Financial Mathematics : Assignment 1

1. $d^{4} = 8\%$ p.a.

we know that,

$$(1-d) = \left(1 - \frac{d^{p}}{p}\right)^{p}$$

$$\Rightarrow \quad d = 1 - \left[\left(1 - \frac{d^{p}}{p}\right)^{p}\right]$$

$$d = 1 - \left(1 - \frac{0.08}{4}\right)^{4}$$

$$d = 0.07763184$$

(i) for equivalent force of interest :

$$\delta = -\ln(1-d)$$

$$= -\ln(1 - 0.07763184)$$

$$\delta = 0.0808108297$$

$$\boxed{\delta = 8.0812\ \%}$$

(ii) for equivalent eff. rate of interest p.a. :

$$i = \frac{d}{1-d} = \frac{0.07763184}{0.92236816}$$

$$i = 0.0841657847 3$$

$$\boxed{i = 8.42\ \%}$$

(iii) we need to find d^{12}

$$1 - d = \left(1 - \frac{d^{12}}{12}\right)^{12}$$

$$d^{12} = 12\left[1 - (1-d)^{\frac{1}{12}}\right]$$

$$= 12\left[1 - (1 - 0.07763184)^{1/12}\right]$$

$$= 12\left[1 - (0.92236816)^{1/12}\right]$$

$$d^{12} = 0.0805393394 4$$

$$\boxed{d^{12} = 8.054\%}$$

2. $\delta(t) = 0.004t + 0.0002t^2$

(i)

$$\overset{1000}{\underset{0 \qquad\qquad\qquad 10}{|\text{————————}|}}$$

$$PV_0 = 1000\, e^{-\int_0^{10} 0.004t + 0.0002t^2\, dt}$$

$$= 1000\, e^{-\left[0.004\frac{t^2}{2} + 0.0002\frac{t^3}{3}\right]_0^{10}}$$

$$= 1000\, e^{-[0.2 + 0.0666667 - 0]}$$

$$= 1000\, e^{[-0.2 - 0.0666667]}$$

$$= 1000\,(0.7659283358 1)$$

$$= \boxed{765.92}$$

(ii) for annual effective rate of interest

$$\frac{1}{(1+i)^{10}} = e^{-\int_0^{10} 0.04t + 0.0002t^2\, dt}$$

$$= (1+i)^{-10} = 0.7659283358 1$$

$1 + i = (0.76592833581)^{-1/10}$

$\boxed{i = 2.70\%}$

3. (i) $d = iv$

Reason - Interest earned on an investment of 'x' amount at the beginning of the term/period is d. Unlikely, interest earned on an investment of 'x' amount at the end of the period is i. Therefore, if we discount i from the end of the period to the beginning with discounting factor v, we obtain d. Hence, $\underline{d = iv}$.

(ii) (a) $\frac{d^4}{4} = 2.25\%$

$1 - d = \left(1 - \frac{d^p}{p}\right)^p$

$1 - d = (1 - 0.0225)^4 = 0.912992193378$

$d = 0.087007806622$

now,

$d^2 = 2[1 - (0.912992193378)^{1/2}]$

$\boxed{d^2 = 8.8988\%}$

(b) $d^{(2)} = 5\%$ p.a. convertibly in 2 yrs.

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - 2d^{(1/2)}\right)^{1/2}$$

$$1 - \frac{d^{(2)}}{2} = \left(1 - 2d^{(1/2)}\right)^{\frac{1}{4}}$$

$$d^{(2)} = 2\left[1 - \left(1 - 2d^{(\frac{1}{2})}\right)^{1/4}\right]$$

$$= 2\left[1 - \left(1 - 2(0.05)\right)^{1/4}\right]$$

$$\boxed{d^{(2)} = 5.199\ \%}$$

(iii) Let d be simple annual discount rate.

