# PROBABILITY AND STATISTICS

Ques 1.

(i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Mean = $\frac{\text{sum of all obs.}}{\text{No. of all obs.}}$

$= \frac{134}{10} = 13.4$

Median = $\frac{n+1^{th}\text{ term}}{2} = \frac{11}{2} = 5.5^{th}\text{ term}$

$= \frac{11+15}{2} = \frac{26}{2} = \boxed{13}$

Mode = 18

$Q1 = \frac{N+1^{th}}{4} = \frac{11}{4} = 2.75^{th}\text{ term}$

$= 7 + 0.75(2)$

$= 7 + 1.5$

$= 8.5$

$Q3 = 3\left(\frac{N+1}{4}\right) = \frac{33}{4} = 8.25^{th}\text{ term}$

$= 18 + 0.25(0)$

$= 18$

$IQR = Q3 - Q1 = 9.5$

$dx = x - \bar{x}$

| x | 4 | 7 | 9 | 9 | 11 | 15 | 18 | 18 | 18 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|
| $dx$ | -9.4 | -6.4 | -4.4 | -4.4 | -2.4 | 1.6 | 4.6 | 4.6 | 4.6 | 11.6 |
| | 88.36 | 40.96 | 19.36 | 19.36 | 5.76 | 2.56 | 21.16 | 21.16 | 21.16 | 134.56 |

Standard deviation $= \sqrt{\frac{dx^2}{N}}$

$= \sqrt{\frac{374.4}{10}}$

$= \sqrt{37.44}$

$= 6.118823416$

(ii)

| $x_i$ | $f_i$ | $x_i f_i$ | $dx$ $(x_i - \bar{x})$ | $dx^2$ | $f dx^2$ |
|---|---|---|---|---|---|
| 0 | 5 | 0 | -2 | 4 | 20 |
| 1 | 11 | 11 | -1 | 1 | 11 |
| 2 | 26 | 52 | 0 | 0 | 0 |
| 3 | 15 | 45 | 1 | 1 | 15 |
| 4 | 3 | 12 | 2 | 4 | 12 |
| | 60 | 120 | | | 58 |

$\bar{X} = \frac{\sum x_i f_i}{f_i}$ $\sigma = \sqrt{\frac{f dx^2}{N}} = \sqrt{\frac{58}{60}}$

$= \frac{120}{60}$ $= \sqrt{\frac{483}{60}}$

$= 2$ $= 2.837225$

$= 0.983192$

| Xi | fi | cf |
|---|---|---|
| 0 | 5 | 5 |
| 1 | 11 | 16 |
| 2 | 26 | 42 |
| 3 | 15 | 57 |
| 4 | 3 | 60 |

$$\text{Median} = \frac{N+1}{2} = \frac{61}{2} = 30.5^{th} \text{ term}$$

$$\text{Median} = 2$$

$$Q1 = \frac{N+1}{4} = \frac{61}{4} = 15.25^{th} \text{ term}$$

$$Q = 1$$

$$Q3 = \frac{3 \times 61}{4} = 45.75$$

$$Q3 = 3$$

$$IQR = Q3 - Q1 = 2$$

$$\text{mode} = 2$$

Ques 2) $\lambda = 0.5$

claims $= 0$

(i) $P(0) = \frac{e^{-0.5}(0.5)^0}{0!}$

$= 0.606530660$

(ii)

(iii)

Ques 3 $\alpha = 2$ $\lambda = 1/2$

$E(X) = \alpha/\lambda = 2/1/2 = 4$

Standard deviation $= \frac{\sqrt{\alpha}}{\lambda} = 2\sqrt{2}$

$= 2.8284271$

(ii) Shape is rightly skewed.

(iii) $F_X(x) = 1 - e^{-\frac{x}{2}}$

Ques 4) ⇒ MALES = 10 QUALIFIED = 6
FEMALE = 8 QUALIFIED = 7

P(males and qualified) = 6/10

P(Female and qualified) = 7/8

$1 - \frac{{}^{10}C_2}{{}^{18}C_2} \times \frac{{}^{8}C_2}{{}^{18}C_2}$

$P = \frac{7}{8} \times \frac{8}{18} \times \frac{7}{8}$

$= \frac{49}{144} = 0.125$

Ques 5 ⇒

Level 1 ⇒ P(delivery) = 0.4 P(late) = 0.2
Level 2 ⇒ " = 0.5 P(late) = 0.4
Level 3 ⇒ " = 0.1 P(late) = 0.1

$P\left(\frac{\text{Level 2}}{\text{Late}}\right) = \frac{P\left(\frac{\text{late}}{\text{Level 2}}\right) \times P(\text{Level 2})}{P\left(\frac{L}{L_2}\right) \times P(L_2) + P\left(\frac{L}{L_1}\right) \times P(L_1) + P\left(\frac{L}{L_3}\right) \times P(L_3)}$

$= \frac{0.4 \times 0.5}{0.4 \times 0.5 + 0.4 \times 0.2 + 0.1 \times 0.1}$

$$= \frac{0.20}{0.20+0.08+0.01}$$

$$= \frac{0.20}{0.29} = \frac{20}{29}$$

$$= 0.68965517$$

Ques 6

$X \rightarrow U(1,2)$ $\quad Y = \frac{3}{6-X}$

$a$ $\quad$ $b$

$a = 1 \quad , b = 2$

$$f_X(x) = \frac{1}{b-a} = \frac{1}{}$$

$$Y = \frac{3}{6-x}$$

$$6 - x = \frac{3}{Y}$$

$$6 - \frac{3}{Y} = x$$

diff w.r.t y.

