1) $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$\lim_{h \to 0} \frac{2(9+2(-3)(h)+h^2) - 18}{h}$

$\lim_{h \to 0} \frac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$

$\lim_{h \to 0} = 2h - 12$

$= 2(0) - 12$

$= -12$

2) $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

(a) $x = 4$

$\therefore gx = 2x \quad x < 6$

$= x-1 \quad x \geq 6$

$\lim_{x \to 4^-} 2x$

$2(4)$

$\cancel{\lim_{x \to 4^-}} = 8 \longrightarrow 1$

$\lim_{x \to 4^+} 2x$

$2(4)$

$\cancel{\lim} = 8 \longrightarrow 2$

$g(4) = 2(4)$

$= 8 \longrightarrow 3$

from 1, 2, 3

$g(x) = 2x \quad x < 6$

$= x-1 \quad x \geq 6$

is continuous at $x = 4$

(b) $x = 6$

$$g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

$\lim\limits_{x \to 6^-} 2(x)$

$2(6)$

$12 \longrightarrow ①$

$\lim\limits_{x \to 6^+} x - 1$

$6 - 1$

$5 \longrightarrow ②$

From ① and ②

~~Limit does~~ $\because \lim\limits_{x \to 6^-} \neq \lim\limits_{x \to 6^+}$

$\therefore$ limit does not exist.

3) $\lim\limits_{x \to 9} \dfrac{x - 9}{3 - \sqrt{x}}$

$\lim\limits_{x \to 9} \dfrac{(\sqrt{x} + 3)\cancel{(\sqrt{x} - 3)}}{-\cancel{(\sqrt{x} - 3)}}$

$\lim\limits_{x \to 9} -(\sqrt{x} + 3)$

$3 + 3$

6

4) (a) $x = -1$

$f(x) = \frac{4x+5}{9-3x}$

$f(-1) = \frac{4(-1)+5}{9-3(-1)}$

$= \frac{-4+5}{9+3}$

$= \frac{5}{12}$ → It is continuous at $y = \frac{5}{12}$

(b) $x = 0$

$g(x) = \frac{4x+5}{9-3x}$

$= \frac{4(0)+5}{9-3(0)}$

$= \frac{5}{9}$ → It is continuous at $y = \frac{5}{9}$

(c) $x = 3$

$f(x) = \frac{4x+5}{9-3x}$

$= \frac{4(3)+5}{9-9}$

$= \frac{17}{0}$

$= \infty^{+}$ → It is continuous at $y = \infty^{+}$

5) $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$$\dfrac{6e^{4x} - \dfrac{1}{e^{2x}}}{8e^{4x} - e^{2x} + \dfrac{3}{e^{x}}}$$

$$\dfrac{6e^{8x^2} - 1}{e^{2x}}$$

5) $\lim_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$ $\quad \because \dfrac{a^{a}}{a^{b}} = e^{}$ (indices)

$$\lim_{x \to \infty} \dfrac{\cancel{e^{4x}}(6 - e^{-6x})}{\cancel{e^{4x}}(8 - e^{-2x} + 3e^{-5x})}$$

$$\lim_{x \to \infty} \left(\dfrac{6 - \dfrac{1}{e^{6x}}}{8 - \dfrac{1}{e^{2x}} + \dfrac{3}{e^{5x}}} \right)$$

$$= \dfrac{6 - \dfrac{1}{\infty}}{8 - \dfrac{1}{\infty} + \dfrac{3}{\infty}}$$

$$= \dfrac{6}{8} \rightarrow \because \dfrac{1}{\infty} = 0.$$

$$= \dfrac{3}{4}.$$

5) $g(y) = y^2+5$ if $y < -2$
$= 1-3y$ if $y \geq -2$

(a) $\lim_{y \to 6} g(y)$

$\lim_{y \to 6^+} = \lim_{y \to 6^-} = 1-3y$

$= 1-3(6)$

$= 1-18$

$= -17$

(b) $\lim_{y \to -2^-} g(y)$

y^2+5

$(-2)^2+5$

$4+5$

$9 \to (1)$

$\lim_{y \to -2^+}$ $\cancel{y^2+5}$ $1-3y$

$1+6$

$7 \to (1)$

from (1) and (2)

