Q1) Let $x = \sqrt{7+\sqrt{7+\sqrt{7+\cdots}}}$ ——— ①

Squaring both sides we get;

$x^2 = 7+\sqrt{7+7}$ ——— ②

Substituting ① in ②

$x^2 = 7+x$

$x^2 - x - 7 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore a = 3.1926, -2.1926$

$\therefore x = 3.1926$

$\therefore \sqrt{7+\sqrt{7+\sqrt{7}}} = 3.1926.$

Q2) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$7x^2 + 11x = -4y^2 + 6y^2$

$18x^2 = 2y^2$

$9x^2 = y^2$

$$\frac{x^2}{y^2} = \frac{1}{9}$$

$$\frac{x}{y} = \frac{1}{3}$$

Q3) $x+3y=4, \quad x^2+y^2-4y=12.$

$x+3y=4 \quad , x^2+y^2-4y=12$

$x+3y=4$

$x=4-3y$ —— ①

Substituting ① in the eq^n we get,

$x^2+y^2-4y=12$

$(4-3y)^2+y^2-4y=12$

$10y^2-28y+4=0$

$5y^2-14y+2=0$

$\therefore y = \frac{14 \pm \sqrt{(-14)^2-4(5)(2)}}{10}$

$= 2.648996, 0.1510004$

When $y = 2.648996$

$x = -3.946998$

When $y = 0.1510004$

$x = 3.546996$

$\therefore$ The ordered pairs that satisfy the integer are

$(-3.94070, 2.6490)$ & $(3.547, 0.151)$

Q4) $d = 2r^2 - 16r + 34$.

d = density of smoke

r = speed of the engine.

a) $r = 3.5$

$d = 2(3.5)^2 - 16(3.5) + 34$.

$= 2.5$ million particles per cubic meter

b) for min smoke $\frac{d}{d(x)} > 0$

$\frac{d}{dx} = 4x - 16 > 0$

$\Rightarrow 4x > 16$

$\therefore x = 4$

no. of revolutions for min smoke

$= 4$ /min

min point $= 211$.

c) $d = 100$

$100 = 2x^2 - 16x + 34$.

$2x^2 - 16x - 66 = 0$

$x^2 - 8x - 33 = 0$

$x^2 - 11x + 3x - 33 = 0$

$(x - 11)(x + 3) = 0$

$x = 11$ or $x = -3$ (invalid)

Speed of engine is 100 revolutions/min

85) Company 1: 1500/day + 25/km

Company 2: 2100/day + 22/km.

n = ~~total~~ no. of km driven in a day.

a) Total cost of a car from company 1 = $1500 + 25n$

b) " " " Company 2 = $2100 + 22n$.

c) $1500 + 25n > 2100 + 22n$.

d) $1500 + 25n > 2100 + 22n$

$$3n > 600$$

$$\underline{\underline{n > 200}}$$

It is less expensive to rent from Company 2 when the no of km driven in a day is greater than 200.

6) $f(x) = x^3 - 7x^2 + 14x - 6 = 0$

a) considering $a = 0, b = 1$.

$f(a) = f(0) = -6$

$f(b) = f(1) = 2$.

since $f(a)$ & $f(b)$ have opp signs the interval is appropriate.

$x = 0.\underline{58}$

b) considering $a = 1$, $b = 3.2$

$f(a) = f(1) = 2$

$f(b) = f(3.2) = 0.112$

the interval is appropriate as $f(a)$ & $f(b)$ have opp signs.

$x = 3.\underline{00}$

c) considering $a = 3.2$, $b = 4$

$f(a) = f(3.2) = -0.112$

$f(b) = f(4) = 2$

$\therefore x = 3.42$

7) $f(x) = 230x^2 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^2 + 54x^2 + 18x - 221$.

Since $f(x)$ has real 0 in $(-1, 0)$ we consider the midpoint -0.5 as x_0

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x)} = -0.5 - \frac{f(0.5)}{f'(0.5)}$

$= 0.15045$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.0418168$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.04065935$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.0406592883\,32.$$

Since $f(x)$ has a real 0 in $(0,1)$ we consider the midpoint 0.5 as x_0.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = -0.7050892.$$

$x_2 = -0.3237911142$

$x_3 = -0.0646081\,3\,103$

$x_4 = -0.04068615115.$

$x_5 = -0.040659288\,35.$

Q8) 70, 86, 81, 83.

Grade on final exam $= x$.

a) Commercial avg $= \frac{70+86+81+83+x}{5}$

$$= \frac{320+x}{5}$$

b) $80 \leq \frac{320 + x}{5} < 90.$

c) $80 \leq \frac{320 + x}{5} < 90$

$400 \leq 320 + x < 450$

$80 \leq x \leq 130.$

If we assume that all marks are out of 100 :- $80 \leq x \leq 100$ //

Q7) $x_1 = 8.54$

$x_2 = -11.81$

$x = 0.$

$y_1 = 4\% = 0.04$

$y_2 = 20\% = 0.20.$

Using linear interpolation

$$y - 0.04 = \frac{0.20 - 0.04}{-11.81 - 8.54}(0 - 8.54)$$

$y = 10.7144963\%$

IRR $= 10.7145\%$

Q10) Revenue $= R(x) = 32x - 0.21x^2$

Cost $= C(x) = 195 + 12x$

To stay profitable $R(x) > C(x)$

$32x - 0.21x^2 > 195 + 12x$

$0.21x^2 - 20x + 195 < 0$

Equating the inequality to 0

$0.21x^2 - 20x + 195 = 0$

$x = 84.21142753, 11.0266771$

Since $x < 11.02666771$ &

$x > 84.211428$ do not

satisfy the original inequality

$11.0266771 < x < 84.2114275$

$\rightarrow$ satisfy original inequality.

$\therefore 12 < x < 84$

No. of orders to remain profitable should be greater than 12 & less than 84, with minimum orders being 12.