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Subject : Numerical Methods and Algebra
Chapter : Error, Dimensions & Interpolation

Homework 1 : Lagrange's Interpolation

1. given points : (7,3), (8,1), (9,1) & (10,9)
To find $f(x_k = 9.5)$

$x_0 = 7$ $f(x_0) = 3$
$x_1 = 8$ $f(x_1) = 1$
$x_2 = 9$ $f(x_2) = 1$
$x_3 = 10$ $f(x_3) = 9$

$$f(x_k = 9.5) = \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \times f(x_0) +$$

$$\frac{(x_k - x_0)(x_k - x_2)(x_k - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \times f(x_1) +$$

$$\frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \times f(x_2) +$$

$$\frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \times f(x_3)$$

$$\therefore f(x_K = 9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3 +$$

$$= \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1 +$$

$$\frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1 +$$

$$\frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times a^9$$

$$= 0.1875 + (-0.3125) + 0.9375$$

$$+ 2.8125$$

$$= 3.625$$

∴ The value is 3.625 at $x = 9.5$

2. $f(x_0) = 168$ $\quad f'(x_0) = 3$

$f(x_1) = 120$ $\quad f'(x_1) = 7$

$f(x_2) = 72$ $\quad f'(x_2) = 9$

$f(x_3) = 63$ $\quad f'(x_3) = 10$

$x_k = 6 \qquad f(x_k = 6) = ?$

$$f(x_k=6) = \frac{(x_k-x_1)(x_k-x_2)(x_k-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times f(x_0) +$$

$$\frac{(x_k-x_0)(x_k-x_2)(x_k-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times f(x_1) +$$

$$\frac{(x_k-x_0)(x_k-x_1)(x_k-x_2)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times f(x_2) +$$

$$\frac{(x_k-x_0)(x_k-x_1)(x_k-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3)$$

$$= \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 +$$

$$\frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 +$$

$$\frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 +$$

$$\frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$= 0.07143 + 180 - 72 + 27 = 147$$

Ans: $f(x_k=6) = 147$

3. $x_0 = 1$ $\quad f(x_0) = 4$
$x_1 = 2$ $\quad f(x_1) = 5$
$x_2 = 7$ $\quad f(x_2) = 5$
$x_3 = 8$ $\quad f(x_3) = 4$

$x_k = 6$ $\quad f(x_k = 6) = ?$

$$f(x_k=6) = \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \times f(x_0) +$$

$$\frac{(x_k - x_0)(x_k - x_2)(x_k - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \times f(x_1) +$$

$$\frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \times f(x_2) +$$

$$\frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \times f(x_3)$$

$$\therefore f(x_k=6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4 +$$

$$\frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5 +$$

$$\frac{(6-1)(6-2)(6-7)}{(7-1)(7-2)(7-8)} \times 5 +$$

$$\frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$= -0.1619 + 1.6666 + 3.3333 - 1.9047$$

$$= 2.3333$$

∴ The value is 2.3333 at $x = 6$

4. $x_0 = 1$ $\quad f(x_0) = f(1) = 2$

$x_1 = 2$ $\quad f(x_1) = f(2) = 4$

$x_2 = 3$ $\quad f(x_2) = f(3) = 8$

$x_3 = 4$ $\quad f(x_3) = f(4) = 16$

$x_4 = 7$ $\quad f(x_4) = f(7) = 128$

$x_k = 5$ $\qquad f(x_k = 5) = ?$

$$f(x_k=5) = \frac{(x_k-x_1)(x_k-x_2)(x_k-x_3)(x_k-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times f(x_0) +$$

$$\frac{(x_k-x_2)(x_k-x_3)(x_k-x_4)(x_k-x_0)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_0)} \times f(x_1) +$$

$$\frac{(x_k-x_3)(x_k-x_4)(x_k-x_0)(x_k-x_1)}{(x_2-x_3)(x_2-x_4)(x_2-x_0)(x_2-x_1)} \times f(x_2) +$$

$$\frac{(x_k-x_4)(x_k-x_0)(x_k-x_1)(x_k-x_2)}{(x_3-x_4)(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times f(x_3) +$$

$$\frac{(x_k-x_0)(x_k-x_1)(x_k-x_2)(x_k-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times f(x_4)$$

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$$= \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 +$$

$$\frac{(5-3)(5-4)(5-7)(5-1)}{(2-3)(2-4)(2-7)(2-1)} \times 4 +$$

$$\frac{(5-4)(5-7)(5-1)(5-2)}{(3-4)(3-7)(3-1)(3-2)} \times 8 +$$

$$\frac{(5-7)(5-1)(5-2)(5-3)}{(4-7)(4-1)(4-2)(4-3)} \times 16 +$$

$$\frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$= -0.6667 + 6.4 + (-24) + 42.6667 + 8.5333$$

$$= 32.933$$

Given:

