Prob & Stats

Assignment - 1

1. P.Q $\rightarrow \dfrac{\hat{\theta} - \theta}{\sqrt{\hat{\theta}/N}} \sim N(0,1)$,

$$\hat{\theta} = \frac{\sum x_i}{n} = \frac{200}{1} = 200$$

C.I = 95%

$$P\left(-1.96 < \frac{\hat{\theta} - \theta}{\sqrt{\hat{\lambda}/n}} < 1.96\right)$$

$$P\left(-1.96\sqrt{\frac{\hat{\theta}}{n}} < \theta - \hat{\theta} < 1.96\sqrt{\frac{\hat{\lambda}}{n}}\right) = 0.9$$

$$P\left(\hat{\lambda} - 1.96\sqrt{\frac{\hat{\theta}}{n}} < \theta < \hat{\theta} + 1.96\sqrt{\frac{\hat{\lambda}}{n}}\right) = 0.9$$

95% CI for θ = (172.28, 227.72)

2 (i) $\bar{x} = \dfrac{\sum x_i}{n}$ $\qquad E(x_i) = \mu$, $V(x_i) = \sigma^2$

$$E(\bar{x}) = E\left(\frac{\sum x_i}{n}\right) = \frac{1}{n} E(x_1 + x_2 + \dots x_n)$$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + \dots + E(x_n)\right]$$

$$= \frac{1}{n}\left[\mu + \mu + \dots \mu\right]$$

$$= \frac{n\mu}{n} = \mu$$

$$V(\bar{x}) = V\left(\frac{\Sigma x_i}{n}\right) = \frac{1}{n^2} V(x_1 + x_2 \dots + x_n)$$

$$= \frac{1}{n^2}\left[V(x_1) + \dots V(x_n)\right]$$

$$= \frac{1}{n^2}\left[\sigma^2 + \sigma^2 + \dots \sigma^2\right]$$

$$= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

ii) $$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$E(S^2) = \frac{1}{n-1} E(\Sigma x_i^2 + \bar{x}^2 - 2\Sigma x_i \bar{x})$$

$$= \frac{1}{n-1}\left[E(\Sigma x_i^2) + \Sigma E(\bar{x})^2 - 2\bar{x} \cdot E(\bar{x})^2\right]$$

$$= \frac{1}{n-1}\left[nE(x_i^2) + nE(\bar{x})^2 - 2E(\bar{x})^2\right]$$

$$= \frac{1}{n-1}\left[nE(x_i)^2 - E(\bar{x})^2\right]$$

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$$E(x_i^2) = \left[V(x_i) + [E(x_i)]^2\right]$$

$$= \sigma^2 + \mu^2$$

$$E(\bar{x}^2) = V(\bar{x}) + [E(\bar{x})]^2$$

$$= \frac{\sigma^2}{n} + \mu^2$$

$$= \frac{1}{n-1}\left[n\sigma^2 + n\mu^2 - n\mu^2 - \sigma^2\right]$$

$$= \frac{1}{n-1}\left[\sigma^2(n-1)\right] = \sigma^2$$

$$\therefore E(s^2) = \sigma^2$$

iii) $X_i \sim N(\mu, \sigma^2)$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$V(s^2) = V\left[\frac{(n-1)S^2}{\sigma^2}\right] \times \frac{\sigma^4}{(n-1)^2}$$

$$= V(\chi^2_{n-1}) \times \frac{\sigma^4}{(n-1)^2}$$

$$= \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{n-1}$$

(iv) $P[-0.5\,E(s^2) < s^2 < 0.5\,E(s^2)]$

$E(s^2) = \sigma^2 = 100$, $n = 100$

$P(-50 < s^2 < 50)$

$P\left(\frac{9 \times -50}{100} < \frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 50}{100}\right)$

