classmate
Date
Page

SRM Assignment
Unit 1 & 2

Q1. $_u q_x = P[T_x \le u]$

$= P[K_x = 0 \text{ and } S_x \le u]$
$= P[K_x = 0] * P[S_x \le u] \Rightarrow P[K_x = 0] = q_x$
As K_x & S_x are independent.

$P[S_x \le u] = \int_0^u 1\,dx = u$, ∵ Uniform distn

$_u q_x = u \cdot q_x$

Q2. a) Period of exposure is 1/6/2000 to 25/10/2000
$= 30 + 31 + 31 + 30 + 25 = 147$ days
$= 147/7 = 21$ weeks

b) Period of exposure is 1/6/2000 to 31/5/2001
∴ As this is exactly a year, there are 52 weeks

Q3.
i) Left Data will be right censored if observations are lost in progress, so that we are not able to see if policy is surrendered. Also if policy is terminated in middle of investigation then that data will be right censored.

ii.

Date________ Page________

i) Left censoring: If we dont know when the policy holder joined the company then the data will be left censored.

) Right censoring: Data will be right censored if observation are lost in progress, so that we are not able to see if policy is surrendered. Also if policy is terminated in middle of investigation then that data will be right censored.

Interval censoring: If we only given callender year of surrender then data is interval censored.

Complete expectation of life $\mathring{e}_x$

$$\mathring{e}_x = E[T_x] = \int_0^{\omega-x} {}_tp_x \, dt$$

This represent probabilities of survival at each future age.

) The curate expectation of life.

$$e_x = \sum_{K=1}^{\infty} {}_Kp_x = \sum_{K=1}^{\infty} e^{-0.0325K}$$

$$= \frac{e^{-0.0325}}{1-e^{-0.0325}} = 30.27$$

P of life aged 36 will survive to age 45

$${}_9p_{36} = e^{\left[-\int_0^9 0.0325\,dt\right]} = e^{-0.2925} = 0.7464 = 75\%$$

iv) A exact age x representing median of life time T of new born.

i.e $_xP_0 = 0.5$

$\therefore {}_xP_0 = 0.5 \Rightarrow e^{(x\epsilon 0.0325)} = 0.5$

$$\Rightarrow x = \frac{-\log 0.5}{0.0325} = 21.33$$

Q5.

i) Gompertz law is suitable model for human mortality for middle to older ages say 35 and older

ii) As $_tP_x = e^{\left(-\int_0^t \mu_{x+s} ds\right)}$

$\because \mu_x = Bc^x$

$_tP_x = e^{\left(\int_0^t Bc^{x+s} ds\right)}$

we can write $c^{x+s} = c^x e^{s \log c}$

$\therefore {}_tP_x = e^{\left[-\int_0^t Bc^x \cdot e^{s\log c} ds\right]}$

$$= \frac{Bc^x}{\log c}\left[e^{s \log c}\right]_0^t$$

$$_tP_x = \frac{Bc^x}{\log c}\left[c^s\right]_0^t$$

$$= \frac{Bc^x}{\log c}(c^t - 1)$$

$\therefore\ {}_tP_x = g c^x (c^t - 1)$

ii). Female smokers aged 30 at entry

$$\frac{h_j(t)}{h_i(t)} = \frac{e^{(-0.4)}}{e^{(0.1)}} = 0.60653066$$

j is male smoker aged 30 at entry
i is female smoker aged 30 at entry

As, $s(t) = e^{(-\int_0^t h(s)\,ds)}$

$S_j(t) = [S_i(t)]^{0.2607}$

which shows $S_j(t) > S_i(t)$ for all $t > 0$.

iii) $h_j(t) = \dfrac{e^{(0.2)}}{e^{(0.1)}} = 1.1051$

j is male smoker aged 30 at entry
i is female smoker aged 40 at entry

As, $s(t) = \int_0^t h(s)\,ds$

$\therefore\ S_j(t) = [S_i(t)]^{1.1051}$

$\therefore\ S_j(t) < S_i(t)$ for all $t > 0$.

Q7

i) The most appropriate rate interval to use is Policy years rate interval starting on Policy anniversary where lives are aged x next bday.

The average age at start of rate interval is $x - 1/2$ assuming that birthday are uniformly distributed.

ii) $P_x(t) = 1/2\,(P_{x,t} + P_{x+1,t})$

we assume that Policy anniversary are uniformly distributed over the calendar year.

