Assignment - 2

1) $$I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2} \cdot dw \quad - (1)$$

Put $t = \frac{2}{w}$ - (2)

$$\therefore \frac{dt}{dw} = -2w^{-2}$$

$$\frac{dt}{-2} = w^{-2} \cdot dw \quad - (3)$$

Subs (2) & (3) in (1)

$$I = \int_{100}^{1} \frac{e^t}{-2} \cdot dt$$

when $w = 1/50$

$t = \frac{2}{1/50} = 100$

$$= -\frac{1}{2} \int_{100}^{1} e^t \cdot dt$$

when $w = 2$

$t = \frac{2}{2} = 1$

$$-\frac{1}{2} \left[e^t \right]_{100}^{1}$$

from (2)

$$= -\frac{1}{2} \times (2.718 - 2.68117 \times 10^{43})$$

$$= 1.30585 \times 10^{43}$$

2) $$I = \int \frac{2t^2 + 1}{(t^4 + 2t)^3} \cdot dt \quad - (1)$$

Put $u = t^4 + 2t$

$$\frac{du}{dt} = 4t^3 + 2$$

$$= 2(2t^3 + 1)$$

$$\frac{du}{2} = 2t^3 + 1 \; dt \quad -- (2)$$

from ① and ②.

$$I = \frac{1}{2}\int \frac{1}{u^3} \cdot du$$

$$= \frac{1}{2}\int u^{-3} \cdot du$$

$$= \frac{1}{2} \times \frac{u^{-2}}{-2} + c$$

$$= -\frac{1}{4} u^{-2} + c.$$

from a, $-\frac{1}{4}(t^4 + 2t)^{-2} + c.$

3) $f'(x) = x^4 + 3x - 9.$

$$f(x) = \int f'(x)$$

$$= \int x^4 + 3x - 9 \cdot dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c.$$

4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$$\int_{-2}^{3} f(x) \cdot dx.$$

$$= \int_{-2}^{1} 3x^2 \cdot dx + \int_{1}^{3} 6 \cdot dx.$$

$$\left(\frac{3(2x^3)}{3}\right)_{-2}^{1} + (6x)_{1}^{3}$$

Date
Page

$= 1 + \frac{8}{3} + 18 - 6$

$= \frac{3+8}{3} + 12 = \frac{11+36}{3} = \frac{47}{3}$

5. $I = \int 3(8y-1) \cdot e^{4y-y} \cdot dy$ — ①

put $t = 4y^2 - y$ — ②

$\frac{dt}{dy} = 8y-1$

$dt = (8y-1)\, dt$ — ③

$I = 3\int 1 \cdot e^t \cdot dt$

$= 3 \cdot [e^t] + c$

$I = 3 \cdot [e^{4y^2-y}] + c$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6, \quad f(1) = \frac{-5}{4}$

$f(4)$

$f'(x) = \int f''(x) \cdot dx = \int 15^{1/2} + 5x^3 + 6$

$= \left[\frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x\right] + c.$

$f'(x) = 10x^{3/2} + \frac{5x^4}{4} + 6x + c$

$f(x) = \int f'(x) \cdot dx$

$$= \int \left[10x^{3/2} + \frac{5}{4}x^4 + 6x + c \right] dx$$

$$= \frac{10x^{5/2}}{5/2} + \frac{5}{4} \cdot \frac{x^5}{5} + \frac{6x^2}{2} + c_1 x$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c_1 x + c_2$$

Put $x = 1$

$$f(1) = 4 + \frac{1}{4} + 3 + c_1 + c_2 = \frac{-5}{4}$$

$$16 + 1 + 12 + 4c_1 + 4c_2 = -5$$

$$29 + 5 + 4(c_1 + c_2) = 0$$

$$4(c_1 + c_2) = -34$$

$$c_1 + c_2 = -17/2$$

$$c_2 = \frac{-17}{2} - c_1 \quad - (2)$$

$$f(4) = 404$$

$$404 = 4(4)^{5/2} + \frac{(4)^3}{4} + 3(4)^2 + c_1(4) + c_2$$

$$128 + 256 + 48 + c_1(4) + c_2 = 404$$

$$c_1(4) + c_2 = -28$$

$$4c_1 + \left(\frac{-17}{2} - c_1 \right) = -28$$

$$4c_1 - \frac{17}{2} - 2c_1 = -28$$

$$8c_1 - 17 - 2c_1 = -56$$

$$6c_1 = -39$$

$$c_1 = -6.5$$

$C_2 = \frac{-17}{2} - (-6.5)$

$C_2 = -8.5 + 6.5$

$= -2$

from ①, $f x =$

$4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2.$

7. Formulate PDE from $f(x+iy) + g(x-iy)$

Let $u(x,y) = f(x+iy) + g(x-iy)$

Let

$z = x + iy$

$\bar{z} = x - iy$ where $i = \sqrt{-1}$

$\therefore u = f(z) + g(\bar{z}).$ $\partial =$ del

Partially diff. 'u' wrt 'x'

