Q1)(i) A stochastic model recognises the random nature of the input components.
It provides with distribution of results rather than single best estimate and allows to reflect reality as accurate as possible.

(ii) If age profile is assigned a function $f(a)$, what we do is set this function to roughly constant for all employees so our average rate of healthy employee falling sick which is equal to $\int f(a)\,\rho(a)\,da$, also will be constant for all employees at time 'a' & so we can still use time-homogeneous models.

(iii) $P[X_t = i \mid \underbrace{X_1 = x_1, X_2 = x_2, \ldots\ldots X_n = x_n}]$

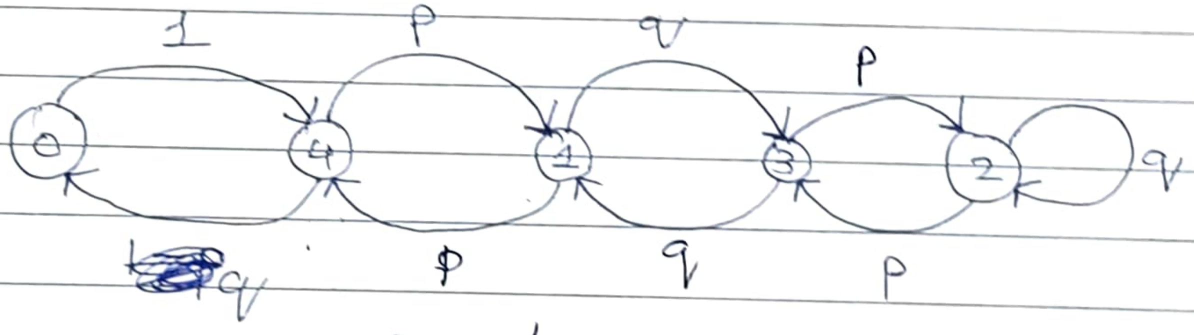
$\Rightarrow P[X_t - X_n - x_n = i \mid \hookrightarrow F_n]$

as $[X_t - X_n]$ is independent, $X_t - X_n + x_n$ will also be independent so.

$\Rightarrow P[X_t - X_n - x_n = i \mid X_n = x_n] \Rightarrow P[X_t = i \mid X_n = 0]$.

$\therefore$ proven that X_n is stochastic process.

Q2)(i)



$q = 1 - p$

(ii)
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q & p \\ 0 & 0 & q & p & 0 \\ 0 & q & p & 0 & 0 \\ q & p & 0 & 0 & 0 \end{bmatrix}$$

JYOTI

(ii) $[\pi_0, \pi_1, \pi_2, \pi_3, \pi_4] = [\pi_0, \pi_1, \pi_2, \pi_3, \pi_4] \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & q & p \\ 0 & 0 & q & p & 0 \\ 0 & q & p & 0 & 0 \\ q & p & 0 & 0 & 0 \end{bmatrix}$

$\pi_0 = q \cdot \pi_4$ — (1)

$\pi_1 = q \cdot \pi_3 + p \cdot \pi_4$ — (2)

$\pi_2 = q \cdot \pi_2 + p \cdot \pi_3$ — (3)

$\pi_3 = q \cdot \pi_1 + p \cdot \pi_2$ — (4)

$\pi_4 = 1 \cdot \pi_0 + p \cdot \pi_1$ — (5)

$\pi_4 = q \cdot \pi_4 + p \cdot \pi_1 \Rightarrow p\pi_4 = p\pi_1$

so $\pi_4 = \pi_1$

$\pi_1 = q \cdot \pi_3 + p \cdot \pi_1 \Rightarrow \pi_3 = \pi_1$ so $\pi_4 = \pi$

~~$\pi_3 = q \cdot \pi_1 + p \cdot \pi_4$~~

$\pi_2 = \pi_2 \cdot q + p \cdot \pi_1 \Rightarrow \pi_2 = \pi_1$.

so $\boxed{\pi_1 = \pi_2 = \pi_3 = \pi_4}$

also, $\pi_0 = q \cdot \pi_4$ so $\pi_4 = \pi_0 / q$.

