Date

Financial Mathematics 1 (Unit 3 & 4)

Assignment 2

Ques. 3

(i) $980000 = 12 \times a_{\overline{25}|}^{(12)}$

$980000 = 12 \times \left(\frac{1 - (1+0.07)^{-25}}{i^{(12)}} \right)$

$980000 = 12 \times \left(\frac{1 - (1.07)^{-25}}{12[(1.07)^{1/12} - 1]} \right)$

$\frac{980000}{12} \times \left(\frac{1-(1.07)^{-25}}{12(1.07)^{1/12}-1} \right)^{-1} = X$

$\boxed{X = 6848 \text{ Rs}}$

$\therefore$ Monthly payment is 6848 Rs.

(ii) loan O/S. On 10^{th} march 2020

$X a_{\overline{34/3}|}^{(12)} = 648,600$ @ 7%.

Spiral

Date

(iii) ~~Loan~~ o/s

Capital repayed on 10^{th} oct, 2026

$= \left[\text{loan o/s just after repayment on } 10^{th} \text{ sept 2026} - \text{loan o/s just after repayment on } 10^{th} \text{ oct 2026} \right]$

$= X\, a^{(12)}_{\overline{8\frac{3}{16}|}} @ 7\% - X\, a^{(12)}_{\overline{13.75|}} @ 7\%$

$= X \left[a^{(12)}_{\overline{8\frac{3}{16}|}} @ 7\% - a^{(12)}_{\overline{13.75|}} @ 7\% \right]$

$= 6848 \left[0.003922 \right]$

$\boxed{\text{Ans} = 26.86}$

(iv) (i) $\left[X\, a^{(12)}_{\overline{24|}} - X\, a^{(12)}_{\overline{19|}} \right] @ 7\%$

$= X \left[a^{(12)}_{\overline{24|}} - a^{(12)}_{\overline{19|}} \right] @ 7\%$

$= 6848 \left[0.07538 \right]$

$\boxed{\text{Ans} = 516.2}$

Spiral

Date

(v) Total interest = 305.56.

Q.12 (i)
The no. of years after which the cumulative discounted cashflows covers the initial investment is termed as discounting Payback Period.

(ii) (a) DPP: $-800(1+i)^t - 50(1+i)^{t-1} + 100 s_{\overline{5.7}|}$ @ 7%, = 0

$t = 17$; NPV = -7.5
$t = 18$ NPV = 22.85

$\boxed{\text{By interpolation } t = 17.767}$

(b) Accumulated profit after 22 years.

$100 \, s_{\overline{4.23}|}$ @ 6% = 480

$\boxed{\text{Profit} = 10\,480{,}000}$

(ii) When rental income is accumulated each year @ 6%.

$t = 18$

$-800(1.06)^{18} - 50(1.06)^{17} + 100.9715 \, s_{\overline{6}|}$ @ 7%

$= 9.7714$

Accumulated profit after 22 years.
$9.7714(1.06)^4 + 20 \, s_{\overline{4}|}$ @ 6%

$\boxed{\text{Ans} = 462{,}790}$

Spiral

Date

Ques 11 (i) TWRR : $(1+i)^2 = \frac{500}{460} \times \frac{550-50}{500+40} \times \frac{\cancel{500}\, X}{550 \times \cancel{500}}$

$0.2 = \frac{500}{460} \times \frac{500}{540} \times \frac{X}{550}$

$$X = \frac{0.2 \times 460 \times 540 \times 550}{500 \times 500}$$

$\boxed{X = 786.9312}$

(ii) MWRR : $460(1+i)^2 + 40(1+i)^{1/2} - 50(1+i) = 786.9312$

$\boxed{i = 30.909\%}$

Ques 10 (i) Let the loan amount be X.

$X = 180000\, a_{\overline{15|}} + 20000\,(Ia)_{\overline{15|}}$ @ 12%.

X = Loan amount from bank = 20,40,588 ..

Total cost of house = 2040588/0.8

$\boxed{\text{Ans} = \text{Rs } 25,50,735}$

Spiral

Date

(ii) loan O/s at beginning of 9th year,

$x = (18000\,v + 18000\,v^2 + \dots 18000\,v^7) + (7 \times 20000\,v^i \; 20000\,v^i)$

$X(9) = 340000\,a_{\overline{7}|} + 20000(Ia)_{\overline{7}|}$ @ 12%.

$\boxed{X(9) = 1875850.33}$

Loan Schedule

Year	Loan O/S	Installment	Interest	capital repay
9	1875850.33	360000	2,25,102.04	1,34,897.8
10	1740952.37	3,80,000	2,08,914.08	1,21,085.72
11	1569866.65			

(iii) Let the new installment be P' Rs

$1569866.66 = Pa_{\overline{5}|}$

$\boxed{P = 4,14,126.87 \text{ Rs}}$

→ Extra payment if loan continued from loan 1

Total amount − loan O/s at 10th yr

$\boxed{Rs = 6,30,133.3.742,}$

Spiral

Date

→ In case of Ban 2.

