Rightarrow 1 - P(z < 0.71842)$

$1 - P(z < 0.72)$

$1 - 0.76424$

$\Rightarrow \boxed{0.23576}$

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Ques (2) $N \sim$ Neg. Binomial (k, p)

$$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^n .$$

$$\Rightarrow {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n .$$

So, $\cancel{k=n}$, $\cancel{p=0.1}$.
$k=3$, $p=0.9$.

$$M_y t = M_N(\log M_x t)$$

$X \sim$ gamma $(6, 2)$

$Y \to$ Total claim amount.

(ii) MGF of Y $\cancel{M_Y(t) = }$

$$M_Y(t) = E(E(e^{Yt}/N=n))$$

$$E(e^{Yt}/N=n) = E(e^{x_1t + x_2t + \cdots x_nt})$$

$$= E(e^{tx_1}) \cdot E(e^{tx_2}) \cdots E(e^{tx_n}).$$

$$\Rightarrow \left(1-\frac{t}{2}\right)^{-6n}$$

So, $$E(E(e^{Yt}/N=n)) = E\left(\left(1-\frac{t}{2}\right)^{-6n}\right)$$

$$= E\left(e^{N\log\left(1-\frac{t}{2}\right)^{-6}}\right)$$

$$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k \Rightarrow \left(\frac{pe^{\log\left(1-\frac{t}{2}\right)^{-6}}}{1-qe^{\log\left(1-\frac{t}{2}\right)^{-6}}}\right)^k$$

3-

$$\left[\frac{p\left(1-\frac{t}{2}\right)^{-6}}{1-q\left(1-\frac{t}{2}\right)^{-6}}\right]^3 \Rightarrow \left[\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

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Q. $V(Y) = \cancel{E(X)\,Q}$

$V(Y) = E(N)\,V(X) + (E(X))^2 \cdot V(N)$.

$\Rightarrow \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left(\frac{36}{2}\right)^2 \times \frac{(3)(0.1)}{(0.9)^2}$.

~~V(Q)=2~~

St. deviation $\rightarrow$ 2.887.

Ques (3) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

$MGF = E(e^{tx}) \Rightarrow \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)}\, dx$

$M_x t = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda\alpha}\, dx \Rightarrow e^{\lambda\alpha} \lambda \int_{\alpha}^{\infty} e^{-(\lambda-t)x}\, dx.$

$\frac{e^{\alpha\lambda}\lambda}{t-\lambda}\left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty} \Rightarrow \frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[-e^{-(\lambda-t)\alpha}\right]$

$\frac{\cancel{e^{\lambda\alpha}}\lambda}{\lambda-t}\,\cancel{e^{-\lambda\alpha}}\, e^{t\alpha} \Rightarrow e^{t\alpha}\frac{\lambda}{\lambda-t}$

(ii) $\boxed{M_x t = e^{t\alpha}\frac{\lambda}{\lambda-t}}$

$\Rightarrow E(x) = M_x'(t) \Rightarrow \lambda\, \frac{e^{t\alpha}}{(\lambda-t)}$

$\lambda\left[\alpha e^{t\alpha}(\lambda-t) + e^{t\alpha}(-1)\right]$

$\Rightarrow \lambda\left[\alpha e^{t\alpha}(\lambda-t) - e^{t\alpha}\right].$

$\Rightarrow \lambda\alpha e^{t\alpha}(\lambda-t) - \lambda e^{t\alpha}$

Put $t=0$ we get: $\lambda^2\alpha - \lambda \Rightarrow \lambda(\lambda\alpha - 1)$.

(iii) $M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda - t} \Rightarrow \lambda e^{t\alpha}(\lambda-t)^{-1}$

$M_x'(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-2}(+1) + (\lambda-t)^{-1} e^{t\alpha}(\alpha)\right]$

$\Rightarrow \lambda\left[(\lambda-t)^{-1} e^{t\alpha}(\alpha) + e^{t\alpha}(\lambda-t)^{-2}\right]$

$\Rightarrow \lambda e^{t\alpha}(\lambda-t)^{-2}\left[\alpha(\lambda-t) + 1\right]$

$\boxed{t=0}$ $\Rightarrow \lambda(\lambda)^{-2}\left[\alpha(\lambda)+1\right]$ $\qquad \alpha + \frac{1}{\lambda}$

$\Rightarrow \cancel{\lambda}\ \frac{1}{\lambda}\left[\lambda\alpha+1\right] \Rightarrow \frac{\lambda\alpha+1}{\lambda}$

