

Numerical Methods & Algebra

Homework - 1

$$\begin{array}{l} 1) \quad x \rightarrow 7 \quad 8 \quad 9 \quad 10 \\ y = f(x) \rightarrow 3 \quad 1 \quad 1 \quad 9 \end{array}$$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$\Rightarrow 0.1875 + (-3.125) + 0.9375 + \cancel{2.18125}$$

$$\Rightarrow 3.625$$

$$\begin{array}{rcccc}
 2) \ x \rightarrow & 3 & 7 & 9 & 10 \\
 y \rightarrow & 168 & 120 & 72 & 63
 \end{array}$$

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168$$

$$+ \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120$$

$$+ \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72$$

$$+ \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63$$

$$\Rightarrow 12 + 180 + (-72) + 27$$

$$\Rightarrow 147$$

$$\begin{array}{rcccl}
 3) x & \rightarrow & 1 & 2 & 7 & 8 \\
 f(x) & \rightarrow & 4 & 5 & 5 & 4
 \end{array}$$

$$f(6) = \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} \times 4$$

$$+ \frac{(6-1)(6-7)(6-8)}{(2-1)(2-7)(2-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-8)}{(7-1)(7-2)(7-8)} \times 5$$

$$+ \frac{(6-1)(6-2)(6-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$\Rightarrow (-0.380952) + 1.66667 + 6.66667 + (-1.904762)$$

$$\Rightarrow 6.047619$$

$$4) \begin{array}{l} x \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 7 \\ y \rightarrow 2 \quad 4 \quad 8 \quad 16 \quad 128 \end{array}$$

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

$$+ \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$\Rightarrow -\frac{2}{3} + \frac{32}{5} + (-24) + \frac{128}{3} + \frac{128}{15}$$