Total installment + 1% processing fee − loan o/s at end of 10^{th} yr

= 5,16,466.35

~~Ans~~ Bank II is more preferable than Bank 1.

Ques 9 TWRR

$(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{33+6} \times \frac{41.05-4.5}{38.5} \times \frac{45}{41.05} \times \frac{45.6}{45} \times \frac{47}{45.6}$

$(1+i)^3 = 1.228313$

$1+i = 1.070955$

$\boxed{i = 7.0955\%}$

MWRR

$30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$

At $i = 7\%$ LHS = 46.74

$i = 7.5\%$ LHS = 47.37

By interpolation $\boxed{i = 7.206\%}$

$\boxed{MWRR = 7.206\%}$

Spiral

Date

(ii) Linked rate of return

year 2011 → eqn of value.

$30.5(1+i) + 6(1+i)^{0.5} = 38.50$

$\boxed{i = 6.266\%}$ — (1)

$38.5(1+i) + 4.5(1+i)^{0.5} = 45$

year 2012 $\boxed{i = 4.92\%}$ — (2)

year 2013

$45(1+i) - 2.5(1+i)^{0.5} = 47$

$\boxed{i = 10.276\%}$ — (3)

Limited rate of return = $(1+i)^3 = (1.06266)(1.0492)(1.10276)$

$\boxed{i = 7.13\%}$

~~Prob~~ PV of outlay is $2.7 + 0.22\overline{a}_{\overline{3}|}$ @ i.

Expected PV of income

$= 0.1v^3 + \overline{a}_{\overline{10}|}(0.05v + 0.2v^2 + 0.22v^3)$ @ i

=

Date

Equating (i) & (i)

$$2.7 + 0.2 a_{\overline{3}|} = 0.1 v^3 + \bar{a}_{\overline{10}|}(0.25 v + 0.2 v^2 + 0.1 v^3)$$

so $$2.7 + 0.2\left(\frac{1-v^3}{v^{-1}-1}\right) = 0.1 v^3 + \frac{1-v^{10}}{\log v}\left[0.25 v + 0.2 v^2\right]$$

$$\boxed{i = 9.3\%}$$

(ii) $NPV = 0.1 v^3 + 0.85 \bar{a}_{\overline{10}|} v^3$

$$0.1 v^3 + 0.85 \bar{a}_{\overline{10}|} v^3 - 2.7 - 0.2 a_{\overline{3}|} \quad @ \; i = 10\%$$

$$0.1(0.7513) + 0.85 \times 0.7513 \times 6.4469 - 2.7 - 0.2 \times 2.486852$$

$$4.19 - 3.19$$

$$\boxed{NPV = 1.0089.}$$

Since NPV is +ve.
hence worth to invest in project.

Spiral

Date

Ques 6

(a) $160000 = X a_{\overline{10}|} @ 8\%$

$$\frac{160000}{\left(\frac{1-(1.08)^{-10}}{0.08}\right)} = X$$

$$\boxed{X = 23844.65209}$$

(b) loan o/s at $t=4$

$PV_4 = 23844.65209 \, a_{\overline{6}|} @ 8\%$.

$$\boxed{PV_4 = 110231.4422}$$

Let the revised payment be X'

$110231.4422 = X' a_{\overline{6}|} @ 10\%$.

$$\frac{110231.4422}{\left(\frac{1-(1.10)^{-6}}{0.10}\right)} = X'$$

$$\boxed{X' = 25309.72428}$$

Spiral

Date

(iv) $PV_7 = 25309.72428 \, a_{\overline{7}|} @ 10\%$.

$$\boxed{PV_7 = 62942.7533\%}$$

Let Y be the final amount

$62942.7533 = Y \, a_{\overline{3}|} @ 9\%$.

$$\boxed{Y = 24865.78179}$$

Ques 5 (i) TWRR for property fund.

$$(1+i) = \frac{13.1}{12.4} \times \frac{14.8}{13.1} \times \frac{15.8}{14.0} \times \frac{16.4}{15.8}$$

$$1+i = \frac{16.4}{12.4}$$

$$\boxed{i = 32.2580645\%}$$

TWRR for equity fund.

$$(1+i) = \frac{9.2}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{13.1}$$

$$\boxed{i = 28.0991736\%}$$

Date

(ii) Project → Property fund

$12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4}$

$= 4 \times 16.4$

when $i = 0.2932$ or 29.32%.

Project → Equity fund.