$E(x^2) = M_x''(t) \Rightarrow \lambda\left[\alpha e^{t\alpha}(\lambda-t)^{-2} + \alpha^2(\lambda-t)^{-1} e^{t\alpha} + e^{t\alpha}(+2)(\lambda-t)^{-3} + (\lambda-t)^{-2}\alpha e^{t\alpha}\right]$

Put $t=0$

$\lambda\left[\alpha(\lambda)^{-2} + \alpha^2(\lambda)^{-1} + (2)(\lambda)^{-3} + \alpha(\lambda)^{-2}\right]$

$\lambda\left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3}\right]$

$E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$

$V(x) = E(x^2) - \left[E(x)\right]^2 \Rightarrow \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]^2$

$\frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left(\alpha^2 + \frac{1}{\lambda^2} + \frac{2\alpha}{\lambda}\right)$

$\cancel{\frac{2\alpha}{\lambda}} + \cancel{\alpha^2} + \frac{2}{\lambda^2} - \cancel{\alpha^2} - \frac{1}{\lambda^2} - \cancel{\frac{2\alpha}{\lambda}} \Rightarrow \boxed{\frac{1}{\lambda^2}}$

Ans (4) $G_V(t) = \left(\frac{p}{1-qt}\right)^k$

$G_V t = \sum t^v \times P(V=v)$.

~~Pt~~ $P(V=4) \Rightarrow \frac{\overline{|k+4}}{\overline{|5}\ \overline{|k}} p^k q^k$.

$= \frac{\overline{|k+4}}{4!\ \overline{|k}} p^k q^k$.

$\Rightarrow \frac{(k+3)!}{(k-1)!\ 4!} p^k (1-p)^k$.

Ans (5) $f(x,y) = ax(x+y)$ $0<x<5$, $10<y<20$

(i) $P\left(y > \frac{1}{3}x + 10\right)$

$a\int_{10}^{20}\int_{0}^{5} (x^2+xy)\, dx\, dy = 1$

$\int_{10}^{20} \left[\frac{x^3}{3} + \frac{x^2 y}{2}\right]_0^5 dy = \frac{1}{a}$

$\int_{10}^{20} \left(\frac{125}{3} + \frac{25}{2}y\right) dy = \frac{1}{a} \Rightarrow \left(\frac{125}{3}y + \frac{25y^2}{4}\right)_{10}^{20} = \frac{1}{a}$

$833.33 + 2500 - (416.667 + 625) = \frac{1}{a}$.

$\boxed{a = 0.00043636}$

$a\left(\frac{2500}{3} + 2500 - \frac{1250}{3} - 625\right) =$

$a\left(\frac{1250}{3} + 1875\right) = 1$

$a\left(\frac{6875}{3}\right) = 1$

~~$a = 0.000436$~~.

$\boxed{a = \frac{3}{6875}}$

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(iii) $\cancel{P\left(Y > \frac{X}{10} + 10\right)}$ $E(Y/X)$

$$f_{Y/X} = \frac{f(x,y)}{f(x)} \Rightarrow$$

$$f(x) = \frac{3}{6875}\int_{10}^{20}(x^2+xy)\,dy \Rightarrow \frac{3}{6875}\left[x^2y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$\Rightarrow \frac{3}{6875}\left[20x^2 + x(200) - 10x^2 - 50x\right]$$

$$= \frac{3}{6875}\left[10x^2 + 150x\right] \Rightarrow \frac{30x}{6875}\left[x+15\right]$$

$$f_{Y/X} = \frac{\cancel{3}}{6875}\,\frac{x(x+y)}{\cancel{30}\,x} \times 68\cancel{75}(x+15) \quad \Bigg| \quad \frac{\cancel{3}\,(\cancel{x}(x+y))\,(\cancel{6875})}{\cancel{6875}\,(10\cancel{x})\,(x+15)}$$

$$f_{Y/X} \Rightarrow \frac{(x+y)(x+15)}{10} \qquad f_{Y/X} = \frac{(x+y)}{10(x+15)}$$

$$So,\quad E(Y/X) = \int_{10}^{20} y \cdot f_{Y/X}\,dy$$

$$\cancel{\frac{30x(x+}{6875}\int y}$$

$$\left(\frac{x+15}{10}\right)\int_{10}^{20}(x+y)(y)\,dy = \frac{(x+15)}{10}\left(xy + y^2\right)_{10}^{20}$$

$$\frac{x+15}{10}\left[20x + 400 - 10x - 100\right]$$

$$\frac{x+15}{10}\left[10x + 300\right] \Rightarrow (x+15)(x+30)$$

So, $E(Y/X) = \int_{10}^{20} y \cdot \frac{(x+y)}{10(x+15)} dy \Rightarrow \frac{1}{10(x+15)} \int_{10} (xy+y^2) dy$