$$\Rightarrow 32.933$$

$$5) \quad x \rightarrow 10 \quad 20 \quad 14 \quad 16$$

$$y \rightarrow 0.25 \quad 0.2 \quad 0.15 \quad 0.10$$

$$f(15) = \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25$$

$$+ \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2$$

$$+ \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15$$

$$+ \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10$$

$$\Rightarrow \frac{-1}{192} - \frac{1}{240} + \frac{5}{632} + \frac{5}{96}$$

$$\Rightarrow 0.120833$$

Homework - 2

1) $X^3 + X - 1$, b/w 0.5 to 1

$$X_1 = 0.5 \quad X_2 = 1$$

$$Y_1 = -0.375 \quad Y_2 = 1$$

$$X_N = \frac{0.5 \times 1 - (-0.375) \times 1}{1 - (-0.375)}$$

$$X_N = 0.636363$$

$$Y_N = -0.105935$$

$$X_1 = 0.636363$$

$$X_2 = 1$$

$$Y_1 = -0.105935$$

$$Y_2 = 1$$

$$X_N = \cancel{0.830252} 0.671195$$

$$Y_N = -0.0264299$$

$$X_1 = 0.671195$$

$$X_2 = 1$$

$$Y_1 = -0.0264299$$

$$Y_2 = 1$$

$$X_N = 0.679662$$

$$Y_N = -0.006376$$

$$X_1 = 0.679662$$

$$X_2 = 1$$

$$Y_1 = -0.006376$$

$$Y_2 = 1$$

$$X_1 = 0.636363 \quad X_2 = 1$$

$$Y_1 = -0.515026 \quad Y_2 = 1$$

$$X_N =$$

$$2) \quad X^3 - 2X + 0.5 = 0 \quad \text{b/w } 0 \text{ to } 1$$

$$X_1 = 0$$

$$X_2 = 1$$

$$Y_1 = 0.5$$

$$Y_2 = -0.5$$

$$X_N = \frac{0 \times (-0.5) - 1(0.5)}{-0.5 - 0.5} = 0.5$$

$$Y_N = -0.375$$

$$X_1 = 0$$

$$X_2 = 0.5$$

$$Y_1 = 0.5$$

$$Y_2 = -0.375$$

$$X_N = 0.285714$$

$$Y_N = -0.048105$$

$$X_1 = 0$$

$$X_2 = 0.285714$$

$$Y_1 = 0.5$$

$$Y_2 = -0.048105$$

$$X_N = 0.260638$$

$$Y_N = -0.003570$$

$$X_N = 0.681692$$

$$Y_N = -0.001524$$

$$X_1 = 0.681692 \quad X_2 = 1$$

$$Y_1 = -0.001524 \quad Y_2 = 1$$

$$X_N = 0.682176$$

$$Y_N = -0.0003629$$

Root of Eq $\rightarrow X^3 + X - 1 = 0$ is $\bullet 0.682176$

$$2) X^3 - 2X + 0.5 = 0 \quad \text{b/w } 0 \& 1$$

$$X_1 = 0$$

$$X_2 = 1$$

$$Y_1 = -1$$

$$Y_2 = 1$$

$$X_N = \frac{0 \times 1 - (1)(-1)}{1 - (-1)} = 0.5$$

$$Y_N = (0.5)^3 - 2 \times 0.5 + 0.5 = -0.375$$

$$X_1 = 0.5$$

$$X_2 = 1$$

$$Y_1 = -0.375$$

$$Y_2 = 1$$

$$X_N = 0.636363$$

$$Y_N = -0.515026$$

$$X_1 = 0$$

$$X_2 = 0.260638$$

$$Y_1 = 0.5$$

$$Y_2 = -0.003570$$

$$X_N = 0.258790$$

$$Y_N = -0.0002487$$

Root of Eq $X^3 - 2X + 0.5 = 0$ is $+0.258790$

$$3) X^3 + 2X - 20 = 0 \quad b/w \quad 2 \text{ \& } 2.5$$

$$X_1 = 2$$

$$X_2 = 2.5$$

$$Y_1 = -8$$

$$Y_2 = 0.625$$

$$X_N = \frac{2 \times 0.625 - (-8) \times 2.5}{0.625 - (-8)}$$

$$= 2.463768$$

$$Y_N = (2.463768)^3 + 2 \times 2.463768 - 20$$

$$= -0.117014$$

$$X_1 = 2.463768$$

$$X_2 = 2.5$$

$$Y_1 = -0.117014$$

$$Y_2 = 0.625$$

$$X_N = 2.469482$$

$$Y_N = -0.001298$$

$$X_1 = 2.469482 \quad X_2 = 2.5$$

$$Y_1 = -0.001298 \quad Y_2 = 0.625$$

$$X_N = 2.469545$$

$$Y_N = -0.000013$$

Root of $X^3 + 2X - 20 = 0$ is 2.469545

$$4) \quad 3x - \cos x - 1 = 0 \quad \text{b/w } 0 \text{ to } 1$$

$$X_1 = 0 \quad X_2 = 1$$

$$Y_1 = -2 \quad Y_2 = 1.000152$$

$$X_N = \frac{0 \times 1.000152 - (-2) \times 1}{1.000152 - (-2)}$$

$$= 0.666633$$

$$Y_N = -0.000033$$

Root of $3x - \cos x - 1 = 0$ is 0.666633

$$5) X^3 - 3X - 5 \quad b/w \quad 2 \text{ \& } 2.5$$

$$X_1 = 2 \quad X_2 = 2.5$$

$$Y_1 = -3 \quad Y_2 = 3.125$$

