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Calculus Assignment - 2.

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Q1) $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

let $x = 2/w$ $\quad x = 1/2, \; w = 2.$
$x = 50, \; w = 1/50.$

diff w.r.t x.

$\frac{dx}{dw} = -1 w^{-2}$

$-dx = dw \cdot w^{-2}.$

$= -\int_{50}^{1/2} e^{2x} \cdot dx$

$= -\left[e^{2x} \cdot \frac{1}{2}\right]_{50}^{1/2}$

$= -\left[e \cdot \frac{1}{2}\right] + \left[e^{100} + \frac{1}{2}\right].$

$= 1.344058571 \times 10^{43}$

Q2) $\int \frac{2t^3+1}{(t^4+2t)^3} \, dt.$

let $t^4 + 2t = x$

diff w.r.t x.

$(2t^3+1) \, dt = \frac{dx}{2}.$

$\therefore \int \frac{1}{2x^3} \, dx.$

$\frac{1}{2} \int x^{-3} \, dx.$

$\frac{1}{2} \frac{x^{-2}}{-2} + C.$

$= \frac{-1}{4(t^4+2t)^2} + C.$

Q3) $f'(x) = x^4 + 3x - 9.$

$f(x) = \int f'(x) \, dx.$

$= \int x^4 + 3x - 9 \, dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C.$

Q4) $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x) \cdot dx = \int_{-2}^{1} 3x^2 \cdot dx + \int_{1}^{3} 6 \cdot dx$$

$$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= [1 - (-8)] + [18 - 6].$$

$$= 9 + 12$$

$$= 21.$$

Q5) $\int 3(8y - 1)\, e^{4y^2 - y}\, dy,$

Let $4y^2 - y = x.$

diff w.r.t y.

$$8y - 1 = \frac{dx}{dy}.$$

$$\therefore \int 3 \cdot e^{x}\, dx.$$

$$3 \cdot e^{x} + C.$$

$$3 \cdot e^{(4y^2 - y)} + C.$$

Q6. $f(x) = ?$

$$f''(x) = 15\sqrt{x} + 5x^3 + 6.$$

$$f'(x) = \int f''(x) \cdot dx.$$

$$= \int 15\sqrt{x} + 5x^3 + 6 \cdot dx.$$

$$f'(x) = \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + C_1$$

$$f(x) = \int f'(x)\, dx.$$

$$= \int 10x^{3/2} + \frac{5x^4}{4} + 6x + C_1\, dx.$$

$$= \frac{10x^{5/2}}{5/2} + \frac{5x^5}{20} + \frac{6x^2}{2} + C_1 x + C_2$$

$$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1 x + C_2.$$

$$f(1) = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + C_1 x + C_2.$$

$$\therefore -\frac{17}{2} = C_1 + C_2 \quad - (2).$$

$$f(4) = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + 4C_1 + C_2$$

$$-28 = 4C_1 + C_2 \quad - (1)$$

(1) - (2).

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$3c_1 = \frac{-39}{2}$

$c_1 = \frac{-13}{2}$.

$c_2 = \frac{-17}{2} - c_1$

$= -2$

Q7) $f(x+iy) + g(x-iy)$.

$\therefore f(x) + f(iy) + g(x) - g(iy)$.

$\therefore f(x) + i f(y) + g(x) - i g(y)$.

Q8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $r(1) = 2$.

$\therefore \frac{dr}{r^2} = \frac{d\theta}{\theta}$, $r(1) = 1$.

$\therefore \int \frac{1}{r^2} dr = \int \frac{d}{\theta} d\theta$.

$\frac{r^{-1}}{-1} = \log(\theta) + c$.

$\frac{-1}{2} = \log(1) + c$

$\therefore c = -1/2$.

$\boxed{\frac{-1}{r} = \log\theta - \frac{1}{2}}$

Q9) $\frac{dv}{dt} = 9.8 - 0.196v.$

$$\frac{dv}{9.8-0.196v} = dt.$$

$$\therefore \int \frac{1}{9.8-0.196v}\, dv = \int 1 \cdot dt.$$

$$\log(9.8-0.196v)\left(\frac{-1}{0.196}\right) = x + C.$$

$$= \frac{-\log(9.8-0.196v)}{0.196} = x + c.$$

Q10) $ty' + 2y = t^2 - t + 1 \quad \therefore y(1) = \frac{1}{2}.$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$t\frac{dy}{dt} = t^2 - t + 1 - 2y$$

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Q11). $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$

$\dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

$\dfrac{dy}{y^3} = \dfrac{x}{\sqrt{1+x^2}}\, dx.$

$\therefore \int \dfrac{1}{y^3}\, dy = \int \dfrac{x}{\sqrt{1+x^2}} \cdot dx.$

Let $1+x^2 = t$.
diff w.r.t x.

