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Numerical Method & Algebra
Assignment - 1

Q1) Actual length = r_1 = 12.5 cm

Actual area = πr_1^2

= $\pi (12.5)^2$

= 490.87365 cm^2

r_2 = 12.65 cm

Approximate area = πr^2

= $\pi (12.65)^2$

= 502.72551 cm^2

Absolute error = App. area - Actual area

= 11.84886

Relative error = $\frac{\text{Absolute area}}{\text{Actual area}}$

= 0.02413

Percentage error = $\left(\text{Relative error}\right) \times 100$

= 2.418%

(2)

Q2) i) $x_1 = 4$ $\quad y_1 = 0.60206$

$x_2 = 6$ $\quad y_2 = 0.7781513$

Using Linear interpolation,

$$x_2 - x_1 = y_2 - y_1$$

$$(6-4) = 0.7781513 - 0.60206$$

$$2 = 0.1760913$$

$$\text{Charge} = \frac{0.1760913}{2}$$

$$= 0.8804565$$

Thus, $x = 5$

$$y = 0.60206 + 0.8804565$$

$$y = 0.69010565$$

ii) $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$$(5.5 - 4.5) = y_2 - y_1$$

$$= 0.7403627 - 0.6532125$$

$$= 0.0871502$$

$$\text{Charge} = \frac{0.0871502}{1}$$

Thus $x = 5$

$$y = 0.6532125 + 0.0871502$$

$$= 0.6967876$$

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(3)

Q3) $x^4 - x - 10 = 0$ $a = 1.5$ $b = 2$

x	y
$a = x_0 = 1.5$	-6.4375
$b = x_1 = 2$	4

1) $x_2 = \dfrac{x_0 + x_1}{2} = 1.75$

$y = -2.37109$

2) $x_3 = \dfrac{x_1 + x_2}{2}$

$= 1.875$

$y_3 = 0.48462$

3) $x_4 = \dfrac{x_2 + x_3}{2}$

$= 1.8125$

$y_4 = -1.0202$

4) $x_5 = \dfrac{x_4 + x_3}{2}$

$= 1.84375$

$y_5 = -0.28773$

5) $x_6 = \dfrac{x_5 + x_3}{2}$

$= 1.859375$

$y_6 = 0.0953$

4

Q4) $x^3 - x - 1 = 0$

$f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$x_1 = x_0 - f(x_0)$

$= 1 - \left(-\frac{1}{2}\right)$

$= 1.5$

$y_1 = 0.875$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5 - \left(\frac{0.875}{0.75}\right)$

$x_2 = 1.34783$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44998$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.3252$

$y_3 = 0.00209$

$f'(x_3) = 4.26846$

$x_4 - x_3 = f(x_3) / f'(x_3)$

$= 1.32472$

$y_4 = 6.000008712$

Q5) $A^2 = A$

$7A - (I + A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - (I + A^2(A) + 3A + 3A)$

$= 7A - (I + 7A)$

$= -I$

Q6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 0 - 1(6-8) + \cancel{*}(-1)4$

$= -2$

$Adj(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{Adj(A)}{|A|}$

$= -\dfrac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{Adj(A)}{|A|}$

$$= \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} +3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ 2 & 1/2 & -2 \end{bmatrix}$$

Q7) $\vec{a} = 8i - 9j$
$\vec{b} = 7i - 9j$

$\vec{a}.\vec{b} = ab\cos\theta$
$= (8i - 9j)(7i - 9j)$
$= 56 + 81$
$= 137$

$a = \sqrt{64 + 72} = \sqrt{145}$
$b = \sqrt{49 + 81} = \sqrt{130}$

$\vec{a}.\vec{b} = ab\cos\theta$

$\cos\theta = \frac{\vec{a}.\vec{b}}{ab}$

$= \frac{137}{\sqrt{36} \times \sqrt{145}}$

$\cos\theta = 0.9720224206$

Q8) $\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30^\circ$

$R^2 = (75)^2 + (25)^2 + 2(75)(25)\cos 30^\circ$

$R^2 = 9497.595284$ m

Q9) $d = 3$

AP $= 101, 104, 107, \ldots\ldots 998$

$a = 101$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$n = 300$

$S_n = \frac{n}{2}(a + l)$

$= \frac{300}{2}(998 + 101)$

$= 150 \times 1099$

$S_n = 164850$

Q10) $a_6 = 32$

$ar^5 = 32$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = \pm 2$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13$

$f'(5) = 13$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 5 - \left(\frac{13}{13}\right)$

$= 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_2)}$

$x_2 = 6 - 9/32$

$x_2 = 5.71875$

$f(x_2) = 0.84787$

Q14) $(x^2 - x + 1)^4 = ((x-1)^2 + x)^4$

using Pascal's triangle,

$[(x-1)^2 + x]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$

$(x^2 - x + 1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$

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Q11) $(3x+4)^5 = (3x)^5 + 5(3x)^4 4 + 10(3x^3) \times 4^2 +$
$(10(3x^2)4^3) + 5(3x)(4^4) + 4^5$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x$
$+ 1024$

Q12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| = -\lambda^3 + 13\lambda - 12$
$-\lambda^3 + 13\lambda - 12 = 0$

$\lambda^3 - 13\lambda + 12 = 0$
Comparing coefficent
$\Rightarrow 1 - 13 + 12$
$= 0$
$\therefore$ Root $= 1$

$\lambda^2 - \lambda - 12 = 0$
$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$
~~$\lambda = 30$~~
$\lambda = 3$ or -4
Eigen Value $= -4, 1$ & 3

Q15) $e^{-x} = 3\log(x)$

$3\log(x) + e^{-x} = 0$

x	y
0.1	-2.0951625
0.2	-1.278179
0.3	-0.8278180
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	-0.03188

$x_0 = 0.6$ $\quad y_0 = -0.11673$

$x_1 = 0.7$ $\quad y_1 = 0.03188$

$$x_2 = \frac{x_0 + x_1}{2} = 0.65$$

$y_2 = -0.03921$

$$x_3 = \frac{x_2 + x_1}{2}$$

$= 0.675$

$y_3 = -0.00293$

$$x_4 = \frac{x_3 + x_1}{2}$$

$= 0.6875$

$y_4 = 0.01465$

$$x_5 = \frac{x_4 + x_3}{2} = 0.68125$$

$y_5 = 0.0059$