$P(-45 < \chi^2_9 < 4.5)$

$= P(\chi^2_9 < 4.5) - P(\chi^2_9 \leq -4.5)$

$= 0.1245$

3. $X \sim Bin(4, p)$

$L = [P(X=0)]^{86} \cdot [P(X=1)]^{75} \cdot [P(X=2)]^{16}$

$[P(X=3)]^{2} \cdot [P(X=4)]^{1}$

$L = [(1-p)^4]^{86} \cdot [(1-p)^3 \cdot (p)^1]^{75} [(1-p)^2 (p)^2]^{16} \cdot$

$[(1-p) \cdot (p)^3]^2 \cdot (p)^4 \cdot \text{Const}$

$L = (1-p)^{603} (p)^{117}$

$\ell = 603 \log(p) + 117 \log(p)$

$\frac{d\ell}{dp} = \frac{603}{1-p} + \frac{117}{p} (-1) = 0$

$$603p - 117 + 117p = 0$$
$$720p - 117 = 0$$
$$\hat{p} = 0.1625$$

(ii) H_0 : Data for no. of claims confirms to be binomial distribution

H_1 : " " " " " doesn't " "

$p = 0.1625$, $P(X=x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$

$P(X=0) = {}^{4}C_{0} (0.1625)^{0} \cdot (0.8375)^{4} = 0.4920$

$P(X=1) = {}^{4}C_{1} (0.1625)^{1} \cdot (0.8375)^{3} = 0.3818$

$P(X=2) = {}^{4}C_{2} (0.1625)^{2} \cdot (0.8375)^{2} = 0.1111$

$P(X=3) = {}^{4}C_{3} \cdot (0.1625)^{3} \cdot (0.8375)^{1} = 0.0144$

$P(X=4) = {}^{4}C_{4} \cdot (0.1625)^{4} \cdot (0.8375)^{0} = 0.0007$

No. of claims →	0	1	2	3	4
O_i →	86	75	16	2	1

$= 180$

E_i → 180×0.492	180×0.3818	180×0.1111	180×0.0144	180×0.0007

$603p - 117 + 117p = 0$

$720p - 117 = 0$

$\hat{p} = 0.1625$

(ii) H_0 : Data for no. of claims confirms to be binomial distribution

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$P(X=2) = {}^4C_2 (0.1625)^2 . (0.8375)^2 = 0.1111$

$P(X=3) = {}^4C_3 . (0.1625)^3 . (0.8375)^1 = 0.0144$

$P(X=4) = {}^4C_4 . (0.1625)^4 . (0.8375)^0 = 0.0007$

No. of claims →	0	1	2	3	4
O_i →	86	75	16	2	1

$= 180$

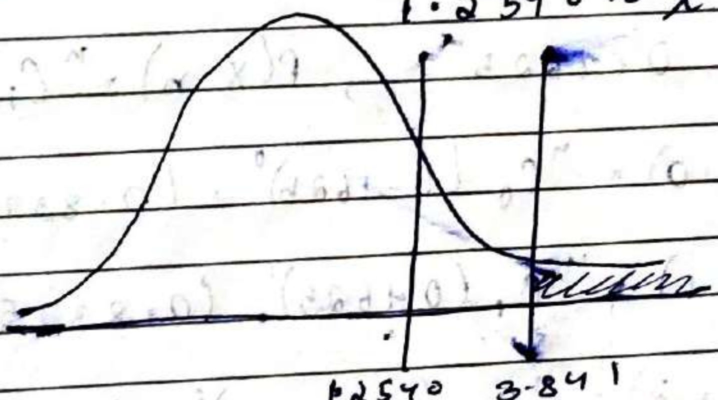
$E_i \to 180 \times 0.492$ 180×0.3818 180×0.1111 180×0.0144 180×0.0007

$E_i \rightarrow$ 88.56 68.72 19.99 2.59 0.13

	0	1	2+
O_i	86	75	19
E_i	88.56	68.72	22.71

$$\sum_{i=1}^{n} \left[\frac{(O_i - E_i)^2}{E_i} \right] = 1.02540 \sim \chi^2_{5-1-1-2}$$

$1.2540 \sim \chi^2_1$



Since, T.S lies in A.R, we have insufficient evidence to reject H_0 at 5% L.S.