$\lim_{y \to -2^-} \neq \lim_{y \to -2^+}$

7) $f(t) = (4t^2-t)(t^3-8t^2+12)$

$= (4t^2-t)\frac{d}{dt} t^3-8t^2+12 +$

$(t^3-8t^2+12)\frac{d}{dt} 4t^2-t$

$= (4t^2-t)\cdot(3t^2-16t)+(t^3-8t^2+12)(8t-1)$

$= 12t^4+64t^3-3t^3+16t^2+$

$-t^3+8t^2-\cancel{12}+8t^4-64t^3$

$f'(t) = 20t^4-2t^3+24t^2$

8) $R(\omega) = \dfrac{3\omega + \omega^4}{3\omega^2 + 1}$

diff w.r.t $x(\omega)$

$$R'(\omega) = \frac{(3\omega^2+1) \cdot \frac{d}{d\omega}(3\omega + \omega^4) - (3\omega + \omega^4)\frac{d}{d\omega}(3\omega^2+1)}{(3\omega^2+1)^2}$$

$$= \frac{(3\omega^2+1).(3 + 4\omega^3) - (3\omega + \omega^4)(6\omega)}{(3\omega^2+1)^2}$$

$$= \frac{9\omega^2 + 3 + 12\omega^5 + 4\omega^3 - 18\omega^2 - 6\omega^5}{(3\omega^2+1)^2}$$

$$R'(\omega) = \frac{+6\omega^5 + 4\omega^3 - 9\omega^2 + 3}{(3\omega^2+1)^2}$$

9) $f(x) = 2e^x - 8^x$

diff w.r.t x

$f'(x) = 2e^x - 8^x \log 8$

10) $y = z^5 - e^z \ln z$

taking log on both side

$\log y = 5.\log z - \log(e^z . \log z)$

$$y = z^5 - e^z \ln z$$

$$= z^5 - e^z \log_{10} z$$

$$= z^5 - e^z \frac{\log z}{\log 10}$$

diff w.r.t z

$$\frac{dy}{dz} = 5z^4 - \frac{e^z}{\log 10}\frac{1}{z} + \frac{\log z}{\log 10} e^z$$

$$= 5z^4 - \frac{e^z}{\log 10}\left(\frac{1}{z} - \log z\right)$$

(1) a $f(x) = (6x^2+7x)^4$

diff w.r.t x

$f'(x) = 4(6x^2+7x)^3 \cdot \frac{d}{dx} 6x^2+7x$

$= 4(6x^2+7x)^3 \cdot (12x+7)$

b $f(t) = 5+e^{4t+t^7}$

diff w.r.t t

$f'(t) = 0+e^{4t+t^7} \cdot \frac{d}{dt} 4t+t^7$

$= (e^{4t+t^7}) \cdot (4+7t^6)$

(4) a

$z = \ln(7-x^3)$

$z = \log(7-x^3)$

diff w.r.t x

$\frac{dz}{dx} = \frac{1}{(7-x^3)} \cdot -3x^2$

b $Q(v) = \frac{2}{(6+2v-v^2)^4}$

dif

$Qv = 2(6+2v-v^2)^{-4}$

diff w.r.t v

$Q'v = 2 \cdot -4(6+2v-v^2)^{-5} \cdot (2-2v)$

$= -8(2-2v) \cdot (6+2v-v^2)^{-5}$

$= -16(1-v) \cdot (6+2v-v^2)$

c $f(t) = \ln(1+t^2)$

Diff w.r.t t

$f'(t) = \frac{1}{1+t^2} \cdot 2t$.