ATQ:

$$\left(1 - \frac{d\,91}{365}\right) = (1+i)^{-91/365}$$

$$1 - \frac{91d}{365} = (1.05)^{-91/365} = 0.98790956075$$

$$d = \frac{365}{91}\left(1 - 0.98790956075\right)$$

$$\boxed{d = 4.8493\%}$$

4. (i) [Already done → 3(i)]

(ii) $i = 0.08$ p.a.

(a) $\left(1 - \frac{d^{12}}{12}\right)^{12} = \frac{1}{1+i} = (1+i)^{-1}$

$= 1 - \frac{d^{12}}{12} = (1+i)^{-1/12}$

$d^{12} = 12\left[1 - (1.08)^{-1/12}\right]$

$d^{12} = 0.076714776 1405$

$\boxed{d^{12} = 7.6715\%}$

(b) $1+i = \left(1 + \frac{i^{365}}{365}\right)^{365}$

$(1.08)^{\frac{1}{365}} = 1 + \frac{i^{365}}{365}$

$i^{365} = 365\left[(1.08)^{1/365} - 1\right]$

$= 0.0769691554076$

$\boxed{i^{365} = 7.697\%}$

(c) $\delta = \ln(1+i) = \ln(1.08)$

$\delta = 0.076961$

$\boxed{\delta = 7.6961\%}$

(d) $1+i = \left(1 + \frac{i^{0.5}}{0.5}\right)^{0.5}$

$i^{0.5} = 0.5\left[(1.08)^2 - 1\right] = 0.0832$

$\boxed{i^{0.5} = 8.32\%}$

Camlin

5 (i) $d^{(4)} = 6\%$ p.a convertible quarterly

(a) $$\frac{1}{\left(1+\frac{i^{(2)}}{2}\right)^{2}} = \left(1-\frac{0.06}{4}\right)^{4}$$

$$\left(1+\frac{i^{(2)}}{2}\right)^{-2} = \left(1-\frac{0.06}{4}\right)^{4} = 0.94133655062$$

$$1+\frac{i^{(2)}}{2} = (0.94133655062)^{-1/2}$$

$$i^{(2)} = 0.0613771552$$

$$\boxed{i^{(2)} = 6.1377\%}$$

(b) $$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^{4} = 0.94133655062$$

$$d^{(12)} = 12\left[1-(0.94133655062)^{1/12}\right]$$

$$d^{(12)} = 0.0603025 2528$$

$$\boxed{d^{(12)} = 6.0303\%}$$

15/4/2005 — 17/7/2005 — 15/4/2006

93 days

(ii) (a) Original term of loan = 1 year
= 365 days.

Amit ~~Repayed~~ repaid the loan early
on 17th July 2005

Actual Term of repayment = 365 − 93
= 272 days early.

Camlin

Amount reduced will be $= \frac{272}{365} \times$ amt. of interest

$$\frac{272}{365} \times 20,000$$

$$= 14904.1096$$

Sum paid will be $= 2,20,000 - 14904.1096$

$$\boxed{= ₹205095.89}$$

6. let retail price ~~by~~ be 100

$\therefore$ cash payment = 70

let d be eff annual disc. rate

$$70 = 100(1-d)$$

$$d = 0.3 = 30\%$$

(ii) $$(1-d) = \left(1-\frac{d^{12}}{12}\right)^{12}$$

$$\boxed{d^{12} = 35.142\%}$$

$$(1-d)^{-1} = \left[1+\frac{i^{2}}{2}\right]^{2}$$

$$(0.7)^{-1.5} = 1+\frac{i^{2}}{2}$$

$$\boxed{i^{2} = 39.04\%}$$

$$\delta = -\ln(1-d) = \boxed{35.667\%}$$

7. $26,\cancel{000} = 20,\cancel{000}\,(1+i)^{17/12}$

$(1.3)^{12/17} = 1+i$

$i = 20.3457\%$

i) $26,\cancel{000} = 20,\cancel{000}\left(1+\frac{i^4}{4}\right)^{4\times 17/12}$