∴ It concludes that data conforms to follow Binomial distⁿ.

4. $f(x) = \dfrac{CB^3}{(x+B)^4}$; $x > 0$, $B > 0$

$$\int_0^\infty f(x)\,dx = 1$$

$$CB^3 \int_0^\infty \frac{1}{(x+B)^4} = 1$$

$$CB^3 \left[\frac{(x+B)^{-3}}{-3} \right]_0^\infty = 1$$

$$\frac{CB^3}{-3} \left[0 - B^{-3} \right] = 1$$

$$C = 3$$

$$E(x) = \int_0^\infty x f(x)\, dx$$

$$= 3B^3 \int_0^\infty \frac{x}{(x+B)^{+4}}\, dx$$

$$= 3B^3 \int_0^\infty \frac{x+B}{(x+B)^{+4}} - \frac{B}{(x+B)^{+4}}\, dx$$

$$= 3B^3 \left[\frac{(x+B)^{-2}}{-2} - \frac{B\,(x+B)^{-3}}{-3} \right]_0^\infty$$

$$= 3B^3 \left[0 - 0 - \left[\left(\frac{B^{-2}}{-2} \right) - \left(\frac{B^{-2}}{-3} \right) \right] \right]$$

$$= 3B^3 \left[\frac{B^{-2}}{2} - \frac{B^{-2}}{3} \right]$$

$$= \frac{B^3}{2} \left[B^{-2} \right] = \frac{B}{2}$$

$$E(x^2) = \int_0^\infty x^2 \cdot f(x)\, dx$$

$$= 3B^3 \int_0^\infty \frac{x^2}{(x+B)^4} \cdot dx$$

Using Pareto Dist^n $\rightarrow \frac{3B^2}{5} \Rightarrow V(x)$

where $\alpha = 3$

$\lambda = B$

ii) $B = X_1, X_2, \dots X_n$

$E(x) = \bar{X}$

$\frac{B}{2} = \bar{X}$

$\hat{B} = 2\bar{X}_n$

$$E(\hat{B}) = E(2\bar{X}_n) = 2E\left(\frac{\Sigma X_i}{n}\right)$$

$$= \frac{2}{n} \Sigma E(X_i) = \frac{2}{n} \times n \times \frac{B}{2} = B$$

$\therefore$ Unbiased.

$$Var(\hat{B}) = Var(2\bar{X}_n) = 4\,Var[\bar{X}_n] = 4\,Var[X]/n$$

$$= 4 \times \frac{3B^2}{4}/n = \frac{3B^2}{n}$$

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Hence $M.S.E[\hat{B}] = \frac{3B^2}{n} + 0 = \frac{3B^2}{n}$

the estimator is consistent since MSE tends to zero as $n \to \infty$

iii) $MSE[b\bar{X}_n] = Var[b\bar{X}_n] + [Bias(b\bar{x}_n)]^2$

$$= b^2 \times \frac{3}{4} \times \frac{B^2}{n} + (E(b\bar{x}_n) - B)^2$$

$$= b^2 \times \frac{3}{4} \times \frac{B^2}{n} + \left[B\left(\frac{b}{2} - 1\right)\right]^2$$

$$= \frac{B^2}{n}\left[\frac{3b^2}{4} + n\left(\frac{b}{2} - 1\right)^2\right]$$

Diff the MSE repect to b:

$$\frac{d(MSE)}{db} = \left[\frac{B^2}{n}\left(\frac{3}{2}b + n\left(\frac{b}{2} - 1\right)\right)\right]$$

Setting this equal to 0 gives $b = \frac{2n}{n+3} = \frac{2}{1+3/n}$

Diff MSE for 2nd time wrt b:

$$\frac{d^2}{db^2}(MSE) = \frac{B^2}{n}\left(\frac{3}{2} + \frac{n}{2}\right) > 0$$

$\therefore$ Min value for MSE at $b = \frac{2}{1+3/n}$

iv) It is seen in (ii) that $B = 2\bar{x}n$ is an unbiased estimator. The estimator $b\bar{x}n$ in iii) where $b = \frac{2}{1 + 3/n}$ is −vely biased estimator for $b < 2$, $n \geq 1$.