10) $y = z^5 - e^z \ln(z)$

$= z^5 - e^z \log_e z$

$= z^5 - e^z \frac{\log z}{\log e}$ → change of base

$= z^5 - e^z \cdot \log z$ → $\log e = 1$

diff w.r.t z

$\frac{dy}{dz} = 5z^4 - \{e^z \frac{1}{z} + \log z \cdot e^z$

$= 5z^4 - e^z \left(\frac{1}{z} + \log z\right)$

13 $x^3 - 3x^2 - 9x + 12$

let $f(x) = x^3 - 3x^2 - 9x + 12$

diff w.r.t x

$f'x = 3x^2 - 6x - 9$

let's Assume

$f'(x) = 0$

$\therefore 0 = 3x^2 - 6x - 9$

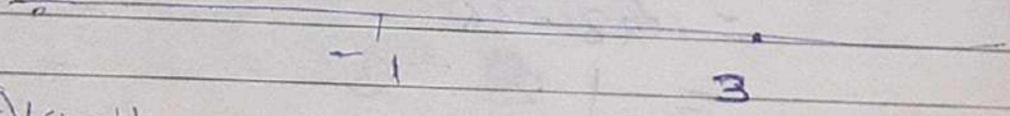
$\therefore 0 = x^2 - 2x - 3$

$\therefore 0 = x^2 - 3x + x - 3$

$0 = x(x-3) + 1(x-3)$

$(x-3) = 0$; $(x+1) = 0$

$\boxed{x = 3}$; $\boxed{x = -1}$



Now

$\therefore f'x = 3x^2 - 6x - 9$

Diff w.r.t x

$\therefore f''x = 6x - 6$ → ①

Now sub $x = 3$ in ①, Now sub x -1

$f''(3) = 18 - 6$

$= 12$

$= +ve$ → ②

$f''(-1) = 6(-1) - 6$

$= -12$

$= -ve$ → ③

from ② and ③ when $x = -1$ &
f(x) is maximum when $x = 3$.
fx is minimum

14.) $4x^3 - 18x^2 + 24x - 7$.

let $F(x) = 4x^3 - 18x^2 + 24x - 7$
diff w.r.t x.
$f'(x) = 12x^2 - 36x + 24$
Now consider
$f'(x) = 0$
$0 = 12x^2 - 36x + 24$
$0 = x^2 - 3x + 2$
$0 = x^2 - 2x - 1x + 2$
$0 = x(x-2) - 1(x-2)$
$(x-2) = 0$; $(x-1) = 0$
$\boxed{x = 2}$ $\boxed{x = 1}$

Now
$\because f'(x) = 12x^2 - 36x + 24$
Diff w.r.t x
$f''(x) = 24x - 36 \rightarrow$ ①
Sub $x = 2$ in ① ; Sub $x = 1$ in ①
$\therefore f''x = 24(2) - 36$; $f''(x) = 24(1) - 36$
$= 48 - 36$; $= 24 - 36$
$= 12 \rightarrow$ ② ; $= -12 \rightarrow$ ③
from ② and ③
f(x) is maximum when $x = -12$ &
f(x) is minimum when $x = 12$.

(d) $f(x) = 3x^2 - 6x$

$y = 3x^2 - 6x \rightarrow$

Now

Put $x = 0$ in $f(x)$

$y = 3(0)^2 - 6(0) =$

$y = 0$

put $x = 1$ in $f(x)$

$\therefore y = 3(1)^2 - 6(1)$

$= 3 - 6 = -3$

put $x = -1$ in $f(x)$

$y = 3(-1)^2 - 6(-1)$

$= 3 + 6$

$= 9$

put $x = 2$ in $f(x)$

$\therefore y = 3(2)^2 - 6(2)$

$= 12 - 12$

$= 0$

~~put~~

put $x = -2$ in $f x$

$y = 3(-2)^2 - 6(-2)$

$= 3 \times 4 + 12$

$= 24$

put $x = 3$ in $f(x)$

$y = 3(3)^2 - 6(3)$

$= 3 \times 9 - 18$

$= 27 - 18$

$y = 9$

put $x = -3$ in $f(x)$

$y = 3(-3)^2 - 6(-3)$

$= 3 \times 9 + 18$

$= 27 + 18$

$= 45$

x	y
0	0
1	-3
-1	9
2	0
-2	24
3	9
-3	45

P $\quad f'x = 0$

$\therefore 3x^2 + 6x = 0$

$= 3x(x+2) = 0$

$\therefore 3x = 0 \; ; \; x+2 = 0$

$x = 0 \; ; \; x = -2$

$\therefore$ The given function is maximum at $x=$ and minimum at $x=0$

Put $x=-1$ in $f'(x)$

$f'(-1) = 3(-1)^2 + 6(-1)$

$= 3 - 6$

$= -3$ which is < 0

$\therefore$ The given function is decreasing between -2 and 0

When you put $x=0$ in fx:

