Tushar
407

Assignment - 2

FE

1) Given

$S_0 = £65$
$\sigma = 25\%$ p.a
$r = 2\%$ p.a
$K = £55$
$T = 6$ months (0.5)

(i) $C_t = S_0 (a_1) - Ke^{-rt} \Phi (a_2)$

$$a_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$a_1 = \frac{\ln\left(\frac{65}{55}\right) + \left(2\% + \frac{(2\%)^2}{2}\right)(0.5)}{(20\%)\sqrt{0.5}}$$

$a_1 = 1.08996$

$a_2 = d_1 - \sigma\sqrt{T}$

$= 0.91318$

$\therefore \quad C_t = 65\,\Phi(1.09) - 55e^{-0.2 \times 0.5}\,\Phi(0.92)$

$C_t = 11.464$

ii) Delta of a call option

$$= \frac{\text{Change in paice of call option}}{\text{change in paice of underlying}}$$

iii) $\Delta_c = \Phi(a_1)$

$\Delta_p = -\Phi(-a_1)$

$\Delta_c = 0.86214$

iv) $\Delta p = -0.13786$

Q2 (i) Delta: its rate of change of option paice with respect to change in paice of underlying asset

$$\Delta = \frac{\text{change in paice of derivative}}{\text{Change in paice of underlying asset}}$$

Vega: It is the rate of change of option paice with respect to change in voltality of underlying asset.

$$\nu = \frac{\text{change in paice of option}}{\text{Change in voltality of underlying asset}}$$

(ii) $c - p = S - Ke^{-rt}$

$$\frac{d}{d\sigma}(c-p) = \frac{d}{d\sigma}(S - Ke^{-rt})$$

$$v_c - v_p = 0$$

~~Since K &~~ Since K & S are constants

$$\therefore v_c = v_p$$

iii) $S_0 = \$55$

$\sigma = 25\%$ p/a

$r = 5\%$

$X = \$50$

$T = 1$

$$a_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{55}{50}\right) + \left(5\% + \frac{1}{2} \times (25\%)^2\right)1}{25\% \times \sqrt{T}}$$

$$a_1 = 0.70624$$

$$a_2 = a_1 - \sigma\sqrt{T}$$

$$= 0.4562$$

$$C_t = S_0\,\Phi(a_1) - Ke^{-rt}\,\Phi(a_2)$$

$$= 55(0.70624) - \cancel{50}\ 50e^{-0.05}(0.4562)$$

$$C_t = \$9.65$$

$$c - p = S - Ke^{-rT}$$

$$9.65 - p = 55 - 50e^{-0.05}$$

$$p = \$\ 2.21$$

(v) We assume

a units of call option
b units of put option
c units of forwards

	CF @ PV	Delta	Vega
Call	$c = 9.65$	Δc	νc
put	$p = 2.211$	Δp	ν_p
forward	-	1	0

Vega Hedging

$$a\nu_c + b\nu_p + c\nu_f = 0$$

$$a\nu_c + b\nu_p = 0 \quad (\because \nu_F = 0)$$

$$\nu_c (a+b) = 0 \quad (\because \nu_c = \nu_p)$$

$$(a+b) = 0$$

$$a = -b \quad | \quad b = -a \quad (i)$$

Delta Hedging

$a\Delta_c + b\Delta_p + c\Delta_f = 0$

$a\Delta_c + b\Delta_p + c = 0 \quad (\because \Delta_f = 1)$

$a\Delta_c + (-a)(\Delta_c - 1) + c = 0$

$(\because$ from (i) & $\Delta p = \Delta_c - 1)$

$a\Delta c + a - a\Delta c + c = 0$

$a = -c$ - (ii)

from (i) & (ii)

$\therefore a = -b = -c$

Portfolio $= 1000$

$a \cdot c + b \cdot P + c \cdot f = 1000$

$ac + bP = 1000 \quad (\because f = 0)$

$9.6525a + 2.2190b = 1000$

$9.6525a - 2.2190a = 1000 \quad (\because b = -a)$

$\Rightarrow 7.4385a = 1000$

$\therefore a = 134.43$

$b = c = -134.43$

$\therefore$ Portfolio

LP on 134 call options
SP on 134 put options
SP on 134 forwards.