As $n \to \infty$, b (tends to 2, so estimator in iii is also consistent as in ii)

5i) $E(x) = \bar{X}$

$\frac{a+b}{2} = \bar{X}$

$a = 2\bar{X} - b$

$V(x) = S^2$

$\frac{(b-a)^2}{12} = S^2$

$b - a = 2\sqrt{3}\,S$

$a = b - 2\sqrt{3}\,S$

$2\bar{X} - b = b - 2\sqrt{3}\,S$

$2b = 2\bar{X} + 2\sqrt{3}\,S$

$\hat{b} = \bar{X} + \sqrt{3}\,S$

$a = 2\bar{X} - \bar{X} - \sqrt{3}\,S$

$\hat{a} = \bar{X} - \sqrt{3}\,S$

iii) Sample $\to 1, 2, 3, 4, 50$, $\bar{X} = 12$, $S = 21.23$

$\hat{a} = 12 - \sqrt{3}\,(21.27) = -24.84$

$\hat{b} = 12 + \sqrt{3}\,(21.27) = 48.84$

In $U(a, b)$, the prob of a sample pt. being less than 'a' or greater that 'b' is zero and we have a sample value so that is greater than our estimate 'b'

It highlights a potential w.r.t. to MOM

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ii) $X \sim U(0, b)$

$$f(x) = \frac{1}{b-a} = \frac{1}{b}$$

$$L = f(x_1) \cdot f(x_2) \cdots f(x_n)$$

$$= \frac{1}{b} \cdot \frac{1}{b} \cdots \frac{1}{b} = \frac{1}{b^n}$$

$$l = -n \log b$$

$$\frac{dl}{db} = \frac{-n}{b} = 0, \; b \to \infty$$

$$\frac{d^2 l}{db^2} = +\frac{n}{b^2} > 0, \text{ min}^m \text{ at } b \to \infty$$

So, using common sense, $L(\theta)$ is max^m when θ is minimum $b \geq \text{Max } X_i$

$$\boxed{\hat{b} = \text{Max } X_i}$$

6 (i) $P(\text{type 1 error}) = P(\text{Reject } H_0 / H_0 \text{ is T})$

X be no. of hits in 1st step 12 missiles

Y be no. of hits in 2nd step 12 missiles

$P(\text{Type 1 error}) = P(X \geq 3 / p = 0.1) +$
$P(X = 0/p = 0.1) \cdot P(Y \geq 5 / p = 0.1) +$
$P(X = 1/p = 0.1) \cdot P(Y \geq 4 / p = 0.1) +$
$P(X = 2/p = 0.1) \cdot P(Y \geq 3 / p = 0.1)$

$= (1-0.8891) + (0.2824)(1-0.9957) +$
$(0.6590-0.2824)(1-0.9944) +$
$(0.8891-0.6590)(1-0.8891)$

$= 0.14727 \approx 15\%$

ii) $P(X \geq 3 / p=0.3) + P(X=0 / p=0.3) . P(Y=5, p=0.3)$
$+ P(X=1 / p=0.3) . P(Y \geq 4 / p=0.3) + P(X=2 / p=0.3)$
$P(Y=3 / p=0.3)$

$= 0.91253 \approx 91\%$

iii) P(type II error) is the prob of accepting H_0 when H_0 is F.

$= 1-0.91253 = 0.08747 \approx 9\% \; (p=0.3)$

iv) 10 space agencies fire 120 missiles and record 40 hits.