5. $x_0 = 10$ $\quad f(x_0) = 0.25$

$x_1 = 20$ $\quad f(x_1) = 0.2$

$x_2 = 14$ $\quad f(x_2) = 0.15$

$x_3 = 16$ $\quad f(x_3) = 0.10$

$x_k = 15$ $\quad f(x_k = 15) = ?$

$$f(x_k=15) = \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} \times f(x_0) +$$

$$\frac{(x_k - x_0)(x_k - x_2)(x_k - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \times f(x_1) +$$

$$\frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \times f(x_2) +$$

$$\frac{(x_k - x_0)(x_k - x_1)(x_k - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \times f(x_3)$$

$$f(x_k=15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25 +$$

$$\frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2 +$$

$$\frac{(15-10)(15-20)(15-14)}{(14-10)(14-20)(14-16)} \times 0.15 +$$

$$\frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$= -0.0052 - 0.0041 - 0.0781 + 0.0520$$

$$= -0.0354$$

Homework 2 - Regula Falsi Method.

1. $x^3 + x - 1 = 0$ lying between 0.5 and 1

x	$y = x^3 + x - 1$
0.5	-0.375
0.6	-0.184
0.7	0.043
0.8	0.312
0.9	0.629
1	1

By Regula Falsi Method, $\bar{x} = \dfrac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$

x	y
$x_1 = 0.6$	$y_1 = -0.184$
$x_2 = 0.7$	$y_2 = 0.043$

$$\bar{x} = \frac{(0.6)(0.043) - (-0.184)(0.7)}{0.043 + 0.184}$$

$$= 0.6811$$

$$\bar{y} = -0.00294$$

∴ Similarly,

x	y
$x_1 = 0.6811$	$y_1 = -0.00294$
$x_2 = 0.7$	$y_2 = 0.043$

$$\bar{x} = \frac{(0.6811)(0.043) - (-0.00294)(0.7)}{0.043 + 0.00294}$$

$$= 0.6823$$

$$\bar{y} = -0.000066$$

∴ Approximate value of root = 0.6823

2. $X^3 - 2x + 0.5 = 0$ lying between 0 & 1

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073
0.4	-0.236
0.5	-0.375
1	-0.5

By Regula Falsi Method, $\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$

x	y
$x_1 = 0.2$	$y_1 = 0.108$
$x_2 = 0.3$	$y_2 = -0.073$

$$\bar{x} = \frac{(0.2 \times -0.073) - (0.108 \times 0.3)}{-0.073 - 0.108}$$

$$= 0.25967$$

$\bar{y} = -0.00183$

Similarly,

x	y
$x_1 = 0.25967$	$y_1 = -0.00183$
$x_2 = 0.2$	$y_2 = 0.108$

$$\bar{x} = \frac{(0.25967 \times 0.108) - (-0.00183 \times 0.2)}{(0.108 + 0.00183)}$$

$$= 0.25868$$

$$\bar{y} = -0.00005$$

$\therefore$ Approximate value of ~~rate~~ root = 0.25868

3. $X^3+2X-20=0$ lying between 2 and 2.5

x	y
2	-8
2.1	-6.539
2.2	-4.952
2.3	-3.233
2.4	-1.376
2.5	0.625

By Regula Falsi Method, $\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$

x	y
$x_1 =$ ~~-1.376~~ 2.4	$y_1 = -1.376$
$x_2 =$ ~~0.625~~ 2.5	$y_2 = 0.625$

$$\bar{x} = \frac{2.4 \times 0.625 - (-1.376 \times 2.5)}{0.625 + 1.376}$$

$$= 2.4688$$

$$\bar{y} = -0.01513$$

Similarly,

x	y
$x_1 = 2.4688$	$y_1 = -0.01513$
$x_2 = 2.5$	$y_2 = 0.625$

$$\bar{x} = \frac{(2.4688 \times 0.625) - (-0.01513 \times 2.5)}{0.625 + 0.01513}$$

$$\bar{y} = f(2.46954) = -0.00011$$

∴ Approximate value of root = 2.46954

4. $3x - \cos x - 1 = 0$ lying between 0 & 1

x	$y = 3x^3 + x\cos x - 1$
0	-2
0.2	-1.399
0.4	-0.799
0.5	-0.499
0.6	-0.199
0.7	0.1
0.9	0.7001
1	1