$\left(1+\frac{i^4}{4}\right) = \left(\frac{26}{20}\right)^{12/68} = 1.0473881364$

$i^4 = 4\,(1.0473881364-1)$

$\boxed{i^4 = 18.955\%}$

(ii) $2\cancel{0},\cancel{000} = 26,\cancel{000}\left(1-\frac{d^2}{2}\right)^{2\times 17/12}$

$\left(\frac{20}{26}\right)^{12/34} = 1-\frac{d^2}{2}$

$1 - 0.9115582345 = \frac{d^2}{2}$

$\boxed{d^2 = 17.69\%}$

(iii) $20,000\left(1+\frac{17\,i}{12}\right) = 26,000$

$1+\frac{17}{12}i = 1.3$

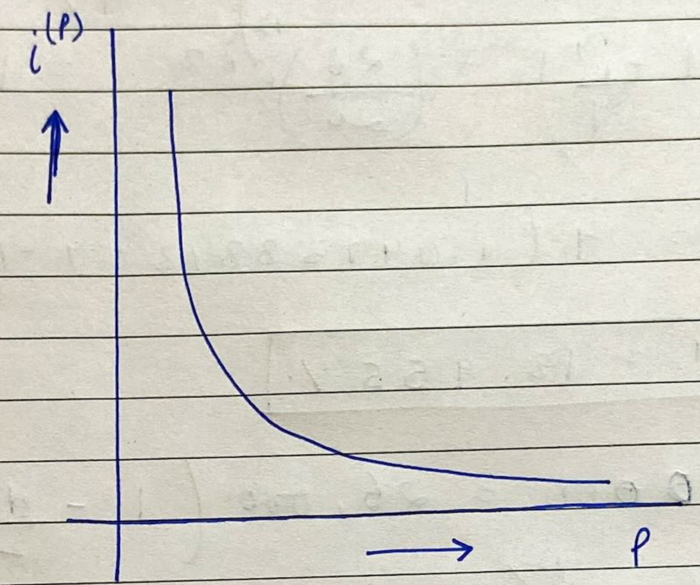
$i = \frac{12}{17}(0.3)$

$\boxed{i = 21.18\%}$

(iv) • $i > i^4$ [interest gets lower with higher frequency of compounding]

• For overall amount, compound interest needs to be lower than simple interest.

8.



(graph attached for 8.)

9. (i) Force of Interest - Measure of interest at individual moments of time.