$X \sim Bin(120, 1/3) \quad \hat{p} = \frac{40}{120} = \frac{1}{3}$

$$\frac{\frac{X}{n} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

$$P\left[-1.2816 < \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} < 1.2816\right]$$

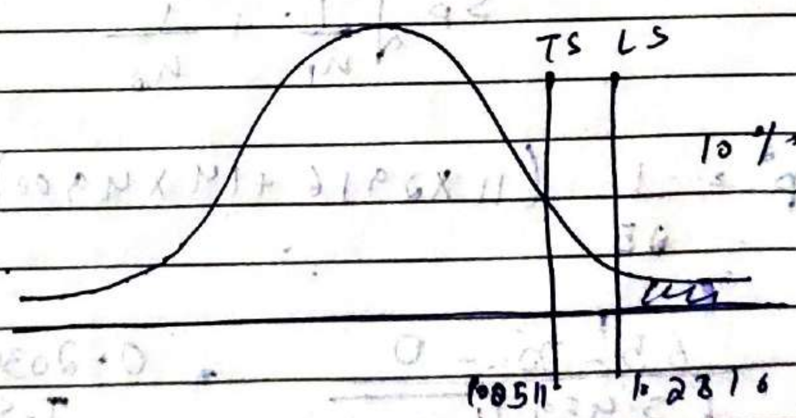
At CI 90% $p \in (0.3333, 0.2782)$

v) $p > 0.278$ at 10% LS

$H_0 : p = 0.278$
$H_1 : p > 0.278$

$$\frac{x - np_0}{\sqrt{np_0q_0}} \sim N(0,1)$$

$$\frac{39.5 - 120 \times 0.278}{\sqrt{120 \times 0.278 \times 0.722}} = 1.0511$$



As T.S < 1.2816.

We have insufficient evidence to reject H_0 at L.S 10%.

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vi) Lower Bound of C I implies that there is only a 10% chances 'p' $\in$ 0.2782, whereas from the hypothesis test, 'p' could be less than $\leq$ 0.2780 with prob. more than 10%. (approx 10.5% corresponding to 1.2511)

The minor disconnect b/w C.I. & H.T at some level is due to.

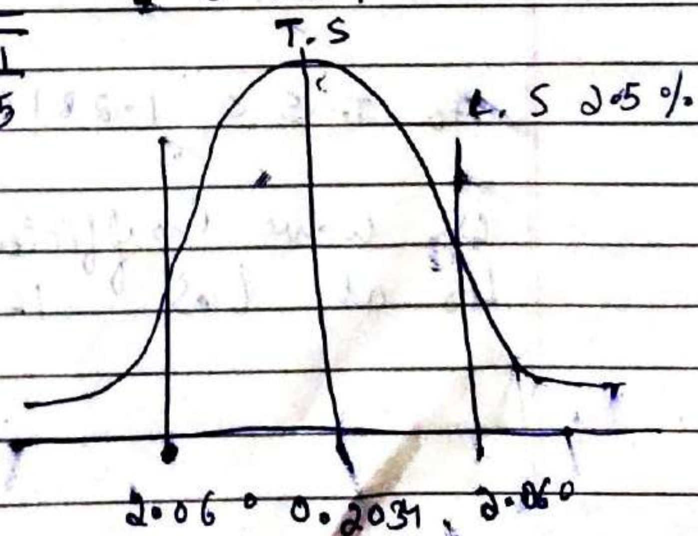
1) Applying continuity correction in H.T
2) Use of sample proportion $\hat{p}$ to estimate pop^n variance in calculating C.I.

vii) $H_0 : \mu_1 = \mu_2 \quad H_1 : \mu_1 \neq \mu_2$

$$P.Q \Rightarrow \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$S_p^2 = \frac{1}{25}(11 \times 2916 + 14 \times 4900) = 4027.04$$

$$\frac{65 - 70 - 0}{63.459\sqrt{\frac{1}{12} + \frac{1}{15}}} = 0.2034$$



Since T.S lies in A R, we have insuff. evidence to reject H_0 at 5% L.S.

