6(a) low volume of data is the main problem ∴
- crude rates may not be suitable
- random sampling errors
- rates at particular age cannot be linked to rates at adjacent age

Age	Z_x	Z_x^2	$\mu^o_{x+1/2}$	$\Delta\mu^o_{x+1/2}$	$\Delta^2\mu^o_{x+1/2}$
18-22	0.5249	0.27552	0.0061	0.0070	0.0061
23-27	0.3615	0.13068	0.0131	0.0131	0.0094
28-32	1.0266	1.05391	0.0262	0.0225	0.0127
33-37	-1.0912	1.19072	0.0487	0.0352	0.0147
38-42	1.2394	1.53611	0.0839	0.0499	0.0138
43-47	0.5949	0.35391	0.1338	0.0637	0.0094
48-52	0.6346	0.40271	0.1975	0.0731	0.0018
53-57	0.1787	0.03193	0.2706	0.0749	
58+	-0.0367	0.001349	0.3455		
		4.976839			

$\Delta^3\mu^o_{x+1/2}$
0.0033
0.0033
0.0020
-0.0009
-0.0044
-0.0076

$\Sigma Z_x^2 = 4.976839 = 4.98$

df ≤ 9.
One sided
Upper 5% exceeds 4.98 except 1.
insufficient evidence to reject Ho.

23	875	11	0·0111	9·7125	0·4731	0·1707
24	950	15	0·0137	13·015	0·5502	0·3027
25	805	10	0·0126	10·143	−0·0449	0·0020
	1390	16	0·0127	17·653	−0·3934	0·1548
	1080	17	0·0122	13·176	1·0535	1·1098
	1310	14	0·0131	17·161	−0·7631	0·582
					0·9197	2·965

$2·9657 < 15·51$

insufficient evidence to reject the Ho.

H_0: graduated scales are the same as the true underlying scales for the beach & business

E_d	CO	q_a	ED	Z_a	Z_a^2
1060	12	0.0125	13.25	0.3[illegible]	[illegible]
1050	14	0.0118	14.75	0.7[illegible]	[illegible]
1010	16	0.0134	16.214	0.0521	0.[illegible]
780	9	0.0102	7.[illegible]4	-0.6961	0.4845
875	11	0.011	9.7125	0.4781	0.[illegible]
950	15	0.0137	13.015	.1802	0.3027
805	10	0.0126	10.143	-0.0449	0.0020
1390	16	0.0127	17.653	-0.3754	0.1408
1080	17	0.0122	13.176	1.0535	1.[illegible]
1310	14	0.0131	17.161	-0.7631	0.582[illegible]
				0.9197	2.96[illegible]

$2.9657 < 15.51$

insufficient evidence to reject H_0.

13) 12)

Age	Z_x	Z_{x+1}	A	B
			-0.40343	-1.04593
60	-0.37165	-1.01415	-1.04593	-0.72496
61	-1.01415	-0.69317	-0.72496	-0.7149
62	-0.69317	-0.68311	-0.7149	0.601412
63	-0.68311	0.6322	0.601412	0.48577
64	0.6332	-0.4598	-0.48577	0.189993
65	-0.45398	0.221781	0.189993	0.553424
66	0.221781	0.585212	0.553424	0.695155
67	0.585212	0.726943	0.695155	0.723721
68	0.726943	0.755509	0.723721	0.61129
69	0.755509	0.643078	0.61129	
70	0.643078			

A^2	AB
0.162758	0.421964
1.093978	0.758262
0.525569	0.518275
0.511083	-0.42995
0.361697	-0.29215
0.23597	-0.09229
0.036098	0.105147
0.306278	0.384716
0.483241	0.503099
0.52372	0.442403
0.373676	0

$r = 0.55296$

$r \times \sqrt{11} = 1.83 \quad > 0$

rates are over graduated.

$1.83 < 1.96$

Ho not rejected.

14) under graduation - when too much emphasis is given to goodness of fit. The undergraduated rates adhere closely to crude rates but there is a lack of smoothness.

over graduation - when too much emphasis is given to smoothness. Lack of adherence to crude data but smoothness from age to age is present.