$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \dfrac{\pi_0}{q}$ (sum = 1)

~~$\dfrac{\pi_0}{q} = 0.25 \dfrac{1}{5} \Rightarrow \pi_0 = \dfrac{q}{5}$~~

$q \, \underbrace{\pi_4}_{\pi_0} + \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$

$\therefore \pi_4 = \dfrac{1}{4+q} = \pi_1 = \pi_2 = \pi_3$ & $\pi_0 = \dfrac{q}{4+q}$

(iv) $P(\text{wet}) = P(0 \text{ umbrella}) \times P(\text{raining})$

$$= p \times \frac{q}{4+q} = \frac{pq}{4+q}.$$

(v) $p = 0.6$

so let n be no. of umbrellas

$$\pi_1 = \pi_2 = \pi_3 = \pi_4 = \frac{\pi_0}{q}.$$

so for n umbrellas, $\pi_1 = \pi_2 = \dots = \pi_n = \frac{\pi_0}{q}.$

so. $\pi_n \cdot q + n \cdot \pi_n = 1.$

$$\pi_n = \frac{1}{n+q} \qquad P(\text{wet}) = \frac{q}{n+q} \times p$$

$$\frac{0.6\times0.4}{n+0.4} < 0.01 \Rightarrow$$

$$\frac{n+0.4}{0.6} > 100. \Rightarrow n+0.4 > \cancel{60}\ 24$$

$$n > \cancel{59.6}\ 23.6$$

So to follow this process & have probablity of getting wet less than 1%, he needs to keep 24 umbrellas atleast.

Q3 (i) & (ii) counting → discrete SS, continuous TD

GRM → continuous SS, continuous TD

poisson process → discrete SS, discrete TD

Markov chain → discrete SS, discrete TD

Markov Jump → discrete SS, continuous TD

(iii) (a) counting process
(b) markov chain
(c) poisson process
(d) markov jump process

Q4) (i) According to markov property, the current event is only dependent on the value of the variable in time t-1, but here a event of going 1 or 2 units up or down not only depends on previous year but also depends on year before that & so it does not follow Markov property.

(ii)

Q5)(i) From the formula, we can see we use past values of Y_i to calculate current Y_i & so we can say that Y_i has dependency on past values & X_i is sum of Y_i & so markov property does not hold in this case

(ii) From the above answer, we saw that the data does not hold the markov property & so ~~does not~~ is not a markov chain.

(iii)

$$\begin{bmatrix} p & 1-p & 0 & 0 & \cdots & & & 0 \\ 0 & pe^{-\lambda} & 1-pe^{-\lambda} & 0 & \cdots & & & 0 \\ 0 & 0 & 0 & pe^{-2\lambda} & 1-pe^{-2\lambda} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iv) (a) as the transition probablities calculated has no time component, it is homogeneous

(b) the no. of errors can't decrease, so irreducible

(c) there are no recurrent states so it does not have stationary probablity distribution

(v) $$[p.e^{-\lambda j(0)}]^n = [pe^{-\lambda j}]^n = p^n e^{-\lambda nj}$$

Q6)(a) Stochastic model are models which predict time dependent random variables which has a level of uncertainity.

Stochastic models replicate reality much more better than deterministic models ~~and~~ and it provides with distribution of results rather than a single best estimate.

(b) (i) transition intensities represent prob. of transition in very small time

$$\sigma_{ij}(t) = \frac{d}{dt} P_{ij}(t) \Big|_{t=0}.$$

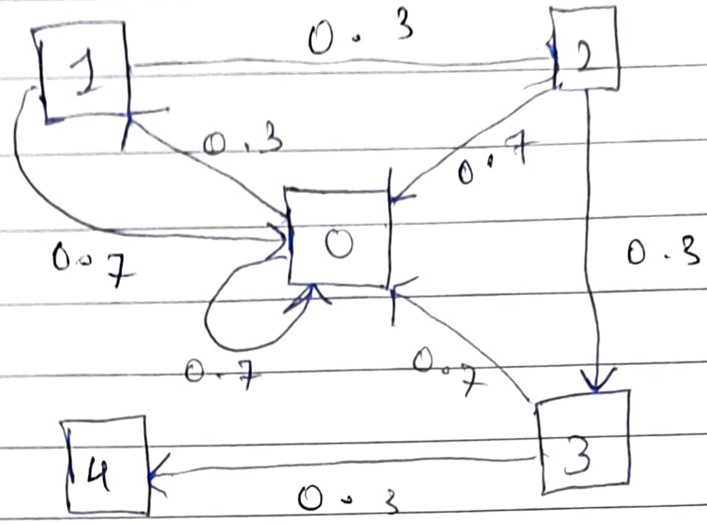
assuming constant transition rates throughout