$\Rightarrow \frac{1}{10(x+15)} \left[\frac{xy^2}{2} + \frac{y^3}{3} \right]_{10}^{20} \Rightarrow \frac{1}{10(x+15)} \left[200x + \frac{8000}{3} - 50x - \frac{1000}{3} \right.$

$\frac{1}{10(x+15)} \left[150x + \frac{7000}{3} \right]$

$20 \times 20 \times 20$

$\frac{8000}{3}$

$\boxed{\frac{1}{x+15} \left[15x + \frac{700}{3} \right]}$

(ii) $P\left(y > \frac{1}{3}x + 10 \right)$,

$f(y) = \frac{3}{6875} \int_0^5 x^2 + xy \; dx \Rightarrow \frac{3}{6875} \left[\frac{x^3}{3} + \frac{x^2 y}{2} \right]_0^5$

$f(y) \Rightarrow \frac{3}{6875} \left[\frac{125}{3} + \frac{25}{2} y \right]$

$\Rightarrow \frac{3}{687}$

So, $\frac{3}{6875} \int_{\frac{1}{3}x+10}^{20} \left(\frac{125}{3} + \frac{25}{2} y \right) dy \Rightarrow \frac{3}{6875} \left[\frac{125y}{3} + \frac{25y^2}{4} \right]_{\frac{1}{3}x+10}^{20}$

$\Rightarrow \frac{3}{6875} \left[\frac{125(20)}{3} + \frac{25}{4}(400) - \frac{125}{3}\left(\frac{1}{3}x+10\right) - \frac{25}{4}\left(\frac{1}{3}x+10\right)^2 \right]$.

$\frac{3}{6875} \left[\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3} - \frac{25}{4} \left[\frac{1}{9}x^2 + 100 + \frac{2x}{3} \right] \right.$

$\frac{3}{6875} \left[\frac{2500}{3} + 2500 - \frac{125}{9}x - \frac{1250}{3} - \frac{25x^2}{36} - 625 - \frac{25x}{6} \right] \Rightarrow \frac{3}{} \left[\frac{1250}{} + 15 \right.$

Sol (6) $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$.

$$M_X(t) = e^{\lambda(e^t-1)} \qquad M_Y(t) = e^{\mu(e^t-1)}$$

So, $Z = X - Y$

$$M_Z(t) = E(e^{tz}) = E(e^{t(x-y)}) = E(e^{tx} \cdot e^{-ty}).$$

$$= E(e^{tx}) \times E(e^{-ty}). \Rightarrow \frac{E(e^{tx})}{E(e^{ty})}$$

$$\Rightarrow M_X(t) \; M_Y(-t)$$

$$\Rightarrow e^{\lambda(e^t-1)} \; e^{\mu(e^{-t}-1)}. \Rightarrow \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$$

$$\Rightarrow e^{\lambda e^t - \lambda + \mu e^{-t} - \mu}$$

$$\Rightarrow e^{(\lambda-\mu)(e^t-1)}$$

$$\Rightarrow e^{e} \; e^{-(\lambda+\mu)} \; e$$

So, By uniqueness MGF of $z = e^{(\lambda-\mu)(e^t-1)}$
$\sim Pois(\lambda-\mu)$

Now: let $z = x + y$

$$E(e^{tz}) = e^{\lambda(e^t-1)} \times e^{\mu(e^t-1)}$$

$$\Rightarrow e^{(\lambda+\mu)(e^t-1)}$$

So, By uniqueness $Z \sim Poi(\lambda+\mu)$.

Ques (7)

			x		
		1	2	3	4
y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

(ii) $V(x/y=2) = E(x^2/y=2) - [E(x/y=2)]^2$

Now: $E[x^2/y=2] \Rightarrow \sum_x x^2 \frac{P(x=x, y=y)}{P(y=2)}$

when $x=1$. when $x=2$ when $x=3$ when $x=4$

$\frac{(1)^2 P(x=1, y=2)}{P(y=2)} + \frac{(2)^2 P(x=2, y=2)}{P(y=2)} + \frac{(3)^2 P(x=3, y=2)}{P(y=2)} + \frac{(4)^2 P(x=4, y=2)}{P(y=2)}$

$\Rightarrow \frac{(1)(0)}{0.6} + \frac{(4)(0.3)}{0.6} + \frac{(9)(0.1)}{0.6} + \frac{16(0.2)}{0.6}$

$\Rightarrow 8.8333.$

Now: $E(x|y=2) \Rightarrow \sum_x \frac{x P(x=x, y=2)}{P(y=2}.$

$\frac{(1)(0)}{0.6} + \frac{(2)(0.3)}{0.6} + \frac{(3)(0.1)}{0.6} + \frac{4(0.2)}{0.6}$

