1) i)

$$\begin{array}{c|cc} & B & O \\ \hline B & -\lambda & \lambda \\ O & \mu & -\mu \end{array}$$

ii) exponentially distributed

state B $\rightarrow \lambda$

state O $\rightarrow \mu$

iii) $\frac{d}{dt} {}_tP_s^{BB} = -\lambda\, {}_tP_s^{BB} + \mu\, {}_tP_s^{BO}$

$\frac{d}{dt} {}_tP_s^{BO} = \lambda\, {}_tP_s^{BB} - \mu\, {}_tP_s^{BO}$

${}_tP_s^{BB} + {}_tP_s^{BO} = 1$

$\frac{d}{dt} {}_tP_s^{BB} = -\lambda\, {}_tP_s^{BB} + \mu(1 - {}_tP_s^{BB})$.

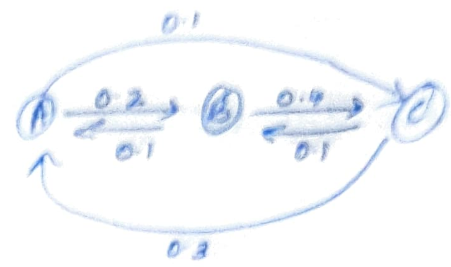
$\frac{d}{dt}\left[\exp((\lambda+\mu)t)\, {}_tP_s^{BB}\right] = \mu \exp((\lambda+\mu)t)$

$\exp((\lambda+\mu)t)\, {}_tP_s^{BB} = \frac{\mu}{\lambda+\mu} \exp((\lambda+\mu)t) + c$.

$c = \frac{\lambda}{\mu+\lambda}$

${}_tP_c^{BB} = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} \exp(-(\lambda+\mu)t)$

2)



$\frac{d}{dt} P_{AA}(t) = -0.3P^{AA} + 0.1P'_{AB} + 0.3P_t^{AC}$

$\frac{d}{dt} P_{AB}(t) = 0.2P_{AA}(t) - 0.5P_{AB}(t) + 0.1P_{AC}(t)$

$\frac{d}{dt} P_{AC}(t) = 0.1P_{AA}(t) + 0.4P_{AB}(t) - 0.4P_{AC}(t)$.

$\frac{d}{dt} P_{\bar{A}\bar{A}}(t) = -0.3P_{\bar{A}\bar{A}}(t)$.

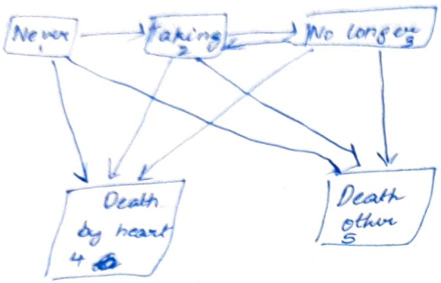
$= \exp(-0.3t)$

$= \exp(-0.6)$

$= 0.5488$

3>

~~Never~~ ~~Taking~~



$${}_{dt+t}P_x = {}_tPa^{31}\, {}_{dt}P_{x+t}^{14} + {}_tPa^{32}\, {}_{dt}P_{x+t}^{24} + {}_tPa^{33}\, {}_{dt}P_{x+t}^{34} + {}_tPa^{34}\, {}_{dt}P_{x+t}^{44} + {}_tPa^{35}\, {}_{dt}P_{x+t}^{54}$$

$${}_{dt}P_{x+t}^{54} = {}_tP_x^{81} = 0.$$

$${}_{dt+t}P_x^{34} = {}_tPa^{32}\, {}_{dt}P_{x+t}^{24} + {}_tPa^{33}\, {}_{dt}P_{x+t}^{34} + {}_tPa\, {}_{dt}P_{x+t}^{44}$$

$${}_{dt}P_{x+t}^{44} = 1$$

By small time probability,

$${}_{dt+t}P_x^{34} = {}_tPa^{32}\, \mu^{24}_{x+t}\, dt + {}_tP_x^{33}\, \mu^{34}_{x+t}\, dt + {}_tP_x^{34} + o(dt).$$

$$\frac{d}{dt}\, {}_tP_x^{34} = {}_tPa^{32}\, \mu^{24}_{x+t} + {}_tP_x^{23}\, \mu^{54}_{x+t}.$$

4> i> mean is 8 calls per hour.

ii> time until next call = 20 minutes.

iii> $P_0(0.5) = \dfrac{e^{-1.5}(1.5)^0}{0!}$

iv> probability phone is engaged = 21/60.

5> $P_{HH}(x, t+dt) = P_{HH}(x,t)\, P_{HH}(t, t+dt) + P_{HS}(x,t)\, P_{SH}(t, t+dt) + P_{HD}(x,t)\, P_{DH}(t, t+dt).$

$P_{DH}(t, t+dt) = 0.$

$P_{HH}(x, t+dt) = P_{HH}(x,t)\, P_{HH}(t, t+dt) + P_{HS}(x,t)\, P_{SH}(t, t+dt)$

$P_{HH}(x, t+dt) = P_{HH}(x,t)(1 - \sigma(t)dt - \mu(t)dt) + P_{HS}(x,t)\, \rho(t,(t_t) + o(d).$