$$X_N = \frac{2 \times 3.125 - (-3) \times 2.5}{3.125 - (-3)}$$

$$= 2.244898$$

$$Y_N = (2.244898)^3 - 3 \times 2.244898 - 5$$

$$= -0.421380$$

$$X_1 = 2.244898 \quad X_2 = 2.5$$

$$Y_1 = -0.421380 \quad Y_2 = 3.125$$

$$X_N = 2.275209$$

$$Y_N = -0.047835$$

$$X_1 = 2.275209 \quad X_2 = 2.5$$

$$Y_1 = -0.047835 \quad Y_2 = 3.125$$

$$X_N = 2.2785908$$

$$Y_N = 0.005293$$

$$X_1 = 2.278598 \quad X_2 = 2.5$$

$$Y_1 = -0.005293 \quad Y_2 = 3.125$$

$$X_N = 2.278972$$

$$Y_N = -0.000589$$

Root of $X^3 - 3X - 5 = 0$ is 2.278972

$$6) X \log_{10} X - 1.2 = 0 \quad \text{b/w } 2 \text{ \& } 3$$

$$X_1 = 2$$

$$X_2 = 3$$

$$Y_1 = -0.597940$$

$$Y_2 = 0.231364$$

$$X_N = \frac{2 \times 0.231364 - 3 \times (-0.59794)}{0.231364 - (-0.59794)}$$

$$= 2.721073$$

$$Y_N = -0.017040$$

$$X_1 = 2.721073$$

$$X_2 = 3$$

$$Y_1 = -0.017040$$

$$Y_2 = 0.231364$$

$$X_N = 2.740207$$

$$Y_N = -0.000383$$

Root of $X \log_{10} X - 1.2 = 0$ is 2.740207

$$7) X^3 - 4X + 1 = 0$$

$$X_1 = 0 \quad X_2 = 1$$

$$Y_1 = 1 \quad Y_2 = -2$$

$$X_N = \frac{0X(-2) - 1X1}{-2-1} = \cancel{0.66}$$

$$= 0.333333$$

$$Y_N = (0.333333)^3 - 4X(0.333333) + 1$$

$$= -0.296295$$

$$X_1 = 0.3$$

$$X_2 = 0.333333$$

$$Y_1 = 1$$

$$Y_2 = -0.296295$$

$$X_N = 0.257143$$

$$Y_N = -0.011569$$

$$\cancel{X_N = 0.257143}$$

$$X_1 = 0$$

$$X_2 = 0.257143$$

$$Y_1 = 1$$

$$Y_2 = -0.011569$$

$$X_N = 0.254202$$

$$Y_N = -0.000382$$

Root of $X^3 - 4X + 1 = 0$ is 0.254202

$$8) X^3 + 2X^2 - 8 = 0$$

$$X_1 = 1.5$$

$$X_2 = 1.82$$

$$Y_1 = -0.125$$

$$Y_2 = 8$$

$$X_N = \frac{1.5 \times 8 - (-0.125) \times 2}{8 - (-0.125)}$$

$$= 1.507692$$

$$Y_N = (1.507692)^3 + 2(1.507692)^2 - 8$$

$$= -0.026542$$

$$X_1 = 1.507692$$

$$X_2 = 2$$

$$Y_1 = -0.026542$$

$$Y_2 = 8$$

$$X_N = 1.509320$$

$$Y_N = -0.005605$$

$$X_1 = 1.509320$$

$$X_2 = 2$$

$$Y_1 = -0.005605$$

$$Y_2 = 8$$

$$X_N = 1.509664$$

$$Y_N = -0.001176$$

$$X_1 = 1.509664 \quad X_2 = 2.11$$

$$Y_1 = -0.0001176 \quad Y_2 = 8$$

$$X_N = 1.509736$$

$$Y_N = -0.000249$$

Root of $X^3 + 2X^2 - 8$ is 1.509736

$$9) X^3 - 9X + 1 = 0$$

$$X_1 = 2.5 \quad X_2 = 3$$

$$Y_1 = -5.875 \quad Y_2 = 1$$

$$X_N = \frac{2.5X_1 - 3X(-5.875)}{1 - (-5.875)} = 2.927272$$

$$Y_N = (2.927272)^3 - 9X(2.927272) + 1$$

$$= -0.261884$$

$$X_1 = 2.927272$$

$$X_2 = 3$$

$$Y_1 = -0.261884$$

$$Y_2 = 1$$

$$X_N = 2.942366$$

$$Y_N = -0.007708$$

$$X_1 = 2.942366$$

$$X_2 = 3$$

$$Y_1 = -0.007708$$

$$Y_2 = 1$$

$$X_N = 2.942807$$

$$Y_N = 0.000222$$

Root of $X^3 - 9X + 1$ is 2.942807

$$10) X^3 - X - 4 = 0$$

$$X_1 = 1.5$$

$$X_2 = 2$$

$$Y_1 = -2.125$$

$$Y_2 = 2$$

$$X_N = \frac{1.5 \times 2 - 2 \times (-2.125)}{2 - (-2.125)}$$

$$= 1.757575$$

$$Y_N = -0.328303$$

$$X_1 = 1.757575$$

$$X_2 = 2$$

$$Y_1 = -0.328303$$

$$Y_2 = 2$$

$$X_N = \frac{1.757575 \times 2 - 2 \times (-0.328303)}{2 - (-0.328303)}$$

$$X_N = 1.791758$$

$$Y_N = -0.039504$$

$$X_1 = 1.791758$$

$$X_2 = 2$$

$$Y_1 = -0.039504$$

$$Y_2 = 2$$

$$X_N = 1.795792$$

$$Y_N = -0.004598$$

$$X_1 = 1.795792$$

$$X_2 = 2$$

$$Y_1 = -0.004598$$

$$Y_2 = 2$$

$$X_N = 1.796260$$

$$Y_N = -0.000537$$

Root of $X^3 - X - 4 = 0$ is 1.796260