3.(i) $C_t = \Sigma \left(e^{-r(T-t)} \quad (t/F_t) \right)$

F_t = Filtration @ $t > 0$
C_T = payoff under derivative at T
C_t = derivation value at time t

ii) $r = 5\%$ p.a
$\sigma = 25\%$ p.a
$T = 6$ months $= 0.5$
$S_0 = £50$
$\alpha = £49$

$$C_t = S_0 \Phi(a_1) - ke^{-rt} \Phi(a_2)$$

$$a_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$a_1 = \frac{\ln\left(\frac{50}{49}\right) + \left(5\% + \frac{1}{2}(25\%)^2\right)0.5}{(25\%)(\sqrt{0.5})}$$

$= 0.399$

$$a_2 = a_1 - 25\% \left(\sqrt{0.5}\right)$$

$$= 0.1673$$

$$\Phi(a_1) = 0.6346$$

$$\Phi(a_2) = 0.5669.$$

$$C_T = S_0 \Phi(a_1) - k e^{-rT} \Phi(a_2)$$

$$C_T = 4.66$$

iii) 4.66

iv) $S_t + p = c + ke^{-rT}$

$$p = 4.66 + 49 e^{-0.03 \times 25} - 50$$
$$= 2.45$$

5) i) $\Delta = \Phi(a_1)$

$$= \frac{\text{Change in price of underlying asset}}{\text{Change in price of Call option}}$$

ii) $$a_1 = \frac{\ln\left(\frac{S_0}{X}\right) + r\left(\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$0.3 = \frac{\ln\left(\frac{40}{45.91}\right) + (0.02 + 0.5 \times \sigma^2)5}{\sigma\sqrt{5}}$$

$$0.3(\sqrt{5}) = \frac{(-1.378) + (0.1 + 2.5\sigma^2)}{\sigma}$$

$$\sigma - 2.5\sigma^2 = 0.05637$$

$$2.5\sigma^2 - 0.7083\sigma - 0.0378$$

$$\sigma = 32\%$$

iii) $R_0 = \$30$

$\sigma = \sqrt{16}$ p/a

Option payoff = C at time T if,

$$\frac{S_1}{S_0} > K_S, \quad \frac{R_1}{R_0} < IR$$

$$\text{Payoff} = Ce^{-rT} Q\left(\frac{S}{S_0} < K_S\right) Q\left(\frac{R_1}{R_0} < K_e\right)$$

iv) Perfect correlation

$$= \frac{R_1}{R_0} = \frac{S_1}{S_0}$$

$$Ce^{-rT} Q\,(R_1 \times R_0 < K_S) = Ce^{-rT} Q\,(S_1/S_0 < K_e)$$

$$\therefore \text{price} = Ce^{-rT} Q\left(\frac{R_1}{R_0} < \min(K_S, K_e)\right)$$

$$= Ce^{-rT} Q\left(\frac{S_1}{S_0} < \min(K_S, K_e)\right)$$

v) Price

$$C e^{-rT} Q\left(\frac{S_1}{50} < K_S\right) \cdot Q\left(\frac{R_1}{K_0} < K_0\right)$$

$$= \cancel{S_0 e^{-0.2 d}}$$

$$= \cancel{50 e^{-0.02}} \cdot \cancel{Q(S)}$$

$$= 50 e^{-0.02} \cdot Q(S_1 < 32) \cdot Q(R_1 < 18)$$

$$= \$1.61$$

6i) a) $\Delta = -\Phi(-a_1)$

$$= \Phi(a_1) = 1$$

b) $\Delta = 0$

$$100{,}000\,\Delta + 24830 = 0$$

$$\Delta = \frac{-24830}{100000}$$

$$= -0.2483$$

ii) $\Phi(-a_1) = 0.2483$

$$1 - \cancel{\phi}\,\Phi(a_1) = 0.2483$$

$$\Phi(a_1) = 0.7517$$

$$a_1 = 0.68$$

$$0.68 = \ln\left(\frac{0.64}{0.63}\right) + \left(0.03 + \frac{1}{2}\sigma^2\right)$$