(ii) sum of transition rates ~~from i~~ in row $= \mu_{ii} + \sum_{j \neq i} \mu_{ij}$

~~to lets derive value of μ_{ii}~~.

we know $\mu_{ii} = -\sum_{j \neq i} \mu_{ij}$.

so sum $= -\sum_{j \neq i} \mu_{ij} + \sum_{j \neq i} \mu_{ij} = 0$!, hence shown (i)(a)



(b) Markov chain is irreducible when you can reach to every other state in chain from all states. The above chain is not irreducible as 4 is an absorbing state so from there you can't go to any other state.

(c)(i) $(0.3)^4 = 0.0081$ M

(ii) O L L L L $\Rightarrow 0.7 \times (0.3)^4$

$= 0.00567$ M

(iii) L L NoL L L L L $= (0.3)^6 \times 0.7 +$

NoL NoL $(0.3)^4 \times (0.7)^3 +$

NoL L $(0.3)^5 \times (0.7)^2 +$

L NoL $(0.3)^5 \times (0.7)^2$

$\hookrightarrow$ fixed.

$= 0.00567$ M

(iv)

MML → L L L L L, NoL L L L L

1	x	L	L	L	NoL	
$^4C_2$	x	L	L	NoL	NoL	↳ fixed.
$^4C_3$	x	L	NoL	NoL	NoL	
1	x	NoL	NoL	NoL	NoL	

$$= 4 \times (0.3)^7 \times (0.7)^2 + {}^4C_2 \times (0.3)^6 \times (0.7)^3 + {}^4C_3 \times (0.3)^5 \times (0.7)^4 + 1 \times (0.3)^4 \times (0.7)^5$$

$$= 0.005624$$

(d) ~~So no of steps the process needs to reach to step $\boxed{4}$ from step $\boxed{0}$~~

~~$E[4] = 0$, $E[3] = 0.7E[0] + 0.3 \times 0$~~

~~$E[2] = 0.7E[0] + 0.3 \times 0.7 E[0] +$~~

~~$E[1] = 0.7E[0] + 0.3 \times 0.3 \times 0.3 \times 0.7 \times E[0] + 0.3 \times 0.3 \times 0.7 \times E[0]$~~

~~$E[0] = 0.7 \times E[0] + 0.3 \times E[1]$.~~

~~let $E[0] = x$.~~

~~$x = 0.7x + 0.21x + 0.063x + 0.0189x$.~~

~~$x = 0.9919x \Rightarrow$~~

(d)

$$Q = \begin{array}{c|c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \hline \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0.7 & 0 & 0.3 & 0 \\ 0.7 & 0 & 0 & 0.3 \\ 0.7 & 0 & 0 & 0 \end{bmatrix} \end{array} \Rightarrow I - Q = \begin{bmatrix} 0.3 & -0.3 & 0 & 0 \\ -0.7 & 1 & -0.3 & 0 \\ -0.7 & 0 & 1 & -0.3 \\ -0.7 & 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

take inverse & add first row

$$N = 175$$

Q8(i) We could model the game w.r.t any of the players. So we will have five states, $-2, -1, 0, 1, 2$ where these show the difference in points. A state of -2 means that particular player is 2 points behind in tie-breaker & $+2$ means the opposite. 0 means both have same number of points in the tie-breaker.

(ii)

w.r.t roger federer

$$
\begin{array}{c|ccccc}
 & -2 & -1 & 0 & 1 & 2 \\
\hline
-2 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0.45 & 0 & 0.55 & 0 & 0 \\
0 & 0 & 0.45 & 0 & 0.55 & 0 \\
1 & 0 & 0 & 0.45 & 0 & 0.55 \\
2 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

(iii)

(a) No because you can't go anywhere from state 2 & -2

(b) No, as states -1 & 1 have period of 2.