$\therefore \frac{\partial u}{\partial x} = \frac{\partial f(z)}{\partial x} + \frac{\partial g(\bar{z})}{\partial x}$

$= f'(z) \cdot \frac{\partial x}{\partial x} + g'(\bar{z}) \frac{\partial(\bar{z})}{\partial x}$

$= f'(z) \frac{\partial z}{\partial x} + g'(\bar{z}) \frac{\partial(\bar{z})}{\partial x}$

$= f'(x+iy) + \frac{\partial(x+iy)}{\partial x} + g'(x-iy)$

$\times \frac{d(x+iy)}{dx}$

$= f'(x+iy)(1+0) + g'(x-iy)(1-0)$

$$\frac{du}{dx} = f'(x+iy) + g'(x-iy)$$

Similarly, P.D u wrt 'y'

$$\frac{du}{dy} = f'(z)\frac{dz}{dy} + g'(\bar{z})\frac{d\bar{z}}{dy}$$

$$= f'(x+iy)(0+1\cdot i) + g'(x-iy)(0-1\cdot i)$$

$$\frac{du}{dy} = i\,[f'(x+iy) - g'(x-iy)]$$

8) $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$ $\quad r(1) = 2$

$$\frac{dr}{r^2} = \frac{1}{\theta}\cdot d\theta$$

On $\int$ on b s

$$\int \frac{1}{r^2}\cdot dr = \int \frac{1}{\theta}\cdot d\theta$$

$$\frac{r^{-2+1}}{-2+1} = \log\theta$$

$$\frac{r^{-1}}{-1} + c = \log\theta$$

$$c = \log\theta + r^{-1} \quad -(1)$$

Page

Put $\theta = 1$ & $r = 2$

$$c = \log 1 + 1/2$$
$$= 0 + 1/2$$
$$c = 1/2$$

$$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\frac{dv}{(9.8 - 0.196v)} = 1 \cdot dt$$

On $\int$

$$\int \frac{1}{1 - 0.02v} \cdot dv = 9.8 \int 1 \cdot dt$$

$$\frac{\log(1 - 0.02v)}{-0.02} = 9.8t + c.$$

Initial value prob (IVP)

$$ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2$$

$$t \frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\therefore \frac{dy}{dt} + \frac{2y}{dt} + Py = Q$$

where $P = 2/t$

$Q = t - 1 + 1/c$

P & Q are funcns and only

$\therefore$ Given ODE is linear

Integrating factor : $(I.f) = e^{\int P dt}$

$e^{\int \frac{2}{t} dt}$

$e^{2\int \frac{1}{t} dt} = e^{2\log t} = e^{\log t^2}$

$= t^2$

General Sol^n :

$$IF = \int Q \cdot IF dt + c.$$

$$= \int \left(t - 1 + \frac{1}{t}\right) \times t^2 \, dt + c$$

$$= \int (t^3 - t^2 + t) \, dt + c.$$

$$= \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$$

IVP ⇒ Put $t = 1$ & $y = 1/2$

$$\frac{1}{2} \times t^2 = \frac{1}{4} - \frac{1}{3} + c$$

$$c = \frac{1}{3} - \frac{1}{4}$$

$$c = \frac{1}{12} \quad (2).$$

(1) & (2) Particular Sol^n is

$$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}$$

$$y' = \frac{xy^3}{\sqrt{1+x^3}} \qquad y(0) = -1 \quad - (1)$$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^3}}$$

$$\frac{1}{y^3} \cdot dy = \frac{x}{(1+x^3)^{1/2}} \times dx$$

$\int$ on both sides

$$\int \frac{1}{y^3} dy = \int \frac{x}{(1+x^3)^{1/2}} dx$$

$$\int y^{-3} dy = \frac{1}{2} \int \frac{2x}{\sqrt{1+x^2}} dx$$

$$\frac{y^{-2}}{2} = \frac{1}{2} \times 2\sqrt{1+x^2} + c$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)} + c$$

$$\frac{1}{2y^2} = \sqrt{1+x^2} + c$$

put $x = 0$, $y = -1$

from (1).