$E_x^c = \int_0^{10} P_x(t)\,dt = \frac{1}{2}\sum_{t=0}^{9} P_x(t) + P_x(t+1)$

Now assuming Population varies linearly b/w of investigation.

$\underline{E}_x = E_x^c + 1/2 \sum_{t=0}^{6} x_t$

Q8.

i) a) Type I censoring

b) Type II censoring

c) Random censoring

ii) The censoring will be informative. The patients who died were probably recovering less well that patient who discharged from hospital classmate

Date ______
Page ______

i) Keplan-Meier estimate of survival function is estimated as follows.

T	n	d	c	d/n	(1-d)/n	s(t)
0	13					
5	13	1	0	0.0769	0.9231	0.92
7	12	1	0	0.0833	0.9167	0.85
14	11	1	2	0.0909	0.9091	0.77
28	8	1	2	0.1250	0.8750	0.67
33	5	1		0.2	0.8000	0.54

∴ The value ~~of~~ of survival function at end of investigation period is 0.54.

9. a) as we name

$$\int_0^1 {}_tp_x \, dt = \int_0^1 (1 - {}_tp_x)\, dt = \left[t - 0.5t^2 q_x\right]_0^1$$

$$= 1 - 0.5 q_x$$

As $q_x = 0.3$, $m_x = \frac{0.3}{1-0.15} = 0.353$

) $q_x = 1 - e^{-\mu}$

$$\int_0^1 {}_tp_x \, dt = \int_0^1 e^{-\mu t} dt = \frac{1}{\mu}\left(1 - e^{-\mu}\right) = q_x/\mu.$$

$\therefore m_x = \mu \ln(1 - q_x) = -\ln 0.7 = 0.3516675$

Q10.

i) For cox model, it is proportional to baseline hazard and constant of proportionality depending on certain measurable quantities called as Cox proportional hazard.

ii) Baseline hazard is annual policy taken through online channel & where premiums are paid by debit.

iii) We have

$$e^{[(B_D \cdot 1)]} / e^{[(B_D \cdot 1) + B_F \cdot 1 + B_m \cdot 1]} = 0.75$$

$\therefore e^{[B_F + B_m]} = 4/3$ — ①

$e^{[(B_D \cdot 1)]} / e^{(B_F \cdot 1)} = \cancel{4/3}\ 1$ — ②

$e^{[B_m \cdot 1]} / e^{[B_D \cdot 2]} = 0.75$ -- ③

from ② & ①

$e^{(B_D + B_m)} = 4/3$

$e^{(B_D)} * e^{(B_m)} = 4/3$

from ③

$e^{(B_D)^2} \cdot 0.75 = e^{(B_M)}$

$\therefore e^{(B_D)} \cdot e^{(B_D)^2} \times 0.75 = 4/3$

$e^{(B_D)^3} = 1.77.78$

$e^{(B_0)} = 1.2114$

$BD = 0.19179$

$BF = 0.19179$

$Bm = 0.0959$

t	$S(t)$	$\lambda(t)$	n_t	d_t	c_t
0	1	0	12	0	
1	0.9167	0.0833	12	1	2
3	0.7130	0.22	9	2	2
6	0.4278	0.4	5	2	3

2 insects died at duration 3 weeks & 2 insects dies at dermation 6 weeks.

ii) Suming all deaths we have = 5.
∴ we stresed with 12, remaining 7 are right censored

Ans.

i) Gromperty law is.
$\lambda_x = Bc^x$.
Where λ_x = Force of mortality.

ii) · Substituting $B = e^{(B_0 + B_1 x_1 + B_3 x_2)}$

$\lambda_x = e^{(B_0 + B_1 x_1 + B_2 x_2)} c^x$ => x is duration since 50th bday.