$2x = \dfrac{dt}{dx}.$

$x\,dx = \dfrac{dt}{2}.$

$\therefore \int \dfrac{1}{y^3}\, dy = \int \dfrac{1}{2\sqrt{t}}\, dt.$

$\dfrac{y^{-2}}{-2} = \dfrac{t^{(1/2)}}{2(1/2)} + c.$

$\dfrac{-1}{2y^2} = \sqrt{t} + c.$

$\therefore \sqrt{1+x^2} + c = \dfrac{-1}{2y^2}.$

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$y(0) = 1$

$\therefore \sqrt{1+0^2} + c = \frac{-1}{2(-1)^2}$

$1 + c = \frac{-1}{2}$.

$c = \frac{-3}{2}$.

Q12) $f(x) = x^3 - 10x^2 + 6$; $x = 3$.

$f(3) = -57$.

$f'(x) = 3x^2 - 20x$. $f'(3) = -33$

$f''(x) = 6x - 20$ $f''(3) = -2$

$f'''(x) = 6$. $f'''(3) = 6$.

$f^{iv}(x) = 0$.

$\therefore f(x) = -57 + -33(x-3) + \frac{(-2)(x-3)^2}{2\times 1}$

$+ \frac{6(x-3)^3}{3\times 2\times 1} + \frac{0\times(x-4)^4}{4!}$

$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3$

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Q1) $f(x) = (1+x)^4$ $\therefore f(0) = 1.$

$f'(x) = 4(1+x)^3$

$f''(x) = 4\times 3(1+x)^2$

$f'''(x) = 4\times 3\times 2(1+x).$

$f^{iv}(x) = 4\times 3\times 2$

$$\therefore f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \frac{x^4 f^{iv}(0)}{4!}$$

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Q14) $f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$$f'(x) = \frac{1}{2\sqrt{1+x}} \qquad f'(0) = \frac{1}{2}.$$

$$f''(x) = \frac{-1}{4(1+x)^{3/2}} \qquad f''(0) = \frac{-1}{4}.$$

$$f'''(x) = \frac{3}{8(1+x)^{5/2}} \qquad f'''(0) = \frac{3}{8}.$$

$$f^{iv}(x) = \frac{-15}{16(1+x)^{7/2}} \qquad f^{iv}(0) = \frac{-15}{16}.$$

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \frac{x^4 f^{iv}(0)}{4!}$$

$$= 1 + x\frac{1}{2} + \frac{x^2}{2}\left(\frac{-1}{4}\right) + \frac{x^3}{6}\left(\frac{3}{8}\right) + \frac{x^4}{24}\left(\frac{-15}{16}\right).$$

$$f(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \frac{-5x^4}{96}.$$

Q16)

$f(x) = \ln(3+4x)$ $\qquad f(0) = \ln(3)$

$f'(x) = \dfrac{4}{3+4x}$ $\qquad f'(0) = \dfrac{4}{3}$

$f''(x) = \dfrac{-16}{(3+4x)^2}$ $\qquad f''(0) = \dfrac{-16}{9}$

$f'''(x) = \dfrac{128}{(3+4x)^3}$ $\qquad f'''(0) = \dfrac{128}{27}$

$f^{iv}(x) = \dfrac{-384 \times 4}{(3+4x)^4}$ $\qquad f^{iv}(0) = \dfrac{-1536}{81}$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \frac{f^{iv}(a)(x-a)^4}{4!}$$

$$f(x) = \ln(3) + \frac{4}{3}(x) - \frac{16(x)^2}{9 \times 2} + \frac{128x^3}{27 \times 3 \times 2} - \frac{1536x^4}{81 \times 4 \times 3 \times 2 \times 1}$$

$$= \ln(3) + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64x^3}{81} - \frac{64x^4}{81}.$$