7. $\Sigma X_{PUt} = 270$, $\bar{X}_{PUt} = 30$

$\Sigma X_{PAi} = 315.5$, $\bar{X}_{PAi} = 35.06$

$\Sigma X^2_{PUt} = 8614.5$, $S_1^2 = \dfrac{8614.5 - 9 \times 30^2}{8}$

$= 64.31$

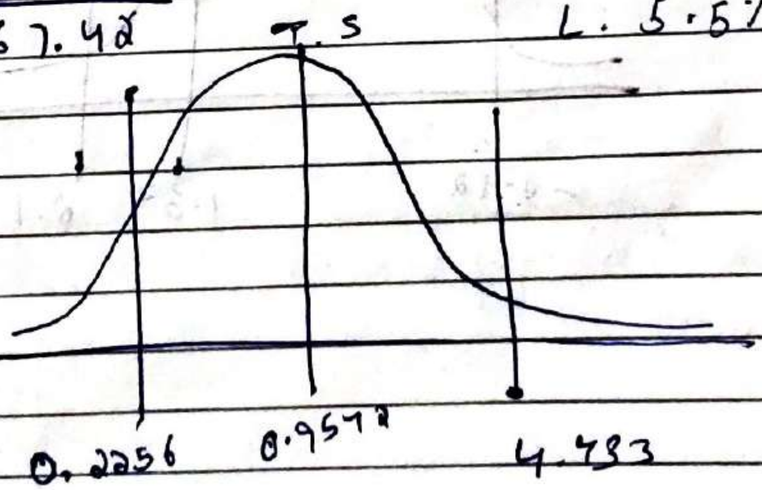
$S_2^2 = \dfrac{11599.25 - 9 \times 35.06^2}{8} = 67.42$

ii) $H_0 : \sigma_1^2 = \sigma_2^2$, $H_1 : \sigma_1^2 \neq \sigma_2^2$

T.S - $\dfrac{S_1^2 / \sigma_1^2}{S_2^2 / \sigma_2^2} \sim F_{n_1 - 1, n_2 - 1}$

$= \dfrac{64.31}{67.42} = 0.9592 \sim F_{8,8}$

T.S L.S 5%



Since T.S lies in A.R, we have insuff. evidence to reject H_0 at L.S 5%.

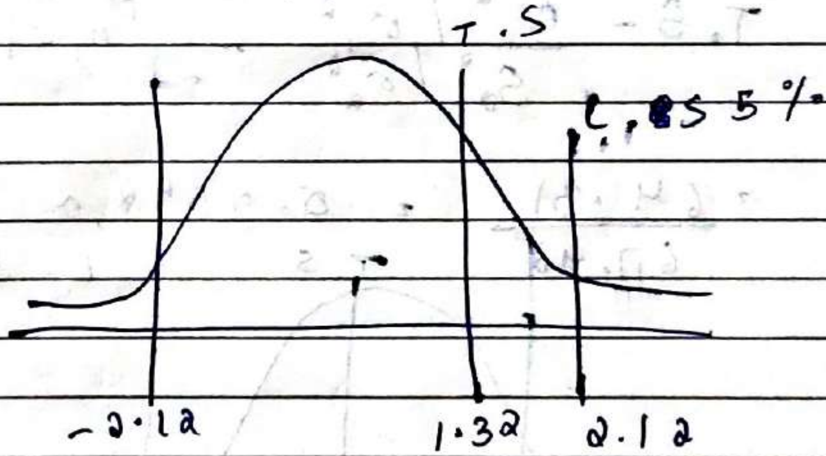
$\therefore$ Pop^n. Var are =.

iii) $H_0 : \mu = \mu_2$; $H_1 : \mu_1 < \mu_2$

$$T.S = \frac{(\bar{X}_2 - \bar{X}_1) - (\mu_2 - \mu_1)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}$$

$$S_p^2 = \frac{64.31 \times 8 + 67.42 \times 8}{16} = 65.86$$

$$T S \rightarrow \frac{(35.06 - 30) - 0}{\sqrt{65.86}\sqrt{\frac{1}{9} + \frac{1}{9}}} = 1.32$$