By Regula Falsi Method, $\bar{x} = \dfrac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$

x	y
$x_1 = 0.6$	$y_1 = -0.199$
$x_2 = 0.7$	$y_2 = 0.1$

$$\bar{x} = \frac{0.6 \times 0.1 - (-0.199) \times (0.7)}{0.1 + 0.199}$$

$$= 0.666555$$

$$\bar{y} = -0.000267$$

∴ Approximate value of root : 0.666555

5. $x^3 - 3x - 5$ lying between 2 and 2.5

x	y
2	−3
2.1	−2.039
2.2	−0.952
2.3	0.267
2.4	1.624
2.5	3.125

By Regula Falsi Method, $\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$

x	y
$x_1 = 2.2$	$y_1 = -0.952$
$x_2 = 2.3$	$y_2 = 0.267$

$$\bar{x} = \frac{2.2 \times 0.267 - (-0.952 \times 2.3)}{0.267 + 0.952}$$

$$= 2.2781$$

$$\bar{y} = -0.01155$$

Similarly, $x_1 = 2.2781 \quad f(x_1) = -0.01155$

$x_2 = 2.3 \quad f(x_2) = 0.267$

$$\bar{x} = \frac{2.2781 \times 0.267 + 0.01155 \times 2.3}{0.267 + 0.01155}$$

$$= 2.279$$

$$\bar{y} = -0.000236$$

Approx. root = 2.279

6. $x \log_{10} x - 1.2 = 0$ lying between 2 & 3

x	y
2	−0.597
2.5	−0.205
2.7	−0.035
2.8	0.052
3	0.2313

By regula Falsi Method, $\bar{x} = \dfrac{x_1 y_2 - y_1 x_2}{y_2 - y_1}$

x	y
2.7	−0.035
2.8	0.052

$$\bar{x} = \frac{2.7 \times 0.052 - (-0.035 \times 2.8)}{0.052 + 0.035}$$

$$= 2.74023$$

$$\bar{y} = -0.000363$$

∴ Approx root of eqn = 2.74023

7. $x^3 - 4x + 1 = 0$

x	y
0	1
1	-2
2	1
3	16

By Regula Falsi Method,

x	y
$x_1 = 0$	$y_1 = 1$
$x_2 = 1$	$y_2 = -2$

$$\bar{x} = \frac{0 \times (-2) - (1)(1)}{-2 - 1}$$

$$= \frac{-0 - 1}{-2 - 1} = \frac{1}{3} = 0.333$$

$$\bar{y} = -0.2962$$

Similarly,

$x_1 = 0$ $\quad y_1 = 1$

$x_2 = 0.333$ $\quad y_1 = -0.2962$

$$\bar{x} = \frac{0 \times (-0.2962) - (1)(0.333)}{-0.2962 - 1}$$

$$= 0.257136$$

$$\bar{y} = -0.011542$$

Similarly,

$x_1 = 0.257136$ $\quad y_1 = -0.011542$

$x_2 = 0$ $\quad y_2 = 1$

$$\bar{x} = \frac{0.257136 \times 1 + 0.011542 \times 0}{1 + 0.011542}$$

$$= 0.25420$$

$$\bar{y} = -0.0003741$$

∴ One root is approximately 0.25420

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8. $x^3 + 2x^2 - 8 = 0$

x	y
0	-8
1	-9
2	-8
3	1
4	24

x	y
2.5	-4.875
2.9	-0.431
3	1

By Regula Falsi Method,

$x_1 = 2.9$ $\quad y_1 = -0.431$

$x_2 = 3$ $\quad y_2 = 1$

$$\bar{x} = \frac{2 \times 1 + 8 \times 3}{1 + 8} = \frac{26}{9} = 2.8889$$

$\bar{y} =$

$$\bar{x} = \frac{2.9 \times 1 + 0.431 \times 3}{1 + 0.431} = 2.932$$

$\bar{y} = 0.0003$

Ans: Approximate value = 2.932

9. $x^3 - 9x + 1 = 0$

x	y
2.5	-5.875
2.9	-0.711
3	1
3.5	

By using Regula Falsi Method

$x_1 = 2.9$ $\quad y_1 = -0.711$

$x_2 = 3$ $\quad y_2 = 1$

$$\bar{x} = \frac{2.9 \times 1 + 3 \times 0.711}{1 + 0.711}$$

$$= 2.94155$$

$$\bar{y} = -0.0215$$

$\therefore$ Approximately root = 2.94155

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10. $x^3 - x - 4 = 0$

x	y
1	-4
1.5	-2.125
1.7	-0.787
1.8	0.032
2	2

By Regular Falsi Method,

$x_1 = 1.7$ $\quad y_1 = -0.787$

$x_2 = 1.8$ $\quad y_2 = 0.032$

$$\bar{x} = \frac{1.7 \times 0.032 + 1.8 \times 0.787}{0.032 + 0.787}$$

$$= 1.79609$$

$$\bar{y} = -0.0020$$

Approx root is ~~1.76900~~ 1.79609.