Page :
Date :

NUMERICAL METHODS OF ALGEBRA.

Assignment 1

Q2) (i) $x_1 = 4$ $y_1 = 0.60206$

$x_2 = 6$ $y_2 = 0.7781513$

using linear interpolation,

$$x_2 - x_1 = y_2 - y_1$$

$$(6-4) = 0.7781513 - 0.60206$$

$$2 = 0.1760913$$

$$\text{charge} = \frac{0.1760913}{2}$$

$$= 0.8804565$$

Thus, $x=5$

$$y = 0.60206 + 0.8804565$$

$$y = 0.69010565$$

(ii) $x_1 = 4.5$ $y_1 = 0.6532125$

$x_2 = 5.5$ $y_2 = 0.7403627$.

$$(5.5 - 4.5) = y_2 - y_1$$

$$= 0.7403627 - 0.6532125$$

$$= 0.0871502$$

$$\text{change} = \frac{0.0871502}{1}.$$

Thus $x=5$

$$y = 0.6532125 + 0.0871502$$

$$= 0.6967876$$

Page :
Date :

Q1) Actual length $= r_1 = 12.5$ cm

Actual area $= \pi r_1^2$

$= \pi (12.5)^2$

$= 490.87365 \text{ cm}^2$

$r_2 = 12.65$

Approximate area $= \pi r^2$

$= \pi (12.65)^2$

$= 502.72551 \text{ cm}^2$

Absolute error $=$ App. area $-$ Actual area

$= 11.84886$

Relative error $= \dfrac{\text{Absolute } \cancel{\text{Approx}} \text{ area}}{\text{Actual area}}$

$= \cancel{1.024138309}\ 0.02413$

Percentage error $= \left(\text{Relative error}\right) \times 100$

$= 2.413\%$

Q3) $x^4 - x - 10 = 0$ $\quad a = 1.5 \quad b = 2$

x	y
$a = x_0 = 1.5$	-6.4375
$b = x_1 = 2$	4

1. $x_2 = \dfrac{x_0 + x_1}{2} = 1.75$

$y = -2.37109$

Page

Date

2] $x_3 = \dfrac{x_1 + x_2}{2}$

$= 1.875$

$y_3 = 0.48462$

3] $x_4 = \dfrac{x_2 + x_3}{2}$

$= 1.8125$

$y_4 = -1.0202$

$x_5 = \dfrac{x_4 + x_3}{2} = 1.84375$

$y_5 = -0.28773$

$x_6 = \dfrac{x_5 + x_3}{2} = 1.859375$

$y_6 = 0.0933$

Q4) $x^3 - x - 1 = 0.$

$f'(x) = 3x^2 - 1.$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2.$

$x_1 = x_0 - f(x_0)$

$= 1 - (-1/2)$

$= 1.5$

$y_1 = 0.875$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 1.5 - \left(\dfrac{0.875}{5.75}\right)$

$x_2 = 1.34783$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44998$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.3252$

$y_3 = 0.00205$

$f'(x_3) = 4.26846$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.32472$

$y_4 = 0.000008712$

Q5) $A^2 = A$

$$7A - (I+A)^3 = 7A - [I^3 + A^3 + 3A^2I + 3AI^2]$$
$$= 7A - (I + A^2(A) + 3A + 3A)$$
$$= 7A - (I + A^2 + 6A)$$
$$= \cancel{7A} - (I + \cancel{7A})$$
$$= -I$$

Q6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$|A| = 0 - 1(6-8) + (-1)4$$
$$= -2.$$

$$Adj(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{Adj(A)}{|A|}$$
$$= -\frac{1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} +3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ 2 & 1/2 & -2 \end{bmatrix}$$

Q7) $\vec{a} = 8\hat{i} - 9\hat{j}$
$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a}.\vec{b} = ab\cos\theta$
$= (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$
$= 56 + 81$
$= 137$
$a = \sqrt{64+72} = \sqrt{145}$
$b = \sqrt{49+81} = \sqrt{130}$

$\vec{a}.\vec{b} = ab\cos\theta$
$\cos\theta = \frac{\vec{a}.\vec{b}}{ab}$
$= \frac{137}{\sqrt{130} \times \sqrt{145}}$
$\cos\theta = 0.9720224206$

Q8) $\vec{R} = \vec{a} + \vec{b}$
$R^2 = A^2 + B^2 + 2AB\cos\theta$
$\theta = 30°$
$R2 = (75)^2 + (25)^2 + 2(75)(25)\cos 30°$
$R^2 = \cancel{97}\ 9497.595284$
$R = 97.4556\ m$

Q9) $d = 3$
AP:- 101, 104, 107 998
$a = 101$
$a_n = a + (n-1)d$
$998 = 101 + (n-1)3$
$n = 300$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{300}{2}(998+101)$$

$$= 150 \times 1099$$

$$S_n = 164850$$

Q10) $a_6 = 32$

$$ar^5 = 32$$

$$ar^7 = 128$$

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$r = \pm 2$$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$$f'(x) = 3x^2 - 14x + 8$$

$$x_0 = 5$$

$$f(5) = -13$$

$$f'(5) = 13$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \left(\frac{13}{-13}\right)$$

$$= 6$$

$$f(x_1) = 9$$

$$f'(x_1) = 32$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - 9/32$$

$$x_2 = 5.71875$$

$$f(x_2) = 0.84787$$

Page :
Date :

Q14) $(x^2-x+1)^4 = ((x-1)^2+x)^4$

using Pascal's triangle,

$$[(x-1)^2+x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$

$$(x^2-x+1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$$

Q11) $(3x+4)^5 = (3x)^5 + 5(3x)^4 4 + 10(3x^3)\times 4^2 + (10(3x^2)4^3) + 5(3x)(4^4) + 4^5$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

Q12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A-\lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$|A-\lambda I| = -\lambda^3 + 13\lambda - 12$

$-\lambda^3 + 13\lambda - 12 = 0$

$\lambda^3 - 13\lambda + 12 = 0.$

comparing coefficents,

$\Rightarrow 1 - 13 + 12$

$= 0.$

$\therefore$ Root $= 1$

Camlin

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda = 3$ or -4

Eigen values = $-4, 1$ & 3.

Q15) $e^{-x} = 3\log(x)$

$3\log(x) + e^{-x} = 0$

x	y
0.1	-2.0951625
0.2	-1.278179
0.3	-0.8278180
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	-0.03188

$x_0 = 0.6 \quad y_0 = -0.11673$

$x_1 = 0.7 \quad y_1 = 0.03188$

$$x_2 = \frac{x_0 + x_1}{2} = 0.65$$

$y_2 = -0.03921$

$$x_3 = \frac{x_2 + x_1}{2}$$

$= 0.675$

$y_3 = -0.00293$

$$x_4 = \frac{x_3 + x_1}{2}$$

$$= 0.6875$$

$$y_4 = 0.01465$$

$$x_5 = \frac{x_4 + x_3}{2} = 0.68125$$

$$y_5 = 0.0059.$$