15) Ho: true underlying rates of withdrawal are the graduated rates.

t	Z_t^2
0	11.77
1	2.69
2	5.01
3	0.45
4	1.07
5	1.62
6	2.01
7	10.11
8	0.47
9	1.88
	37.07

df = 7

37.07 > 14.07

Ho is rejected.

Date :

MON TUE WED THU FRI SAT SUN

47) Ho: standard table rates are the true underlying mortality rates for the term assurance policyholders.

x	E_x	d_x	q_x^s (%)	$E_x \cdot q_x^s$	$d_x - E_x q_x^s$
40	50000	87	0.2053	102.65	-15.65
41	48560	84	0.2247	109.11	-25.11
42	47190	101	0.2418	114.11	-13.11
43	44100	112	0.2602	114.75	-2.75
44	43600	123	0.2832	123.48	-0.48
45	40400	110	0.3110	125.64	-15.64
46	37280	108	0.3438	128.17	-20.17
47	35370	122	0.3816	134.97	-12.97
48	32100	150	0.4243	136.20	13.80
49	29000	139	0.4719	136.85	2.15
50	26200	151	0.5244	137.39	13.61
				1363.32	-76.32

test statistic = |-2.07| > 1.96.

∴ reject Ho.

Date : ....

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1> Mortality rates, as evident from large experiences, vary smoothly with age; ∴, crude estimate of mortality at any age carries info of mortality at adjacent rates. Smoothing experience enables to use data at adjacent ages to improve estimate at each age. This reduces sampling error. Mortality experience can be used in financial calculations. Removal of bias in the data arising from faulty data collection is a main limitation of mortality investigation that graduation may not overcome.

2> i> smooth set of rates that are suitable for a particular purpose.
ii> remove sampling errors.
iii> use info available from adjacent ages.

i> smoothness
ii> adherence to data.
iii> suitability for the purpose in hand.

3> i> - produce a smooth set of rates that are suitable for a practical reason.
- remove random sampling errors.
- use info available from adjacent ages.

ii> - graduation by parametric formula.
- graduation by reference to a standard table.
- graphical graduation

iii> No: graduated rates represent the mortality of members in the pension scheme.

Age group	Pensioners	Actual deaths	Exp Deaths	z_x	z_x^2
50-54	37259	248	227.28	1.3786	1.9005
55-59	28057	892	367.55	1.2839	1.6485
60-64	25654	680	672.13	-0.3074 -0.3290	0.0845
65-69	20475	987	997.13	0.5445	0.1082
70-74	16219	1380	1360.77	1.0906	0.296
75-79	11843	1625	1584.59	2.2121	1.1895
80-84	7535	1564	1487.57	1.3195	4.8933
85-89	3294	925	891.36	-2.5254	1.7409
90+	450	130	155.48		6.3776

$n = 9$

$m < 9$ $\quad$ $m = 8$

reject H_0.

$\Sigma z_x^2 = 18.2497 > CV$

iv) - Signs test
- Individual z_x tests

v) graduated rates are below crude mortality est rates for most ages, the exceptions 65-89, 90+
$\therefore$ graduated rates are under estimate use of them may result in over provisioning of annuity liabilities under the pension scheme.

Date : ....

MON TUE WED THU FRI SAT SUN

5) i) a) Maximum smoothness can be attained by ignoring estimated rates and drawing a very smooth graduation curve. This causes large deviations. So we will have smooth curve but a low adherance to data. On the other hand, joining up a plot of the estimated values will give perfect adherence to data but would not attain a smooth curve.

b) Conflict is resolved through striking a fair balance between adherence to data and smooth progression of rates.

$$\log(q_x/p_x) = a+bx$$

$$(q_x/p_x) = e^{a+bx}$$

$$\frac{q_x}{1-q_x} = e^{a+bx}$$

$$q_x = \frac{1}{1+e^{-(a+bx)}}$$

x	$a+bx$	q_x	E	A	A-E	$(A-E)^2/E$
30	-7.0140	0.000898	224.60	245	20.40	1.85
31	-6.8978	0.001009	252.25	276	23.75	2.24
32	-6.7816	0.001133	283.29	313	29.71	3.12
33	-6.6654	0.001273	318.16	323	4.84	0.07
34	-6.5492	0.001429	357.30	339	-18.30	0.94
			1435.60	1496	60.40	8.22

$\chi^2_3 = 8.22$

$df = 5-2 = 3$

Ho rejected.