$12.1(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 15.1(1+i)^{1/4} = 4 \times 4$

$\boxed{i = 67.31\%}$

Ques 9 $\quad (I\ddot{a})_{\overline{n}|} \quad i > 0$

To Prove $\quad (I\ddot{a})_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

Proof $\quad PV = 1 + 2v + 3v^2 + \cdots + nv^{n-1}$ — ①

multiply by v on both sides

$(v)PV = v + 2v^2 + 3v^3 + \cdots (n-1)v^{n-1} + nv^n$

②

① − ②

$dPV = (1 + v + v^2 + \cdots v^{n-1}) - nv^n$

$dPV = \ddot{a}_{\overline{n}|} - nv^n$

$\boxed{(I\ddot{a})_{\overline{n}|} = \dfrac{\ddot{a}_{\overline{n}|} - nv^n}{d}}$

Date

Qu 3 (i) $v^{3/12}$ and $\ddot{a}_{\overline{24}|}$ PV of annuity - I

$=(1.08)^{-3/12} \times 200 \times a_{\overline{24}|} \times i/d$

$=(1.08)^{-3/12} (200)(10.2007)\dfrac{(0.08)}{0.074074}$

$\boxed{\text{Ans } 2161.365534}$

(ii) PV of annuity 2

$320\, \ddot{a}^{(4)}_{\overline{16.5}|} \times v^{1/6} = (320)\dfrac{(1-v^{16.5})}{i} \times \dfrac{i}{d^{(4)}}$

$= 320 \left(\dfrac{1-(1.08)^{-16.04}}{0.08}\right) \times 1.049579$

$\boxed{A = 2996.04887}$

(iii) PV of annuity (3)

$15\, \ddot{a}_{\overline{24}|}$ @ $a^{(12)}/12$

$\dfrac{15\,(1-(1.08)^{-24/12})}{0.00639215} = 1795.651919$

Spiral

$\boxed{\text{Total annuities} = 6953.005823}$

Revised Annuity

$6953.005823 = x\, \ddot{a}_{\overline{21}|}^{(2)}$ @

$\boxed{i = 0.08}$

$\boxed{X = 649.0135687}$

Ques 2 (i)

(a) Payback Period

It is the no. of years necessary to recover funds invested in Project without considering the interest at all (both on the borrowing and investing).

(b) Discounted Payback Period

It is the no. of years after which the cumulative discounted cash inflow covers the initial ~~amount~~ investment.

(iii) IRR of Project A

~~PV of out~~ $PV_{outlay} = -170 - 20v - 10v^2$

$PV_{inflow} = 20v + 20v^2 + 200v^3$

Spiral

Date

$NPV = PV_{outlay} - PV_{inflow} = 0$

$-170 - 20v - 10v^2 + 20v + 20v^2 + 120v^3 = 0$

$-170 + 10v^2 + 120v^3 = 0$

$-170 + 10(1+i)^{-2} + 120(1+i)^{-3} = 0.$

at $i = 0.07$ $\quad L(i) = 171.44$

$i = 0.08$ $\quad L(i) = 167.33$

By interpolation

$\boxed{i = 7.4\%}$

Project B $\quad PV_{outlay} = -200 \quad PV_{inflow} = 14a_{\overline{6}|} + 200v^6$

$NPV \Rightarrow 200 = 14\left(\frac{1-v^6}{i}\right) + 200(v^6)$

$200 = 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$

At $i = 0.07$ $\quad L(i) = 200.$

By interpolation

$\boxed{i = 7\%}$

(iv) Project A

$NPV = -200 + 14a_{\overline{6}|} + 20v^6$ @ $d = 6\%$

Spiral

Date

$= 5.995851$.

(c) So, project A would be more profitable if the borrowing rate is less than 7.4%. and Project B is less than 7%.

Qs1 ① Loan amount = 20,000

Payments (monthly) = 427.90

$t = 5$ years.

$20,000 = (12)(427.9)\, a_{\overline{5|}}^{(12)}$ @ i

$a_{\overline{5|}}^{(12)} = 3.894991042$

$\frac{1 - v^5}{i^{(12)}} = 3.89499$

$\frac{1 - (1+i)^{-n}}{12((1+i)^{1/12} - 1)} - 3.89 = 0$

if $i = 10\%$. LHS = 3.96254

$i = 11\%$ LHS = 3.87812

By interpolation

$\boxed{i = 10.8\%}$

$\boxed{APR = 10.8\%}$

Spiral

Date

(ii) $PV_1 = 427.9 \times 12\, a_{\overline{4}|}^{(12)}$ at $i = 10.8\%$.

$= 427.9\ \cancel{(12)} - \dfrac{[1 - (1.108)^{-4}]}{\cancel{12}[0.00858300]}$

$\boxed{PV_1 = 16775.97728}$

$16775.97728 = (12)(274.49)\, a_{\overline{T}|}^{(12)}$ @ 10.8%

$\dfrac{1 - v^{-T}}{12(0.00858300)} = 5.09307$

$(1.108)^{-T} = 0.478433238$

$\boxed{T = 7.2499 \text{ years}}$

(iii) Interest paid in I^{st} loan $= 3763.22$

Interest paid in II^{nd} w. $= 7104.3233$

Interest paid more $= 3341.43$