Financial Mathematics – II

Q1> Loan Amount : 20000
Monthly Installment : 427.90
Time : 5years.

∴ APR =

$$20000 = 427.90 \times a_{\overline{60}|} \; @ \; \frac{i^{12}}{12}$$

$$\frac{20000}{427.90} = \frac{1 - v^{60}}{i}$$

$$\therefore \frac{i^{(12)}}{12} = 0.85834\ \%$$

$$\boxed{i = 10.8\ \%}$$

$$\text{flat Rate} = \frac{\text{Total Repaid} - \text{original Amount}}{\text{Term of loan} \times \text{original Amount}}$$

$$= \frac{(60 \times 427.90) - 20{,}000}{5 \times 20000} = 0.05674$$

$$= 5.674\ \%$$

APR = $5.674\ \% \times 2$

$\leftarrow 0.11348$

$\approx 11.348\ \%$

ii) ∴ NPV after 1 year

$$= 427.90 \times a_{\overline{4}|}^{(12)} \quad @ \quad i = 10.8\%$$

$$= 427.90 \times \frac{1 - v^4}{i^{(12)}}$$

$$= 427.90 \times \frac{0.09747292 4}{i^{(12)}}$$

$$= \text{~~16563.0844~~} \text{ ₹}$$

$$16,775.83 \text{ ₹}$$

∴ Term of new loan

$$\frac{\text{~~16563.0844~~}}{16,775.83} = 274.49 \times a_{\overline{n}|} \quad @ \quad \frac{i^{(12)}}{12}$$

$$\frac{16,775.83}{274.49} = \frac{1 - (1.0085843)^{-n}}{0.0085843}$$

$$\boxed{n = 7.25 \text{ 11 years}}$$

ii) Interest with second method =

$$= (427.9 \times 12) + (274.49 \times 12 \times 7.25) - 20000$$

$$= 23880.63 + 16775.83 - 20000$$

$$= 9015.43$$

Interest in 1st method : 5674

More = 9015.43 − 5674

$$\boxed{\text{More} = ₹\ 3341.43}$$

Q2) i) a) Discount Payback period

Discount Payback period is the number of year at which the cummulative discounted cash inflows cover the initial investment.

b) Payback period.

payback period is the number of year necessary to recover funds invested in the project without considering the interest, both on borrowing and investing.

ii) Unlike the NPV, Neither the DPP nor the PP give any Indication of how profitable a project ~~is~~, is, as they ignore cash flows after the accumulated value of Zero is reached.

The pp can give misleading result, as it does not take into account the time value of money.

Hence NPV is Superior measure as compai-red to Dpp or pp.

iii) Project A

pv of outlay = $-170 - 20v - 10v^2$

pv of inlay = $20v + 20v^2 + 200v^3$

NPV = $-170 - 20v - 10v^2 + 20v + 20v^2 + 200v^3$

0 = $-170 + 10v^2 + 200v^3$

IRR $\Rightarrow$ $0 = -170 + 10v^2 + 200v^3$

$170 = 10(1+i)^{-2} + 200(1+i)^{-3}$

At $i = 0.07$, LHS $= 171.99$

$i = 0.08$, LHS $= 167.33$

By interpolating, $i = 7.4\%$

Project B

pv of outlay $= -200$

pv of inlay $= 14\, a_{\overline{6}|} + 200v^6$

$$NPV = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

$$IRR \Rightarrow 0 = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

$$200 = 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200(1+i)^{-6}$$

At $i = 0.07$ LHS $= 200$

By interpolating $i = 7.1\%$

B. $NPV_A = -170 + 10v^2 + 200v^3$

At $d = 6\%$

$NPV_A = -170 + 10(1-0.06)^2 + 200(1-0.06)^3$

$NPV_A = 4.9528$

$NPV_B = -200 + 14\, a_{\overline{6}|} + 200v^6$

$= -200 + 14\left(\frac{1-(0.94)^6}{0.06}\right)(0.94) + 200(0.94)^6$

$NPV_B = 5.9958509$

iii) Each of the project would only be profitable if the ~~rate~~ ~~data~~ of interest is below IRR