$f(0) = 0(0+3) = 0$

$\therefore x = 0, y = 0$

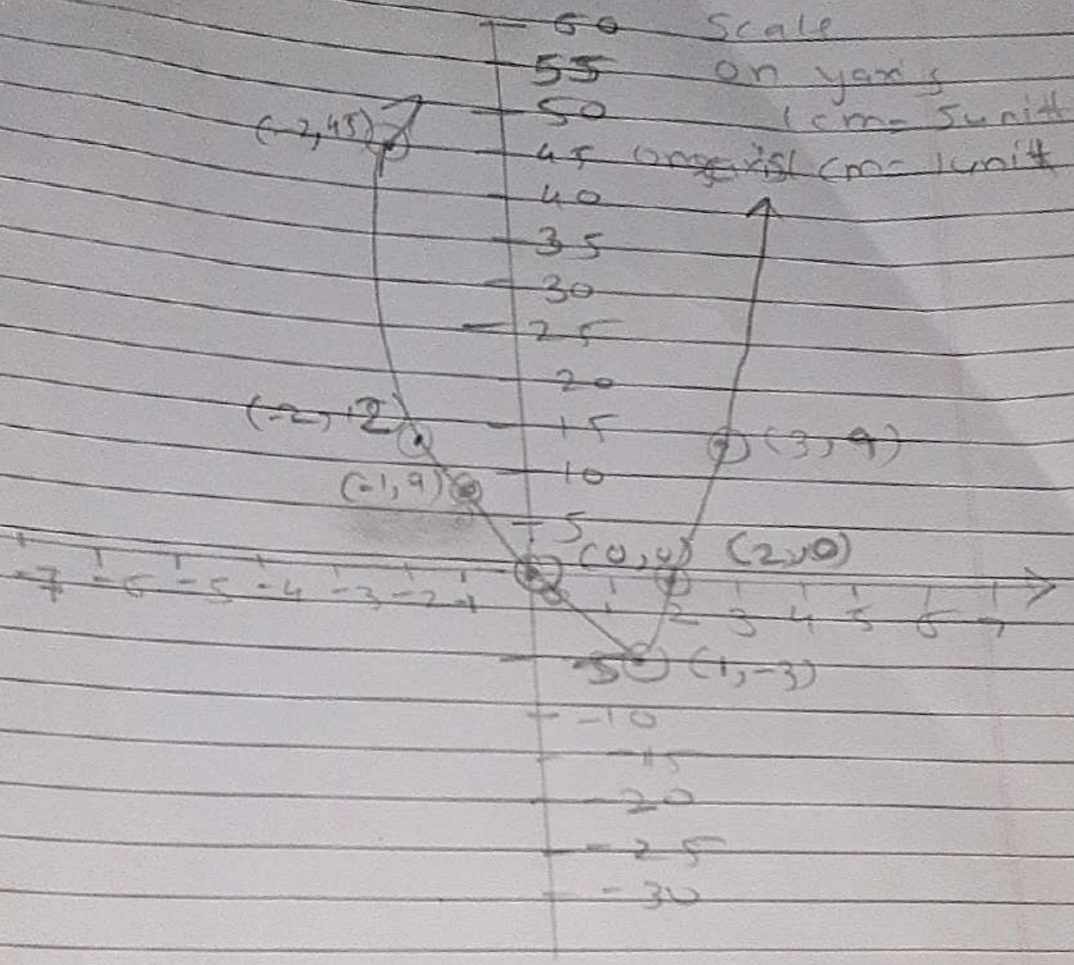
Put $x=-2$ in fx.

$\therefore$ maximum value $f(2) = (-2)^2(-2+3)$

$= 4 \times 1$

$= 4$

$\therefore x = -2, y = 4$



Scale
on y axis
1cm = 5 unit
on x axis 1cm = 1 unit
60
55
50
45
40
35
30
25
20
15
10
5
-5
-10
-15
-20
-25
-30
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
(-2,45)
(-2,12)
(-1,9)
(0,0)
(2,0)
(1,-3)
(3,9)