Since, T. S lies in A. R, we have insuff. evidence to reject H_0 at 5% LS

∴ Cost of claims in private hospitals is similar to that of public hospitals

8) i) $\hat{\theta}$ is said to be unbiased when $E(\hat{\theta}) = \theta$

ii) Measure of bias is given by $E(\hat{\theta}) - \theta$

iii) M.S.E of this estimator $\hat{\theta} = (E(\hat{\theta}) - \theta)^2$

(iv) $\hat{\theta}$ is efficient as an estimator with lower MSE is said to be more efficient than one with higher MSE

v) θ can be estimated using :

a) Method of Moments : The popⁿ moments are equated to the sample moments to estimate the parameters

b) MLE : A maxᵐ likelihood fⁿ $L(\theta) = \prod_{i=1}^{n} f(x_i ; \theta)$ is generated. A MLE of parameter is given by solⁿ to $\frac{dL}{d\theta} (\theta < 0$

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a) i) Variable t_k is defined as $t_k = \frac{N(0,1)}{\sqrt{\chi^2_k/k}}$
where k = degree of freedom

& 2 R.V $N(0,1)$ & χ^2_k are independent.

ii) Mean & Var of t_k for $k > 2$ are 0 and $\frac{k}{k-2}$ resp.

iii) $\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}$, $t_9 = 3.25$

CI for μ is thus $\left(50 - 3.25\sqrt{\frac{48.5067}{10}},\ 50 + 3.2\right)$ i.e $(42.83, 57.17)$

b. $t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}}\right) \sim N\left(0, \sqrt{9/7}\right)$

$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim N\left(0, \sqrt{9}/\sqrt{7}\right)$

$\frac{\bar{x} - \mu}{s/\sqrt{7n/9}} \sim N(0,1)$

CI for μ E $\left(50 - 2.58\sqrt{\frac{48.667}{10} \times \frac{9}{7}},\ 50 + 2.58\sqrt{\frac{48.667}{10} \times \frac{9}{7}}\right)$

i.e $(43.54, 56.45)$

10 i) $P(\text{type I error}) = P(\text{Rej } H_0 / H_0 \text{ is T})$

$X \sim Poi(n\mu)$ i.e $Poi(3000)$

$H_0, X \sim Poi(3000)$

$P(\text{Reject } H_0 / H_0 \text{ is T}) = P(X > 3100$ when $X \sim Poi(3000)$

$X \sim N(3000, 3000)$

$$P(X > 3100) = P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$$

$1 - P(Z < 1.825) = 3.39\%$

ii) $P(\text{type II error}) = P(\text{Accepting } H_0 / H_0 \text{ is F})$

iii) Power of test - Prob of rejecting / H_0 is F

$P(\text{reject } H_0 / H_0 \text{ is F}) = P(X > 3100$ when $X \sim Poi(n\mu) \sim N(n\mu, n\mu)$

$$P(X > 3100) = 1 - \left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$$

$\therefore$ Power of test depends on value of H_1's parameter.

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iv) $\hat{\mu} \sim N(\mu, \hat{\mu}/n)$

$$\frac{\hat{\mu} - \mu}{\sqrt{\hat{\mu}/n}} \sim N(0,1)$$

$$P\left(-2.5758 < \frac{2.9 - \mu}{\sqrt{2.9/1000}} < 2.5758\right)$$

$$= 0.99$$

$$\mu \in (2.7613, 3.0387)$$

11) Revised sample mean $= \dfrac{213200 - 11000 - 48000 + 27500 + 37500}{12}$

$= 17987.67$

$\Sigma x^2 = 4919860000 - 121000000 - 2304000000 + 756250000 + 992250000$
$= 4243360000$

$\therefore$ Revised S.D $= \left[\frac{1}{11}\left(4243360000 - \frac{213200^2}{12}\right)\right.$