$$\frac{(-1)^{-2}}{2} = \sqrt{1+0} + c$$

$$\frac{1}{2} = 1 + c$$

$$c = -1/2$$

Particular soln :

$$\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{1}{2}$$

$$\therefore \frac{1}{2y^2} = \frac{2\sqrt{1+x^2} - 1}{2}$$

$$y^2 = \frac{1}{2\sqrt{1+x^2} - 1}$$

$$y = \pm \frac{1}{\sqrt{2\sqrt{1+x^2} - 1}}$$

Above DE is valid when

$$1+x^2 \geq 0$$

which is true for all real values of 'x'.

and,

$$2\sqrt{1+x^2} - 1 > 0$$

$$2\sqrt{1+x^2} > 1$$

$$\sqrt{1+x^2} > \frac{1}{2}$$

$$1+x^2 > \frac{1}{4}$$

$$x^2 > \frac{-3}{4}$$

which is not possible for real 'x'.

The soln to DE

$$y = \frac{\pm 1}{\sqrt{2\sqrt{1+x^2}-1}}$$ is defined for all real values of 'x'.

12. $f(x) = x^3 - 10x^2 + 6 = -57$

abt $x = 3$

$$f'(x) = 3x^2 - 10(2x) + 0 = 3x^2 - 20x = -33$$

$$f''(x) = 6x - 20 = -2$$

$$f'''(x) = 6$$

$$x = a = 3$$

$$(x) = f(a) + (x-a)\frac{f'(x)}{1!} + \frac{f''(x)\cdot(x-a)^2}{2!} + \frac{f'''(x)(x-a)^3}{3!}$$

$= -57 + (x-3)\cdot(-33) + (x-3)^2\left(\frac{-2}{2}\right) + (x-3)^3 \times \frac{6}{3\times 2}$

$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3 + \dots$

13) Maclaurin series:

$f(x) = (1+x)$

$f'(x) = u(1+x)^{u-1}$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

$f(x) = f(0) + \frac{x^1}{L1} f'(0) + \frac{x^2}{L2} f''(0) + \frac{x^3}{L3} f'''(0)$

$(1+x)^u = 1 + u + (u-1)u + (u)(u-1)(u-2)\dots$

14) $f(x) = \sqrt{1+x}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad f'(0) = 1/2$

$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $\qquad f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{-3}{8}(1+x)^{-5/2}$ $\qquad f'''(0) = -3/8$

$\sqrt{1+x} = \sqrt{1+x} + \frac{x^1}{L1}\left(\frac{1}{2}\right) + \frac{x^2}{L2}\left(\frac{-1}{4}\right) + \frac{x^3}{L3}\left(\frac{-3}{8}\right)\dots$

Q15 $f(x) = e^{-6x}$ $\quad x = +4$

$f(x) = e^{-6x}$

$f'(x) = e^{-6x}(-6)$ $\quad f'(4) = e^{24}(6)^1$

$f''(x) = (-6)e^{-6x}(-6)$ $\quad f''(-4) = e^{24}(6)^2$

$f'''(x) = (-6)^3 e^{-6x}$ $\quad f'''(-4) = e^{24}(6)^3$

$e^{6x} = e^{+24} + (x-4)(e^{24})(6) + (x-4)^2(e^{24})(6)^2 + \frac{(x-4)^3 e^{24}}{(6)^3}$

Q16) $f(x) = \ln(3+4x)$ $\quad x = 0.$

$\ln(3)$

$f'(x) = \frac{4}{3+4x}$ $\quad f'(0) = 4/3.$

$f''(x) = (-4)(4x+3)^{-2}(4)$ $\quad f''(0) = (-1)(4)^2(3)^{-2}$

$f'''(x) = (+8)(4x+3)^{-3}(4)^2$ $\quad f'''(0) = (-1)(-2)(3)^{-2}(4)^3$

$\ln(3+4x) = \ln(3) + \frac{2}{4}\left(\frac{4}{3}\right) + \frac{x^2}{2}(-1)(4)^2(3)^{-2} + x^3(4)^3(2)(3)^{-2}$