1) (a) $y_i = \alpha + \beta x_i$

$\Rightarrow$ $y_1 = \alpha + \beta x_1$ & $y_2 = \alpha + \beta x_2$

$\Rightarrow$ $56 = \alpha + 40\beta$ & $\Rightarrow 62 = \alpha + 10\beta$

$\Rightarrow 128 = 4\alpha + 40\beta$

$\Rightarrow$ ② − ① $\Rightarrow$ $3\alpha = 72$

$\Rightarrow$ $\alpha = \frac{72}{3}$

$\Rightarrow \boxed{\alpha = 24}$

$\Rightarrow$ $10\beta = 62 - 24$

$\Rightarrow$ $\boxed{\beta = 3.8}$

$\Rightarrow$ equation of regression line $\boxed{y_i = \cancel{\alpha}\, 24 + 3.8 x_i}$

(b) Gradient of regression line $= \hat{\beta} = 1.724$

2) $\Sigma x = 385.2$

$\Sigma x^2 = 12,666.58$

$\Sigma y = 1162.5$

$\Sigma y^2 = 119,026.9$

$\Sigma xy = 38,191.41$

$\rightarrow$ $\cancel{\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}$

$S_{xx} = 12666.58 - \frac{(385.2)^2}{12} = 301.66$

$S_{yy} = 119026.9 - \frac{(1162.5)^2}{12} = 6409.7125$

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$S_{xy} = 38191.41 - \frac{(385.2)(1112.5)}{12}$

$= 875.16$

$\hat{\alpha}_1 = 3.748181$ $\quad \hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{875.16}{301.66} = 2.901146$

$\Rightarrow y_i = 3.748181 + 2.901146\, x_i$

$\hat{\sigma}^2 = \frac{1}{10}\left[6409.7125 - \frac{(875.16)^2}{301.66}\right]$

$= 387.0744$

$\rightarrow S.E(\beta) = 1.132$

$\Rightarrow P\left(-2.228 < \frac{2.901 - \beta}{1.132} < 2.228\right) = 0.95$

$\Rightarrow P[\cancel{\ldots} -0.378904 < -\beta < -5.423096] = 0.95$

$\Rightarrow 95\%\ CI = [2.901 - 2.228(1.132),\ 2.901 + 2.228(1.132)]$

$= \{0.378904,\ 5.423096\}$

$\rightarrow SE(\mu_1) = \sqrt{\left[\frac{1}{12} + \frac{(7.9)^2}{301.66}\right] 387.07} = 10.5988$

$\Rightarrow P\left[-2.228 < \frac{119.79 - \mu_1}{10.5988} < 2.228\right) = 0.95$

$\Rightarrow 95\%\ CI = \{119.79 - 2.228\,(10.5988),\ 119.79 + 2.228\,(10.5988)\}$

$= (96.17, 143.40)$

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3) (2) for ANOVA model

→ These must be normal population
→ The variance should be constant
→ The ~~s~~ observations are independent and identically distributed

(3) $SS_{TOT} = SS_{RES} + SS_{REG}$

→ $SS_{TOT} = S_{yy} = 1495 - \frac{103^2}{28} = 1116.11$

→ $SS_{REG} = 516.11$

⇒ $SS_{RES} = SS_{TOT} - SS_{REG} = 600$

ANOVA TABLE

		SUM of SQUARES	M.S.
Regression	3	516.11	172.04
Residual	24	600	25
Total.	27	1116.11	

$$F = \frac{MSS_{REG}}{MSS_{RES}} = \frac{172.04}{25} = 6.88$$

Test statistic = 6.88

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4) Analysis

It is a descending sequence of the ~~g~~ response variables.

4)(a) The mathematical model will be

$y_{ij} = \mu + \tau_i + e_{ij}$; k = no. of treatments
$i = 1, \dots, k$; n = no of response
$j = 1, 2, \dots, n$; y_{ij} = is the response
μ = is the mean response
e_{ij} = is the error term
τ_i = is the ith treatment

b) $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$

$CF = 104385.94$

$SS_{TOT} = 174316 - CF$
$= 69930.06$

$SS_{REG} = 22325.69$

$SS_{RES} = SS_{TOT} - SS_{REG} = 47604.37$

Test Statistic $= \dfrac{MSS_{REG}}{MSS_{RES}} = 3.283$

$F_{4, 28, 0.05} = 2.714$

$\therefore$ H_0 rejected

(C) O_i

	3-5	5-8	8-10	10-12	
0-3	45	20	6	5	= 76
4-6	7	20	9	6	= 42
7-9	5	8	15	12	= 40
	57	48	30	23	158