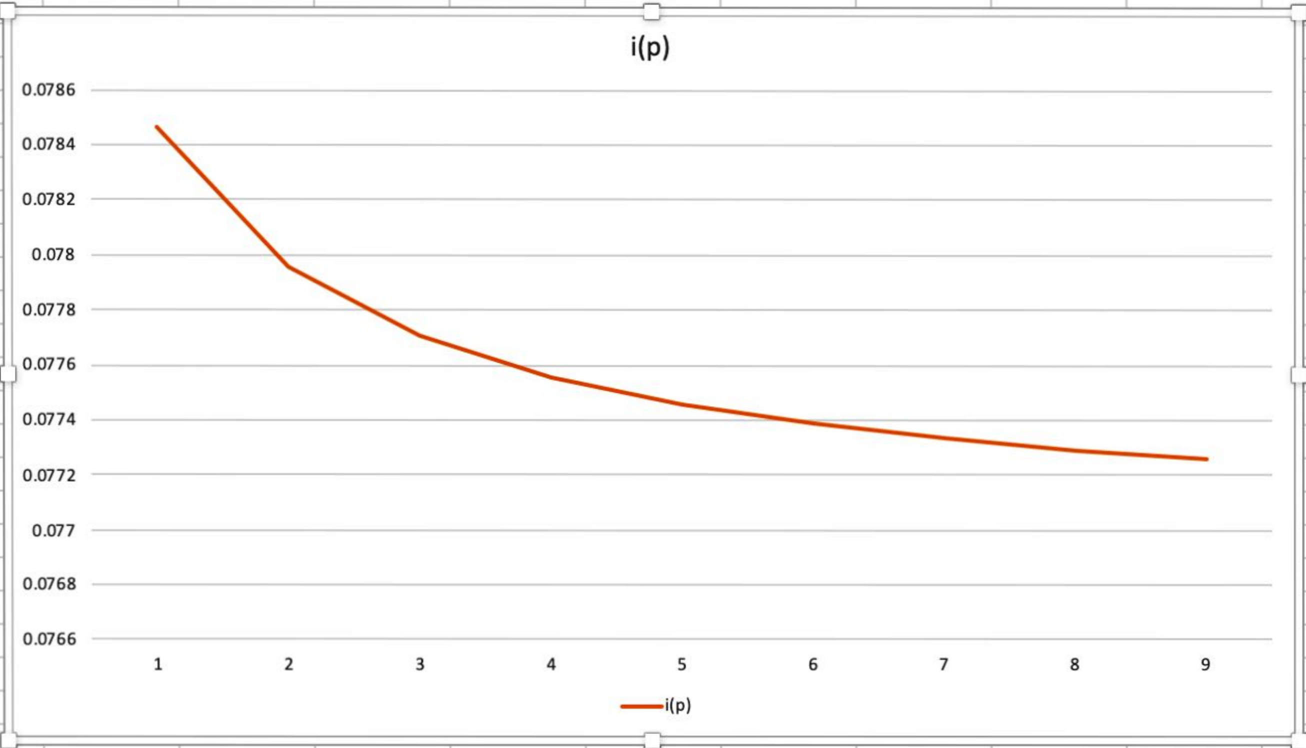
(ii) (a) $i^2 = 0.0775$

$$1 + i = \left(1 + \frac{i^2}{2}\right)^2$$

$$i = \left(1 + \frac{i^2}{2}\right)^2 - 1$$

$$\cancel{i = (1.0775)^2 - 1}$$

Camlin


i(p)
0.0786
0.0784
0.0782
0.078
0.0778
0.0776
0.0774
0.0772
0.077
0.0768
0.0766
1
2
3
4
5
6
7
8
9
i(p)

$$i = \left(1 + \frac{0.0775}{2}\right)^{2} - 1$$

$$\boxed{i = 7.9001\%}$$

$$\cancel{\delta = \ln(1+i) = \ln(1.0775)}$$

$$\cancel{\delta = 7.4644\%}$$

(b) $\delta = \ln(1+i)$

$$\boxed{\delta = 7.603\%}$$

(c) $\left(1 + \frac{i^{2}}{2}\right)^{-2} = \left(1 - \frac{d^{12}}{12}\right)^{12}$

$$\left(1 + \frac{i^{2}}{2}\right)^{-2/12} = 1 - \frac{d^{12}}{12}$$

$$d^{12} = 12\left[1 - \left(1 + \frac{0.0775}{2}\right)^{-2/12}\right]$$

$$\boxed{d^{12} = 7.58\%}$$

(d) $\left(1 + 2i^{1/2}\right)^{1/2} = \left(1 + \frac{i^{2}}{2}\right)^{2}$

$$i^{1/2} = \frac{1}{2}\left[\left(1 + \frac{i^{2}}{2}\right)^{4} - 1\right]$$

$$\boxed{i^{1/2} = 8.2122\%}$$

Camlin

10. $\delta(t) = \begin{cases} 0.08 & 0 \le t \le 5 \\ 0.06 & 5 \le t \le 10 \\ 0.04 & t \ge 10 \end{cases}$

