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Homogeneous D.E

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

$$v = \frac{y}{x}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$f(v) = v + x\frac{dv}{dx}$$

$$\frac{dx}{dy} = f\left(\frac{x}{y}\right)$$

$$v = \frac{x}{y}$$

$$x = vy$$

$$\frac{dx}{dy} = v \cdot 1 + y\frac{dv}{dy}$$

$$f(x) = v + y\frac{dv}{dy}$$

Q $$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{y}{2x} - \frac{x}{2y}$$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{y}{x} - \frac{1}{2}\cdot\frac{1}{\frac{y}{x}} \quad - (1)$$

$$v = \frac{x}{y}, \quad y = vx, \quad \frac{dy}{dx} = v + x\frac{dv}{dx} \quad - (2)$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\cdot v - \frac{1}{2}\cdot\frac{1}{v}$$

$$x\frac{dv}{dx} = \frac{v}{2} - \frac{1}{2v} - v$$

$$x\frac{dv}{dx} = -\left(\frac{v}{2} + \frac{1}{2v}\right)$$

$$= -\frac{1}{2}\left(v + \frac{1}{v}\right)$$

$$= -\frac{1}{2}\left(\frac{v^2 + 1}{v}\right)$$

$$x\frac{dv}{dx} = -\left(\frac{v^2+1}{2v}\right)$$

$$\int \frac{2v}{v^2+1}\,dv = \int -\frac{dx}{x}$$

$$\log(v^2+1) = -\log x + \log c$$

$$\log\left(\frac{y^2}{x^2}+1\right) + \log x = \log c$$

$$\log\left[\frac{y^2+x^2}{x^2}\cdot x\right] = \log c$$

$$\frac{y^2+x^2}{x} = c$$

$$y^2+x^2 = c\cdot x$$

2 Q) $x^2\frac{dy}{dx} = x^2 + xy + y^2$

$$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$$

$$= 1 + \frac{y}{x} + \frac{y^2}{x^2} \quad - (1)$$

$$v = \frac{y}{x}, \quad y = vx, \quad \frac{dy}{dx} = v + x\frac{dv}{dx} \quad - (2)$$

$$v + x\frac{dv}{dx} = 1 + v + v^2$$

$$x\frac{dv}{dx} = 1 + v^2$$

$$\int \frac{1}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\tan^{-1} v = \log x + c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$$

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Ear example

$\log x + \log y = c$

if c has log

$\log x.y = \log c$

$x.y = c$

Assignment - 2

1. $\int \frac{e^{\frac{1}{w}}}{w^2} = \int e^{\frac{1}{w}} \cdot \frac{1}{w^2} \, dx$

let $u = \frac{1}{w^2} \rightarrow w^{-2}$, $du = -x^{-3} dx$

$= -\frac{1}{x^2} dx$

$= -x$

$du = -w^{-3} dx$

$-w^2 du = dw$

let $\pi = \frac{1}{w^2} = w^{-2}$

$\int e^u \cdot \frac{1}{w^2} \cdot -w^2 du$

$d\pi = -2w^{-3} dw$

$-2w^3 d\pi = dw$

$-\int e^u + c$

$[e^1 - e^{\frac{1}{50}}] + c$

0.36885

2. $\int \frac{2t^3+1}{(t^4+2t)^3}$

$\int \frac{2t^3+1}{t^{12}+2t^3+3t^{11}6t^5(t^4+2t)}$

let $u = t^4+2t$

$du = 3t^3+2\,dt$ $\quad du = 4t^3+2$

$\int \frac{3t^3+2-t^3-1}{(t^4+2t)^3}\,dt$

$\int \frac{du}{u^3} - \frac{t^3-1}{(t^4+2t)^3}$

2

$\frac{1}{2}\int \frac{4t^3+12}{(t^4+2t)^3}$

$\frac{1}{2}\int \frac{du}{t^3}$

$\frac{1}{2}\log. 3\log t + C$

3. $f'(x) = x^4 + 3x - 9$

$f(x) = 4x^5 + \frac{3x^2}{2} - 36x$

$f'(x) = 20x^4 + \frac{12}{2}x - 36$

$=$

3. $f'(x) = x^4 + 3x - 9$

4. $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx$$

using $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

$$\int_{-2}^{1} 3x^2\,dx + \int_1^3 6$$

$$\left[\frac{3x^3}{3}\right]_{-2}^{1} + [6x]_1^3$$

$$[1 - (-8)] + [18 - 6]$$

$$[9] + [12]$$

$$21$$

5. $\int 3.(8y-1).e^{4y^2-y}\,dy$

let $u = 4y^2 - y$

$du = 8y - 1\,dy$

$3\int du\,e^u$

$3.e^u + c$

$3e^{4y^2-y} + c$

3. $\frac{dx}{dt} \propto \sqrt{x}$

$\frac{dx}{dt} = -k\sqrt{x}$

$\int \frac{dx}{\sqrt{x}} = \int -k\,dt$

$2\sqrt{x} = -kt + c$

$t = 0 \qquad t = 2$

$x = 10^6 \qquad x = 10^4$

$2\sqrt{10^6} = -k(0) + c$

$c = 2 \times 10^3$

$2\sqrt{10^4} = -k(2) + 2 \times 10^3$

$10^2 = -k + 10^3\sqrt{10}$

$k = 1000 - 100 = 900$

$2\sqrt{x} = -900t + (2 \times 10^3)$

$2\sqrt{x} = -900\left(\frac{8}{3}\right) + 2 \times 10^3$

$\sqrt{x} = -300(7) + 1000 = -200$

$x = 40000$

Question is wrong

Show: $t = \frac{8}{3}$, $x = 0$

$t = 2\frac{2}{9}$

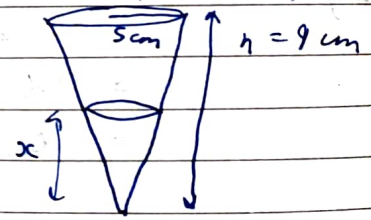
4. $h = 9\text{ cm}$, $r = 5\text{ cm}$

$\frac{dx}{dt} = \frac{\pi}{A}$

$= \frac{\pi}{\pi r^2}$

$= \frac{1}{r^2}$

$r^2\,dx = dt$



$\frac{9}{x} = \frac{5}{r}$

$r = \frac{5}{9}x$