E_i

	3-5	5-8	8-10	10-12
0-3	27.41772	23.08861	14.43038	11.06329
4-6	15.15189	12.75949	7.97468	6.11392
7-9	14.43038	12.15189	7.59494	5.82278

$$TS = \frac{\Sigma (O_i - E_i)^2}{E_i} = 49.91148$$

degree of freedom = $2 \times 3 = 6$

$$\chi^2_{6, 5\%} = 12.59$$

$\therefore H_0$ is rejected

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10) $\Sigma x = 0.93$ $\Sigma y = 0.23$

$\Sigma x^2 = 0.2925$ $\Sigma y^2 = 0.0189$

$n = 3$ $\Sigma xy = 0.0735$

$S_{xx} = 0.0042$

$S_{yy} = 0.00126$

$S_{xy} = 0.0022$

$SS_{TOT} = 0.00126$

$SS_{REG} = S^2_{xy} / S_{xy} = 0.00115$

$SS_{RES} = 0.00011$

ANOVA

	DOF	SS	MSS
REG	1	0.00115	0.00115
RES	1	0.00011	0.00011
TOT	2	0.00126	

$H_0 = \beta = 0$ vs. $H_1 : \beta \neq 0$

Test statistic $= \dfrac{0.00115}{0.00011} \sim F_{1,1}$

$= 10.4545 \sim F_{1,1}$

$\therefore F_{1,1,5\%} = 161.4$

$\therefore$ H_0 is not rejected.

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(1) $\Sigma x = 45$ $\quad$ $\Sigma y^2 = 43956$

$\Sigma x^2 = 277.5$ $\quad$ $\Sigma xy = 2602$

$\Sigma y = 644$ $\quad$ $n = 10$

i) Negative

ii) $S_{xx} = 75$ $\quad$ $S_{yy} = 2482.44$

$S_{xy} = -296$

$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{xy}}} = -0.686002$

(iv) $\hat{\beta} = \frac{S_{xy}}{S_{xx}} = -3.9467$

$\hat{\alpha} = \bar{y} - \bar{x}\hat{\beta} = 82.16$

$\hat{y} = 82.16 - 3.9467x$

(v) $r^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = 47.0598\%$

vi) $x_0 = 1$

change $= \beta \cdot x_0 = -3.9467$

6) $\Sigma x = 39$

$\Sigma x^2 = 207$

$\Sigma y = 562$

$\Sigma y^2 = 40508$

$\Sigma xy = 2853.$

$n = 10$

i) $S_{xx} = 54.9$

$S_{xy} = 661.2$

$S_{yy} = 8923.6$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}\,S_{yy}}} = \frac{661.2}{\sqrt{(54.9)(8923.6)}} = 0.94466$$

(ii) $$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = \frac{661.2}{54.9} = 12.04371$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 56.2 - 12.04371\,(3.9) = 9.2295$$

(iii) $SS_{TOT} = 8923.6$

$SS_{REG} = 7963.30492$

$SS_{RES} = 960.29508$

(IV) $$R^2 = \frac{S^2_{xy}}{S_{xx} \cdot S_{xy}} = 89.24\% > 75\%$$

$\rightarrow \therefore$ It is a good fit

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8) Assumptions under ANOVA

- Population is normal
- Variance is constant
- These are independent obs.

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7) $S_{xy} = 4789.4221$

$S_{yy} = 1189.20767$

$S_{xy} = 2176.84$

→ $\hat{\beta} = \dfrac{S_{xy}}{S_{xx}} = 0.45451$

$\alpha = y - \hat{\beta} x = 10.78056$

→ $\hat{y} = 10.78056 + 0.45451 x$

(ii) $H_0 : \beta = 0$ v.s $H_1 : \beta \neq 0$

$T.S = \dfrac{\hat{\beta} - \beta}{SE(\hat{\beta})} \sim t_{n-2}$

$$\hat{\sigma}^2 = \frac{1}{n-2}\left(S_{yy} - \frac{S_{xy}^2}{S_{xx}} \right)$$

$= 22.20136$

$SE(\hat{\beta}) = 0.004635$

$T.S = \dfrac{0.45451}{0.00463549}$

$= 6.675 \sim t_9$

⇒ $t_{9, 2.5\%} = 2.262$

∴ H_0 is rejected.

(iii) $r = \dfrac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = 0.91213$

(iv) $x_0 = 25$

$$\mu_0 = \frac{\hat{\mu} - \mu}{SE(\hat{\mu})} \sim t_{n-2}$$

$\rightarrow$ $SE(\hat{\mu}) = 1.55$

$\Rightarrow$ 95% $CI = \hat{\mu} \pm t_{9,\,2.5\%} \; SE(\hat{\mu})$

$= (18.64313, 25.66349)$