

Calculus

Q.1) $\int_{1/50}^2 \frac{e^{2/w}}{w^2} dw$

let $t = e^{2/w}$

diff. w.r.t w

$$\frac{dt}{dw} = -2e^{2/w} \frac{1}{w^2}$$

$$= \int_{1/50}^2 \frac{t w^2 dt}{-2w^2 e^{2/w}}$$

$$= \frac{-1}{2} \int_{1/50}^2 dt = \frac{-1}{2} (t^2) \Big|_{1/50}^2$$

$$= \frac{-1}{2} (2 - 0.02)$$

$$= -0.99$$

Q.2 $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} dt$

let $x = (t^4 + 2t)^3$

diff. w.r.t t

$$\frac{dx}{dt} = 3(t^4 + 2t)^2 (4t^3 + 2)$$

$$\int \frac{2t^3 + 1}{x(3)(t^4 + 2t)^2 (4t^3 + 2)}$$

Multiply and divide by 2

$$\int \frac{4t^3 + 2}{2x(3)(4t^3 + 2)(x)^{2/3}}$$

$$= \frac{1}{6} \int x^{-2/3} dx$$

$$= \frac{1}{6} \frac{x^{-2/3} + 1}{-5/3 + 1} + \frac{3}{6(-1)} x^{-2/3} + C$$

$$= \frac{-1}{4} (x^4 + 2x) + C \quad 2 - \frac{1}{4} x^{2/3} + C$$

Q.3 $f'(x) = 2x^4 + 3x - 9$ $f(x) = \int f'(x) dx$

$$f(x) = \int (2x^4 + 3x - 9) dx$$

$$= \frac{2x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Q.4 $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^3 f(x) dx = \int_{-2}^1 3x^2 dx + \int_1^3 6 dx$$

$$\int_{-2}^1 3x^2 dx + \int_1^3 6 dx$$

$$[x^3]_{-2}^1 + [6x]_1^3$$

$$1 + 8 + 18 - 6 = 21$$

Q.5) $\int 3(8y-1)e^{4y^2-4y} dy$

$u = e^{4y^2-4y}$

$\int \frac{3(8y(-1))e^{4y^2-4y}}{e^{4y^2-4y}(8y-1)} dy$

diff w.r.t.
 $\frac{du}{dy} = (e^{4y^2-4y})(8y-1)$

$3u + C$

$3(e^{4y^2-4y}) + C$

Q.6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6 dx$

$= \frac{15x^{5/2}}{3/2} + \frac{5x^4}{4} + 6x + k$

$f(x) = \int \frac{15x^{5/2}}{3/2} + \frac{5x^4}{4} + 6x + k dx$

$f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + C$

$f(1) = \frac{-5}{4} = 4 + 1 + 3 + k + C$

$k + C = -8.5$

$f(4) = 404 = 4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + 4k$

$4k + C = 164$

$$\begin{array}{r} 4k + c = 164 \\ -12 + c = 88 \\ \hline \end{array}$$

$$k = 57.5$$

$$c = 66$$

$$\therefore f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$$

Q.9. $\int \frac{dw}{9.8 - 0.196w} = \int \frac{2}{a} dt$

$$\log(9.8 - 0.196w) = -0.196t + c$$

Q.12) Taylor series for $f(x) = x^3 - 10x^2 + 6$ at x

$$f(x) = x^3 - 10x^2 + 6 \quad f(3) = -57$$

$$f'(x) = 3x^2 - 20x \quad f'(3) = -33$$

$$f''(x) = 6x - 20 \quad f''(3) = -2$$

$$f'''(x) = 6 \quad f'''(3) = 6$$

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2} f''(3) + \frac{(x-3)^3}{6} f'''(3) \dots$$

$$\frac{(x-3)^3}{6} f'''(3) \dots$$

$$x^3 - 10x^2 + 6x - 5 + (x-3) - 3 + \frac{(x-3)^2}{2} (-2) + \frac{4(x-3)^3}{6}$$

Q.13) MacLaurian series for $f(x) = (1+x)^4$

$$f'(x) = 4(1+x)^{4-1}$$

$$f''(x) = (4)(4-1)(1+x)$$

$$f'''(x) = -(4)(4-1)(4-2)(1+x)$$

$$f(x) = f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$$(1+x)^4 = 1 + 4x + (4)(3)x^2 + (4)(3)(2)x^3 + \dots$$

Q.14) $f(x) = \sqrt{1+x}$

$$f'(x) = \frac{1}{2} (1+x)^{-1/2} \quad f'(0) = 1/2$$

$$f''(x) = -1/4 (1+x)^{-3/2} \quad f''(0) = -1/4$$

$$f'''(x) = 3/8 (1+x)^{-5/2} \quad f'''(0) = 3/8$$

$$\sqrt{1+x} = \sqrt{1+x} + \frac{x^1}{1!} \left(\frac{1}{2}\right) + \frac{x^2}{2!} \left(-\frac{1}{4}\right) + \frac{x^3}{3!} \left(\frac{3}{8}\right)$$

Taylor series.

Q.15)

$$f(x) = e^{-6x}$$

$$x = -4$$

$$f(x) = e^{-6x}$$

$$f(-4) = e^{+24}$$

$$f'(x) = e^{24} (6)$$

$$f'(x) = e^{-6x} (-6)$$

$$f''(-4) = e^{24} (6)^2$$

$$f''(x) = (-6) e^{-6x} (-6)$$

$$f'''(-4) = 6^{24} (6)^3$$

$$f'''(x) = (-6)^3 e^{-6x}$$

$$e^{-6x} = e^{+24} + \frac{(x+4)}{1!} e^{24} (6) + \frac{(x+4)^2}{2!} (6)^2 + \frac{(x+4)^3}{3!} (6)^3 + \dots$$

Q.6)

$$f(x) = \ln(3+4x)$$

$$x = 0 = (3)$$

$$f'(x) = \frac{4}{3+4x}$$

$$f'(0) = 4/3$$

$$f''(x) = -4(4x+3)^{-2} (4)$$

$$f''(0) = -1(4)^2 (3)^{-3}$$

$$f'''(x) = (+8)(4x+3)^{-3} (4)$$

$$f'''(0) = (-1)(-2)(3)^{-4}$$

$$\ln(3+4x) = \ln(3) + \frac{x}{3} + \frac{x^2}{2} (-1)(4)^2 (3)^{-3} + \frac{x^3}{3!} (4)^3 (2)(3)^{-4} + \dots$$