$= 6435$

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12. i) The C.R.L.B result holds under very general conditions except where range of distⁿ involves the parameters. Such as uniform distⁿ in this case

ii) $Y = \max_i X_i$

Since X_i lies b/w 0 & θ, the support for Y will be 0 & θ. for $0 < y < \theta$

$$P(Y < y) = P[\max X_i < y]$$
$$= P\left[\bigcap_{i=1}^{n} (X_i < y)\right]$$
$$= \prod_{i=1}^{n} P[X_i < y] \quad [X_i \text{ is indep}]$$
$$= \prod_{i=1}^{n}\left[\int_0^y \frac{dx}{\theta}\right] = \left(\frac{y}{\theta}\right)^n$$

$$= \frac{n}{\theta^n} y^{n-1} \text{ for } 0 < y < \theta$$

Now for $k \geq 0$

$$E[Y^k] = \int_0^\theta y^k \frac{n}{\theta^n} \cdot y^{n-1}\, dy$$
$$= \frac{n}{n+k}\int_0^\theta \frac{n+k}{\theta^n} y^{n+k-1}\, dy$$

$$= \frac{n\,\theta^k}{n+k}$$

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iii) Bias of estimator $\hat{\theta}(c)$ is given as →

$$\begin{aligned} \text{Bias}[\hat{\theta}(c)] &= E[\hat{\theta}(c) - \theta] \\ &= E[cY - \theta] \\ &= cE[Y] - \theta \\ &= c \cdot \frac{n\theta}{n+1} - \theta \\ &= \left(\frac{cn}{n+1} - 1\right)\theta \end{aligned}$$

MSE of estimate $\hat{\theta}(c)$ is given as:

$$\begin{aligned} \text{MSE}[\theta(c)] &= E[(\theta(c) - \theta)^2] \\ &= E[(cY - \theta)^2] \\ &= E[c^2y^2 - 2c\theta Y + \theta^2] \\ &= c^2E(Y^2) - 2c\theta E(Y) + \theta^2 \\ &= c^2 \cdot \frac{n\theta^2}{n+2} - \frac{c \cdot 2n\theta^2}{n+1} + \theta^2 \end{aligned}$$

iv) For $\hat{\theta}(c)$ to be an unbiased estimator of θ, we need $\text{Bias}[\hat{\theta}(c_u)] = 0$

$$\Rightarrow \left(\frac{c_u \cdot n}{n+1} - 1\right)\theta = 0$$

$$\Rightarrow c_u = \frac{n+1}{n}$$

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v) In order to minimise MSE $[\hat{\theta}(c)]$, we need

$$0 = \frac{d}{dc} MSE[\hat{\theta}(c)] = \frac{d}{dc}\left[\frac{c^2 \cdot n\theta^2}{n+2} - \frac{2n\theta^2}{n+1} \cdot c + \theta^2\right]$$

$$= \frac{2cn\theta^2}{n+2} - \frac{2n\theta^2}{n+1}$$

$$\text{Thus } c_n = \frac{2n\theta^2}{n+1} \cdot \frac{n+2}{2n\theta^2} = \frac{n+2}{n+1}$$

$$\frac{d^2}{dc^2}\left[MSE[\hat{\theta}(c)]\right] = \frac{d}{dc}\left[\frac{2c\,n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}\right]$$

$$= \frac{2n\theta^2}{n+2} > 0 \Rightarrow \text{min}^m$$

vi) In order to minimize errors in estimation, it is preferred to opt for an estimator which has lower MSE among diff. competing estimator. So here $\hat{\theta}(c_m)$ will be preferred over $\hat{\theta}(c_u)$. As n becomes large $\hat{\theta}(c_u) = \frac{n+1}{n} Y \to Y$ ||| ly, $\hat{\theta}(c_m) = \frac{n+2}{n+1} Y \to Y$, the two estimator becomes one & same as $n \to \infty$.