For $v(t)$

Case 1

$0 \le t \le 5$

$\cancel{v(0,5)} = e^{-\int_0^5 0.08\,dt}$

$= \cancel{e^{-[0.08t]_0^5}} = e^{-0.4}$

$PV(0,t) = 1 \times e^{-\int_0^t 0.08\,dt} = e^{-0.08t}$

Case 2

when $5 \le t \le 10$

$PV(5,10) = 1 \times e^{-\int_5^{10} 0.06\,dt} \cdot (e^{-0.08 \times 5})$

$= e^{-0.7}$ $(t=10)$

$\cdot\ e^{-(0.06t + 0.1)}$ $(0 \le t \le 10)$

$PV(t \ge 10) = e^{-0.7} \cdot e^{\int_{10}^{t} 0.04\,dt}$

$= e^{-(0.04t + 0.3)}$ $t \ge 10$

11. (a) $PV_0 = 50{,}000\left(1 - 0.18\right)^{\frac{9}{12}}$

$$\boxed{PV_0 = 43085.43}$$

amt. initially lent to the borrower

(b) i. $\delta = 6\%$

$$d = 1 - e^{-\delta} = 1 - e^{-0.06}$$

$$d = 5.823546641587\%$$

$$d^{12} = 12\left[1 - (1-d)^{1/12}\right]$$

$$\boxed{d^{12} = 5.9851\%}$$

ii. $d^{4} = 6\%$

$$\left(1 - \frac{d^{4}}{4}\right)^{4} = \left(1 + \frac{i^{12}}{12}\right)^{-12}$$

$$\left(1 - \frac{d^{4}}{4}\right)^{-4/12} = 1 + \frac{i^{12}}{12}$$

$$i^{12} = 12\left[\left(1 - \frac{0.06}{4}\right)^{-4/12} - 1\right]$$

$$\boxed{i^{12} = 6.0607\%}$$

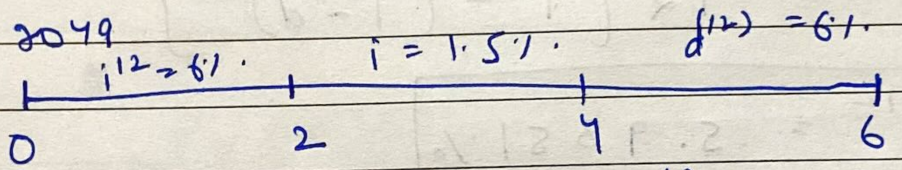
$$i^4 = 4\left[\left(1-\frac{0.06}{4}\right)^{-4/4} - 1\right]$$

$$\boxed{i^4 = 6.09137\%}$$

(c) $d < \delta < i^4 < i$

(d) done already.

12.



$$\left(1+\frac{i^{12}}{12}\right)^{12} = 1+i$$

$C = 7049$

$$i = 0.0616778322$$

$$AV_2 = 7049(1.0616778322)$$

$$= \underline{\underline{7945.35}}$$

$i_{SI} = 1.5\%$ (10 quarters.)

$$Af = 1+ni = 1 + ((10)(0.015))$$
$$= 1.15$$

$$AV_{4.5} = 7945.349282 \times 1.5$$

$$= \underline{\underline{9137.15}}$$

$d^{12} = 6\%$ (1.5 yrs)

$$\left(1 - \frac{d^{12}}{12}\right)^{12} = 1-d$$

$$d = 0.05837793$$

$$AF = \frac{1}{(1-d)^{n}} = 1.094421325$$

$$AV_6 = 9137.151651 \times 1.094421325$$

$$= \boxed{9999.894}$$

13. i) 0 to t, x to $2x$, $i = 10\%$. let 't' be no. of yrs.

$$2\cancel{x} = \cancel{x}(1+i)^{t}$$

$$2 = (1.1)^{t}$$

$$t = \frac{\ln(2)}{\ln(1.1)}$$

$$t = 7.27254$$

or $t = 7.27$ yrs.

$$i^{12} = 10\%$$

(ii) $$2x = x\left(1 + \frac{0.1}{12}\right)^{12 \times t}$$

$$2 = (1.10471306744)^{t}$$

$$t = \frac{\ln(2)}{\ln(1.10471306744)}$$

$$t = 6.96031123$$

or

$$t = \underline{\underline{6.96}} \approx 7 \text{ yrs.}$$