$\therefore$ For A Less than 7.4 %
for B Less than 7 %

Q3) PV of annuity 1 $= 200\ \ddot{a}_{\overline{22}|}\ (v^{3/12})$

$= (1.08)^{-1/4}\ (200)\ (10.7007)\ \frac{(0.08)}{(0.074074)}$

$= 2161.365534$

PV of annuity 2 $= 320\ \ddot{a}^{(4)}_{\overline{16.25}|}\ v^{2/12}$

$= 320 \left(\frac{1-(1.08)^{-16.25}}{0.08} \right) (1.049519)$

$(1.08)^{-1/5}$

$= 2996.048870$

PV of annuity 3 $= \left(15\ \ddot{a}_{\overline{226}|}\ @\ \frac{d^{(12)}}{12} \right)$

$= 15 \left(\frac{1-(1.08)^{-226/12}}{0.006392917} \right)$

$= 1795.651326$

Q5) For Property Fund

i) $1+i = \frac{13.1}{12.4} \times \frac{14.8}{13.1} \times \frac{15.8}{14.8} \times \frac{16.4}{15.8}$

TWRR = i = 32.2580645 %

For Equity Fund

$1+i = \frac{9.2}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{12.1}$

TWRR = i = 28.0991736 %

ii) $12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4} = 4(16.4)$

i = 29.32 %

iii) $(12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 13.1(1+i)^{1/4} = 4 \times 15.5$

i = 67.31 %

Q6)

i) 160,000 = X $a_{\overline{10}|}$ @ 8% P.a.

$\therefore X = \frac{160000}{6.7101}$

X = 23844.6520g

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Total P.V. = 6953.06573

Revised ~~arm~~ annuity $\doteq$ $6953.06573 = X\ \ddot{a}^{(2)}_{\overline{21.5}|} @ d^2$

$$6953.06573 = X\left(\frac{1-(1.08)^{-21.5}}{0.08}\right)(1.059615)$$

$$X = 649.0135571$$

Q4) To Derive: $(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

$$P.V = 1 + 2v + 3v^2 + \dots + nv^{n-1} \quad \text{—— ①}$$

$$(1-d)(PV) = v + 2v^2 + 3v^3 + \dots + (n-1)v^{n-1} + nv^n \quad \text{—— ②}$$

eqⁿ ① − eqⁿ ②

$$PV - (1-d)(PV) = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$$

$$d(PV) = \ddot{a}_{\overline{n}|} - nv^n$$

$$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

ii) Loan o/s after 4th payment

$= 23,844.65 \times a_{\overline{6}|} @ 8\cdot1\cdot$

$= ₹\ 1,10,231.4325$

Annual Repayment,

$110231.4325 = Y \times a_{\overline{6}|} @ 10\cdot1\cdot$

$\therefore Y = ₹\ 25,309.722$

iii) Loan o/s 7th repayment

$= 25309.722 \times a_{\overline{3}|} @ 10\cdot1\cdot$

$= ₹\ 62,942.747$

$\therefore 62,942.747 = X \times a_{\overline{3}|} @ 9\cdot1\cdot$

$\therefore X = ₹\ 24865.779$

$\therefore$ Total Repayment
$= (4 \times 23844.65) + (3 \times 25309.722) + (3 \times 24865.779)$

$= ₹\ 2,45,905.907$

$1,60,000\ (1+i)^{10} = 245905.907$

$\therefore i = 4.3914\ \cdot1\cdot$

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Q7> i> Government securities provide fixed yeild market condition. Whereas, in property, Income depends upon market condition and there could be no income sometime Government securities are also safest and most secure while property which was proven in 2008 crash.

ii> a> a loan by tender could involve the government Asking investor to submit prices at which they will buy the fix security from government.

b> The Advantaget to government is it can be choose the higest price to offer and sell the security.
Disadvantage could be that the Security might not be hold by a wide variety of investors and be concantated

Advantage to investor are. They can be ~~B~~Set at their ideal prize.
Disadvantage will be. If Their price is lower they won't get any sell.

iii> Share have uncertain income, as the dividant is given by ~~diston~~ decision of directors.

For indey linked bond, The absolute income ~~ins~~ isn't known and there could be deflation too.

Saving interest as interest rate change very frequently.

Q8) i) P.V of outlay = PV of inflow.

$$= -2.7 - 20\, a_{\overline{3}|} @ i + 0.1v^3 + \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3)$$

$\therefore i = 9.3\%$

ii) $0.1v^3 + 0.85\,\bar{a}_{\overline{10}|}v^3 - 2.7 - 0.2\,a_{\overline{3}|}$ @ $i = 10\%$

$= 4.19 - 3.19$

$= 1.0089$

As the NPV is positive the project can be consider for investing.