$\sigma = 7.1\%$

iii) a) $Xe^{-rT} \, \Phi(-d_2) - S_0 \, (\Phi(-d_1))] = 0.2983$

$= 0.63 \times e^{-0.03} \times (1 - \Phi(0.609)] - 0.64 \times 0.2483$

$= 6.894$

b) Cash ~~Hold~~ Holding in Hedging Portfolio

$= 24830 \times 0.64 + 100{,}000 \times 0.0894$

$= £\,165{,}806$

Q7 (i) $\Delta = \frac{da}{db} = \frac{da}{db}(t, b_t)$

a = derivative
b = underlying asset

$\gamma = \frac{d^2a}{db^2}$

$\nu = \frac{\partial a}{\partial \sigma_b}$

ii) $r = 3\%$ p/a
$\nu = \$29$
$x = 50$
$c = 17.91$
$S_0 = 60$
$\sigma = 25\%$ p.a
$T = 3$ years

iii) 0.801 shares

$C - \Delta \times S_0$ short cash = 30.15 share in cash

iv) $a(b, \sigma + \delta) = a(b, \sigma) + \delta \frac{da}{d\sigma}$

$= 17.91 + 29 \times 0.02$

$\boxed{= 18.49}$

Q8) i) $\Delta = \frac{da}{db}$

ii) $S_0 = \$100$

$r = 3\%$

$x = 109.42$

$T = 1$

$\Delta = 0.42079$

$\Phi(a_1) = 0.42079$

$a_1 = 0.2$

$a_1 = \ln\left(\frac{S_0}{x}\right) + \left(r + \frac{1}{2}\sigma^2\right)T$

$\Rightarrow -0.2 = \ln\left(\frac{100}{109.42}\right) + \left(0.03 + \frac{\sigma^2}{2}\right)$

$\Rightarrow -0.2\sigma + 0.09 - 0.03 - \frac{\sigma^2}{2} = 0$

$\Rightarrow -\frac{\sigma^2}{2} - 0.2\sigma + 0.06 = 0$

$\sigma = 0.2$

Q9) ii) Price of derivative @ time t

$$D_T = g(t, S_t) = \frac{S_t^n}{S_0^{n-1}} e^{\mu(T-t)} \quad , \quad (n > 1)$$

Then, $D_T = g(T, S_T)$

$$= f(S_T) = \frac{S_T^n}{S_0^{n-1}}$$

$$\frac{\partial g}{\partial t} = -\mu \frac{S_t^n}{S_0^{n-1}} e^{\mu(T-t)} = -\mu g$$

$$\frac{\partial g}{\partial S_t} = \frac{n(n-1) S_t^{n-2}}{S_0^{n-1}} e^{\mu(T-t)} = \frac{n}{S_t} g$$

$$\frac{\partial^2 g}{\partial S_t^2} = \frac{n(n-1) S_t^{n-2}}{S_0^{n-1}} e^{\mu(T-t)} = \frac{n(n-1)}{S_t^2} g$$

$$-\mu + (r-q)n - r + \frac{1}{2}\sigma^2 n(n-1)$$

If $q = 0$

$$\mu = rn - r + \frac{1}{2}\sigma^2 n(n-1)$$

$$= r(n-1) + \frac{1}{2}\sigma^2 n(n-1)$$

$$= \left(r + \frac{1}{2}\sigma^2 n\right)^2 (n-1)$$

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