$\Rightarrow \frac{0.6+0.3+0.8}{0.6} \Rightarrow 2.8333.$

$\Rightarrow \boxed{0.807}$

Ques ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$

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$0 < u < 1 \; ; \; \frac{u}{2} < v < 2$

$E(u/v=v) = ?$

$\frac{f(u,v)}{f(v)} \Rightarrow f\; u/v=v$

$f(v) \Rightarrow \frac{48}{67}\int_0^1 (2uv - u^2)\,du \Rightarrow \frac{48}{67}\left[u^2v - \frac{u^3}{3}\right]_0^1$

$\frac{48}{67}\left[v - \frac{1}{3}\right] \Rightarrow \frac{\cancel{48}^{16}}{67}[3v-1]$

$f(v) = \frac{16}{67}[3v-1]$

$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}}{\cancel{67}} \frac{(2uv^2 - u^2) \times \cancel{67}}{\times \cancel{16}(3v-1)}$

$\Rightarrow \frac{3}{3v-1}(2uv^2 - u^2)$. $2u^2v^2 - u^3$

Now: $E(u/v=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - u^2)\,du$

$\frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1 \Rightarrow \frac{3}{3v-1}\left[\frac{2v^2}{3} - \frac{1}{4}\right]$

$\frac{\cancel{3}}{3v-1}\left[\frac{8v^2-3}{\cancel{12}\,4}\right] \Rightarrow \boxed{\frac{8v^2-3}{4(3v-1)}}$

Q8..

$X \sim \text{Gamma}(3, 2)$ $\qquad E(X) = \frac{3}{2} \qquad V(X) = \frac{3}{4}$.

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$E(Y/X=x) = 3x + 1 \qquad var(Y/X=x) = 2x^2 + 5$.

$E(Y) = E(E(Y/X=x)) \Rightarrow E(3x+1)$

$= 3E(X) + 1$

$\Rightarrow 3\left(\frac{3}{2}\right) + 1 \Rightarrow \frac{9}{2} + \frac{1}{1} = \frac{9+2}{2}$

$= 11/2$

$V(Y) = E(V(X/Y)) + V(E(X/Y))$.

$\Rightarrow E(2x^2+5) + V(3x+1)$

$= 2E(x^2) + 5 + 9V(x)$

$V(X) = E(X^2) - [E(X)]^2$

$\frac{3}{4} + \frac{9}{4} = E(x^2)$

$12/4 = E(x^2) \Rightarrow \boxed{E(x^2) = 3}$

$V(Y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$

$= \cancel{6} \; 11 + \frac{27}{4} = \frac{44+27}{4}$

$= \boxed{71/4}$

44
27
71

Ques (9)

$E(x) = 3 \quad , \quad V(x) = 4.$

$E(y) = 4 \quad , \quad V(y) = 1$

Correlation coeff. $(x, y) = -0.3\sqrt{4} \Rightarrow cov(x,y)$

$= -0.6$

$Z = x + y.$

(i) $cov(x, x+y) = cov(x,x) + cov(x,y)$

$= V(x) + cov(x,y)$

$\Rightarrow 4 + (-0.6)$

$= 3.4.$

(ii) $Var(z) = cov(x+y, x+y)$

$= cov(x,x) + 2cov(x,y) + cov(y,y)$

$= V(x) + 2cov(x,y) + V(y)$

$\Rightarrow 4 + 2(-0.6) + 1$

$\Rightarrow 5 - 1.2$

$\Rightarrow 3.8$

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Q10

$$f(x,y) = kx^{-\alpha}e^{-y/\beta}$$

$1 < x < \infty$, $1 < y < \infty$

$\alpha > 1$, $\beta > 0$.

$$k\int_1^\infty\int_1^\infty x^{-\alpha}e^{-y/\beta}\,dx\,dy = 1$$

$$k\int e^{-y/\beta}\left[\frac{x^{-(\alpha-1)}}{-\alpha+1}\right]_1^\infty = 1$$

$$\frac{k}{1-\alpha}\int e^{-y/\beta}\left[-1\right] \Rightarrow \frac{k}{\alpha-1}\int e^{-y/\beta}dy = 1$$

$$\frac{k}{\alpha-1}\frac{\left[e^{-y/\beta}\right]_1^\infty}{\left(\frac{-1}{\beta}\right)} \qquad \frac{k}{\alpha-1}(\beta)\left(+e^{-1/\beta}\right) = 1$$

$$\boxed{k = \frac{\alpha-1}{(\beta)\left(e^{-1/\beta}\right)}}$$