$$\frac{3}{Y^2} = \frac{dx}{dy} = \frac{d}{dy}(w^{-1}(y))$$

$$f_Y(y) = f_Y(w^{-1}(y)) \left| \frac{d}{dy}(w^{-1}(y)) \right|$$

$$f_Y(y) = 1 \times \frac{3}{y^2}$$

$$f_Y(y) = 3y^{-2}$$

Ques 7 ⇒

| $X =$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X=x) =$ | 0.05 | 0.15 | 0.35 | 0.4 | 0.05 |

(i) $F(3) = P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

(ii) $E(X) = \sum_{1}^{5} x \cdot P(X=x)$

$= 0.05 + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)!$

$= 0.05 + 0.30 + 1.05 + 1.6 + 0.25$

$= 0.6 + 1.6 + 1.05$

$= 3.25$

iii $V(X) = E(X^2) - (E(X))^2$

$= 11.45 - (3.25)^2$

$= 11.45 - 10.5625 = 0.8875$

$E(X^2) = 0.05 + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$

$= 0.05 + 0.60 + 3.15 + 6.4 + 1.25$

$= 11.45$

iv Skewness $= E(x_i - \bar{X})^3$

| | | | | | |
|---|---|---|---|---|---|
| | | | | | $= X$ |
| | | | | | $= (X - \bar{X})^3$ |

Ques 8 ⇒

$P$(claim exceeding 1000) $= 0.2$

No of claims $= 15 = n$

$x = 3$

$= {}^{15}C_3 (0.2)^3 (0.8)^{12}$

$= \frac{15!}{3!\,12!} (0.2)^3 (0.8)^{12}$

$= 0.250138895$

Ques 9 ⇒ $P(W) = 0.01$ $P(L) = 0.99$

$n = 7$ No. of winning = 3

$$P(X=3) = {}^{7}C_{3} (0.01)^3 (0.99)^4$$
$$= 0.00003362$$

Ques 10 ⇒ $\frac{2}{5}$

Total no of working days in a year = 5×52
= 260

P(more than two claim) = 1 − P(less than 2 claim)
= 1 − P(X ≤ 2)
= 1 − P(X=2
n=5 p=0.4
= 1 − 0.6826
= 0.3174.

Ques 11

$$P(M) = P(F) = 1/2$$

Boys = 5

Girls = 2

P(All five children are boys) = B × B × B

$= 1/2 \times 1/2 \times 1/2 = 1/8$

Ques 12

$\lambda = 10$

$x = 2$

$$\frac{e^{-\lambda} (\lambda)^x}{x!}$$

$$\frac{e^{-10} (10)^2}{2!}$$

$= 0.00226\ 9996$

Ques) 3

$a = 75$ and $b = 120$

$f_X(x) = \frac{1}{b-a}$

$F_X(x) = \frac{x - 75}{45}$

Prob that x lies bet 80 and 100

$F_X(80 \le x \le 100) = \left[\frac{x-75}{45}\right]_{80}^{100}$

$= \left[\frac{25}{45} - \left[\frac{80-75}{45}\right]\right]$

$= \frac{25}{45} - \frac{5}{45}$

$= \frac{20}{45} \quad \frac{4}{9}$

$= 0.4444444.$

Ques 14

$\alpha$ ↑ , $\lambda$ ↑

$X \sim$ Gamma $(5, 0.4)$ Calculate $P(X < 22.89)$

$$P(X < 22.89) = 1 - P(X > 22.89)$$

$$f_X(22.89) = \frac{(0.4)^5 \, e^{-0.4(22.89)} \, (22.89)^4}{4!}$$

$$= 0.012367206$$

$$P(X < 22.89) = 1 - 0.012367206$$

$$= 0.98763279 4$$

Ques 15

$$\int_0^\infty \frac{k}{(x+5)^4} \, dx = 1$$

$$k \int_0^\infty \frac{1}{(x+5)^4} \, dx = 1$$

Let $x + 5 = t$

$$dx = dt$$

$$\frac{1}{} = \frac{dt}{dx}$$

$$K \int_5^{\infty} \frac{1}{t^4}\, dt = 1$$

$$K \int_5^{\infty} t^{-4}\, dt = 1$$

$$K \left[\frac{t^{-4+1}}{-4+1}\right]_5^{\infty} = 1.$$

$$\frac{-K}{3} \left[t^{-3}\right]_5^{\infty} = 1$$

$$\frac{-K}{3} \left[\frac{1}{(x+5)^3}\right]_0^{\infty} = 1$$

$$\frac{-K}{3} \left[\frac{1}{(\infty)^3} - \frac{1}{(5)^3}\right] = 1$$

$$\frac{K}{3} \left(+\frac{1}{(5)^3}\right) = 1$$

$$K = \frac{(5)^3}{3}$$

$$K = \frac{125 \times 3}{3}$$

375

$F(10)$

$$-\frac{375}{3}\left[\frac{1}{(x+5)^3}\right]_0^{10}$$

$$-125\left[\frac{1}{(15)^3}-\frac{1}{(5)^3}\right]$$

$$=0.96296296$$

(iii) $E(X) = \int x \cdot \frac{375}{(x+5)^4}\,dx$

$$= 375\int \frac{x}{(x+5)^4}\,dx$$

$x+5=t$

$dx=dt$

$$= 375\int (t-5)\,t^{-4}\,dt$$

$$= 375\int (t^{-3}-5t^{-4})\,dt$$

$$= 375\left[\frac{t^{-2}}{(-2)}-\frac{5t^{-3}}{(-3)}\right]+C$$

$$= 375\left[\frac{5}{3}\times\frac{1}{(x+5)^3} - \frac{1}{2}\times\frac{1}{(x+5)^2}\right] + c$$

$$= \frac{375}{(x+5)^2}\left[\frac{5}{3(x+5)} - \frac{1}{2}\right] + c$$