~~$P_{HH}(x, t+dt) =$~~

$\frac{d}{dt} P_{HH}(x,t) = P_{HH}(x,t)(-\sigma(t) - \mu(t)) + P_{HS}(x,t)\, \rho(t,(t)$

$\frac{d}{dt} P_{H\bar{H}}(0,t) = -(\sigma(t) + \mu(t))\, P_{H\bar{H}}(t).$

$\frac{1}{P_{HH}(0,t)} \frac{d}{dt} P_{H\bar{H}}(0,t) = -(\sigma(t) + \mu(t)).$

$= -(\sigma(t) + \mu(t))$

$\frac{d}{dt} \ln P_{H\bar{H}}$

$$\int_0^t \ln P_{\overline{AA}}(0,t) = \int_s^t -(\sigma(s)+\mu(s))\, ds$$

$P_{\overline{AA}}(0)=1$.

$$P_{\overline{AA}}(0,t) = \exp\left(\int_s^t (\sigma(s)+\mu(s))\, ds\right)$$

7) i) The maximum likelihood estimate of transition intensity from state i to j divided by total waiting time in i

Entry and exit time from each individual status.

Total no. of transitions of each type made.

$$\frac{d}{dt} P_{AA}^{(s,t)} = -P_{AA}^{(s,t)}\mu$$

$$\begin{bmatrix} -\mu & \mu \\ 0 & 0 \end{bmatrix}$$

$$\frac{1}{P_{AA}(t)} \frac{d}{dt} P_{AA}(t) = -\mu$$

$$\frac{d}{dt} \ln(P_{AA}(t)) = -\mu$$

$$P_{AA}(t) = \exp(-\mu t + c)$$

$P_{AA}(0) = 1 \quad c = 0$

$$P_{AA}(t) = \exp(-\mu t).$$

claim occurs with cost £ c if moves to state theft claim.

$c(1-\exp(\mu t))$

$$\begin{tikzcd} \boxed{\text{Lapsed}} & \boxed{\text{active}} \arrow[l, "\lambda"'] \arrow[r, "\mu"] & \boxed{\text{theft}} \end{tikzcd}$$

8) $(0,1,2,3,4)$

9) i) transition rates of moving from state to each other state must be non-negative.

The transition rates off the leading diagonal must be non-negative.

The transition rates on the leading diagonal must be ~~be~~ non-positive.

Sum of each row → 0.

Finite square matrix.

$\{2, 1, 0\}$

$$\boxed{2} \xrightarrow{0.4} \boxed{1} \xrightarrow{0.2} \boxed{0}$$

$$\frac{d}{dt} P_{22}(t) = -0.4 P_{22}(t)$$

$$\frac{d}{dt} P_{21}(t) = 0.4 P_{22}(t) - 0.2 P_{21}(t)$$

$$\frac{d}{dt} P_{20}(t) = 0.2 P_{21}(t)$$

10) $\frac{d}{dt} P_{\bar{A}\bar{A}}(t) = -2t \times P_{\bar{A}\bar{A}}(t)$

$\frac{d}{dt}\left[\ln P_{\bar{A}\bar{A}}(t)\right] = -2t$.

$P_{\bar{A}\bar{A}}(s) = -s^2 + C$

$P_{AA}(0) = 1 \qquad C = 0$.

$P_{AA}(s) = \exp^{-s^2}$

P(in first visit to B at time T | in state A at t=0

$= \int_0^T$ P(remains in A to time s) x

P(transitions to B in time (s, s+ds)) x

P(remains in B to time T) ds.

$= \int_0^T P_{\bar{A}\bar{A}}(s) \times 2(s) \times P_{BB}(s,T)\, ds$.

$= \int_0^T e^{-s^2} \times 2s \times e^{-T^2+s^2}\, ds$.

$= e^{-T^2} \times T^2$.

$\frac{d}{dt} e^{-T^2} \times T^2$

$2T \times e^{-t^2} - 2t^3 \times e^{-t^2} = 0$.

$\therefore$ maxima

11)

$$P(T_0 < y \mid N_s = 1) = \frac{P(T_0 < y, N_s = 1)}{P(N_s = 1)}$$

$$= \frac{P(N_y = 1, N_s - N_y = 0)}{P(N_s = 1)}$$

$$\frac{(\lambda y e^{-\lambda y})\, e^{-\lambda(s-y)}}{\lambda s e^{-\lambda s}} = \frac{y}{s}.$$

joint density

$$\lambda^n e^{-\lambda(t_1 + t_2 + \dots + t_n)}\, 1_{\{t_1, t_2, \dots, t_n\}}$$

$$P(N_s = k \mid N_t = n) = \frac{\dfrac{e^{-\lambda s}(\lambda s)^k}{k!} \cdot \dfrac{e^{-\lambda(t-s)}\lambda^{n-k}(t-s)^{n-k}}{(n-k)!}}{\dfrac{e^{-\lambda t}(\lambda t)^n}{n!}}.$$

$$= \frac{e^{-\lambda t}\lambda^n s^k (t-s)^{n-k}}{k!\,(n-k)!} \cdot \frac{n!}{e^{-\lambda t}\lambda^n t^n}.$$

$$= \frac{n!}{k!\,(n-k)!} \cdot \frac{s^k (t-s)^{n-k}}{t^k t^{n-k}}.$$

$$= {}^nC_k \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}.$$

Binomial$(n, s/t)$.