Q9) i) TWRR for fund.

$$(1+i)^3 = \frac{33}{30.5} \times \frac{38.65}{39} \times \frac{45.6}{41.05} \times \frac{47}{(45.6-2.5)}$$

$\therefore i = 7.0951\%$

MWRR

$$47 = 30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5}$$

$\therefore i = 7.206\%$

iii) TWRR ~~Takes~~ Takes time of Income and outflow and fund into consideration and isn't affected by quantity of money that

and goes out or grow unlike in MWRR which is affected by such quantity.

Q10) Loan = 180 k [9 ⌐15] @ 12 %] + 20 k [I a

= 180 k [6.8109] + 20 k [40.731]
= 1225.962K + 814.62K
= ₹ 20, 40, 582

Total cost = Loan + Sell
= 80 % of ⌐Cost + 20 % of ⌐

$\frac{80}{100} \times ⌐C = 2040582$

Total cost = ₹ 25 50 727.5

loan o/s after t = 8

34000 x 9 ⌐7 @ 12 % + 20000 (Ia) ⌐7

= 15, 51, 692 + 324160
= ₹ 18, 75, 852

Term	Loan at Start	Repaid	Int	Cap. loan
9	18 15852	360000	225702.24	1348 97.76
10	1740 954	380000	208914.5	171085.49

loan o/s
1740954
1569869

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YOUVA

Sol> i) TWRR

$$(1+i)^2 = \frac{500}{460} \times \frac{590}{540} \times \frac{\times}{540}$$

$$(1.2)^2 = 1.44$$

$$X = (1.44 \times 550) / (1.086975 \times 1.1111)$$

$$X = 655.78$$

ii) MWRR

$$460(1+i)^2 + 40(1+i)^{1.25} - 850(1+i) =$$

$$= 655.78$$

$$\therefore i = 19.85\%$$

iii) Effective Interest Rate of 2016

$$\frac{655.78}{550} = 1.1923$$

$\therefore$ LWRR

$$(1+i)^2 = 1.1923 \times 1.2044$$

$$i = 19.83\%$$

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Q12) i) DPP is when NPV turns position first time taking interest into account

ii) a)

$$-800(1+i)^{t} - 50(1+i)^{t-1} + 100\,\bar{s}_{\overline{t-2}|} = 0$$

$$\therefore \quad t = 17.767$$ ---- by using trial and error.

b) accumulated profit after 22 years.

$$\therefore \quad 100\,\bar{s}_{\overline{4.233}|} \quad @ \; 6\% = 480$$

$$\therefore \quad ₹ \; 480000$$

iii) $100\,\bar{s}_{\overline{11}|} = 102.971$ at end of year

$$-800(1+i)^{t} - 50(1+i)^{t-1} + 102.971\,s_{\overline{t-2}|}$$

$$@ \; 7\% \geq 0$$

By using Trial and error.

$t = 18$ years.

$$-800(1+i)^{18} - 50(1+i)^{17} + 102.971\,s_{\overline{16}|}$$

$$= 9.7714$$

Hence AV 22

$$= 462.79$$

$$\therefore \quad ₹ \; 462.790$$

Q13) i) $X \; a^{(12)}_{\overline{25|}} = 9880$ @ 7.1.

$X = 821.76$

$X/12 = 68.48$

ii) Loan o/s after 10/03/29

$821.76 \; a^{(22)}_{\overline{11\,1/3|}} = 648,600$

iii) Loan o/s on 10th spt 2016

$821.76 \times a^{(12)}_{\overline{13\,5/6|}}$

Loan o/s on 10th oct 2016

$821.76 \times a^{(12)}_{\overline{13.75|}}$

$\therefore \; 821.76 \left[a^{(12)}_{\overline{13\,5/6|}} - a^{(12)}_{\overline{13.75|}} \right]$

$= 26.86$

iv) i) Capital Repayment in 12 th month

$821.76 \left[a^{(12)}_{\overline{7\,1/3|}} - a^{(12)}_{\overline{6\,1/3|}} \right] = 516.20$

ii) Interest $= 821.76 - 516.20$

$= 305.56$