∴ c^x is basline which depend on time.
and $e^{(B_0 + B_1 x_1 + B_2 x_2)}$ is depend on covarciates.
So ratio does not depend on time hence its perportiond nazzard model.

iii) Baseline hazard of model is related to non smoker female.

iv) for female smoker we have
$x_1 = 0$ & $x_2 = 1$, $x = 4$.

hazard is given by.

$$\lambda x = e^{(B_0 + B_1 \cdot 0 + B_2 1)} c^4$$
$$= e^{(-4 + 0.65)} \times 1.05^4$$
$$= 0.0351 \times 1.2155$$
$$\lambda x = 0.04266.$$

v) hazard for non smoker is

$$\lambda_s = e^{(B_0 + B_1 x_1)} c^5$$

hazard for smoker at duration t is,

$$\lambda_t = e^{(B_0 + B_1 x_1 + 0.65)} c^t$$

if smoker & non hazard are same then
$\lambda_s = \lambda t$.

$$e^{(B_0 + B_1 x_1)} = e^{(B_0 + B_1 x_1 + 0.65)} c^t$$

if smoker & non hazard are then.
$\lambda_s = \lambda t$.

$$e^{(B_0 + B_1 x_1)} = e^{(B_0 + B_1 x_1 + 0.65)} c^t.$$
$$c^5 = e^{(0.65)} c^t$$
$$c^{5-t} = e^{(0.65)} = 1.9155$$

As $c = 1.05$.

$$1.05^{5-t} = 1.9155$$

$$S-t = \frac{\ln(1.9155)}{\ln(1.05)} = \frac{0.65}{0.04879}$$

$S-t = 13.32$

When two payout are equal non smoker is approx 13.32 yrs older than smoker.

Q13 E_x^c is central exposed to risk at age label x.
$E^c x = \int_{t=0}^{2} P'(x)\,dt$, $P(x)$ is nearest bday at time t

Assuming $P'_{56}(x)$ is linear.

$E^c_{56} = \frac{1}{2}\left(P'_{56}(2015) + P'_{56}(2016) + \frac{1}{2} P(_{56}(2016)\right) + P'_{56}(2017))$

$= \frac{1}{2} P'_{56}(2015) + P_{56}(2016) + \frac{1}{2} P_{56}(2017)$

∵ no. of policyholder aged label is 56 nearest bday lie bet bet 55.5 & 56.5 i.e. bet in age of 55 last last bday & 56 last bday.
Assuming that bday are uniformly distributed.

$P_{56}(2015) = \frac{1}{2}\left(P_{55}(2015) + P_{56}(2015)\right)$

Also, $P'_{56}(2016) = \frac{1}{2}\left(P_{55}(2016) + P_{56}(2016)\right)$
$= 20800$

$P_{56}(2017) = \frac{1}{2}\left(P'_{55}(2017) + P_{56}(2016)\right)$
$= 19250$

$P_{56}(2017) = 1/2\, P(_{55}(2017) + P_{56}(2016))$

$P'_{56}(2017) = \frac{1}{2}(P_{55}(2017) + P_{56}\cdot(2016))$

$= 19250.$

$E^c_{56} \cancel{(2017)} = \frac{1}{2} \times 20050 + 20800 + \frac{1}{2}\, 19250$

$= 40450.$

$\mu_{56} = d_{56} / E^c_{56} = 1380/40450$
$= 0.0341$

ii) We can estimate initial ~~ro~~ rates of mortality using estimated values of μ from i).

$q_{55.5} = 1 - e^{(-\mu_{56})} = 0.0335$
And $q_{56.5} = 1 - e^{(-\mu_{57})} = 0.0352$

Q 14

i) Hazard is given by
$\lambda(t, z) = \lambda_0(t)\, e^{[0.3z_1 + 0.2z_2 + 0.3z_3}$
$+ 0.5z_4 - 0.1z_5 + 0.7z_6$
$+ 0.15z_7 + 0.4z_8]$

ii) The prob of person who has been looking for life partner for one year will stay single for next 2 years is
$e^{(-\text{integral}_1^3\, \lambda(t, z)\, dt)}$.

If the person is female then
$P_F = e^{[-e^{0.3 + 0.1\cdot 2.5 + 0.4\cdot 20}\, \text{integral}_1^3\, \lambda_0(t)\, dt]}$

~~If the person is female of 37 then~~
~~$PF = e^{-e^{-0.37}} - 0.12$~~

Q15 Advantages :-

- compared to initial exposure to risk, it is easy to calculate.
- It is difficult to model exposed risk in terms of underlying process being modelled.

ii Rita → 3 months exposed to risk.
Sita → 1 yr exposed to risk.
Nita → 9 months exposed to risk.
Outa → None exposed to risk.

iii) Total exposed to risk.
Central → $0.25 + 1 + 0.75 + 0 = 2$ yr.
Initial → $1 + 1 + 0.75 + 0 = 2.75$ yr.