(iv)

(v)

$[\pi_1, \pi_2, \pi_3, \pi_4, \pi_5] \quad [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5]$

$$P\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.45 & 0 & 0.55 & 0 & 0 \\ 0 & 0.45 & 0 & 0.55 & 0 \\ 0 & 0 & 0.45 & 0 & 0.55 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\pi_1 = \pi_1 + 0.45\pi_2$

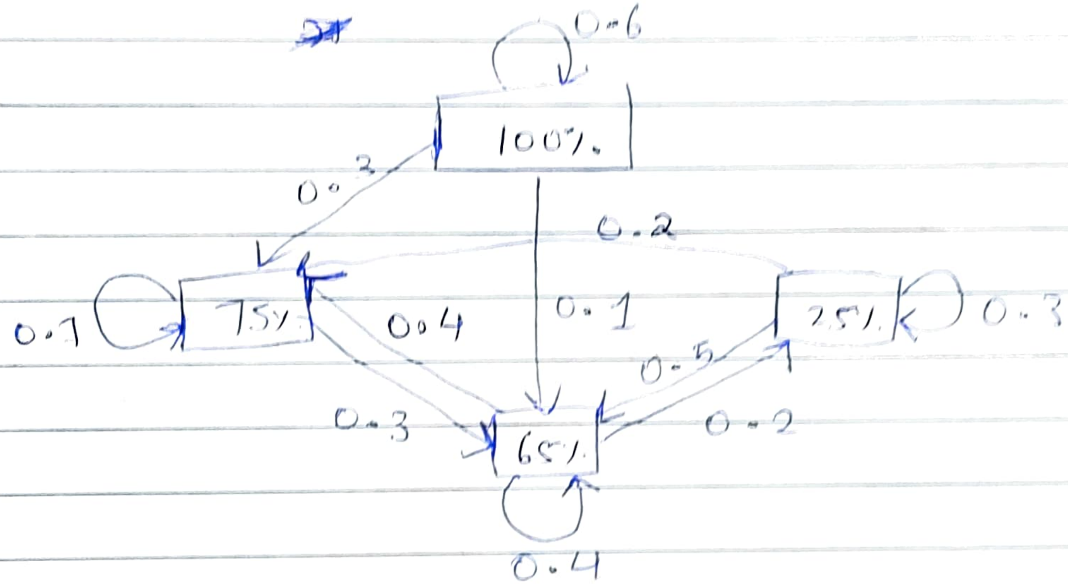
$\pi_2 = 0.45\pi_3$

$\pi_3 = 0.55\pi_2 + 0.45\pi_4$

$\pi_4 = 0.55\pi_3$

$\pi_5 = \pi_5 + 0.55\pi_4$

Q9)(a)



(b) $S_0 = [0.05, 0.3, 0.45, 0.2]$

(i) in 1 year $= [0.05, 0.3, 0.45, 0.2]\begin{bmatrix} 0.6 & 0.3 & 0.1 & 0 \\ 0 & 0.7 & 0.3 & 0 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0 & 0.2 & 0.5 & 0.3 \end{bmatrix}$

$= [0.03, 0.445, 0.375, 0.15]$

in 2 year $= [0.03, 0.445, 0.375, 0.15]\begin{bmatrix} 0.6 & 0.3 & 0.1 & 0 \\ 0 & 0.7 & 0.3 & 0 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0 & 0.2 & 0.5 & 0.3 \end{bmatrix}$

$= [0.018, \overset{0.5005}{\cancel{0.278}}, 0.3615, 0.12]$ 36.15%

(ii) $[x_1, x_2, x_3, x_4] = [x_1, x_2, x_3, x_4] \times \begin{bmatrix} 0.6 & 0.3 & 0.1 & 0 \\ 0 & 0.7 & 0.3 & 0 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0 & 0.2 & 0.5 & 0.3 \end{bmatrix}$

so $x_1 = 0.6x_1$, $x_2 = 0.3x_1 + 0.7x_2 + 0.4x_3 + 0.2x_4$,

$x_3 = 0.1x_1 + 0.3x_2 + 0.4x_3 + 0.5x_4$,

~~$x_4 = 0.1x_1 + 0.3x_2 + 0.4x_3 + 0.5x_4$~~, O.

$x_4 = 0.2x_3 + 0.3x_4$.

so $x_1 = 0$ //

$0.7x_4 = 0.2x_3$ $\qquad x_1 + x_2 + x_3 + x_4 = 1$

$x_2 = 0 + 0.7x_2 + \overset{1.4x_4}{\cancel{0.4x_3}} + 0.2x_4$

$0.3x_2 = 1.6x_4$.

$x_2 = 5.333x_4$. $\qquad x_3 = 3.5x_4$ //

$x_1 = 0$.

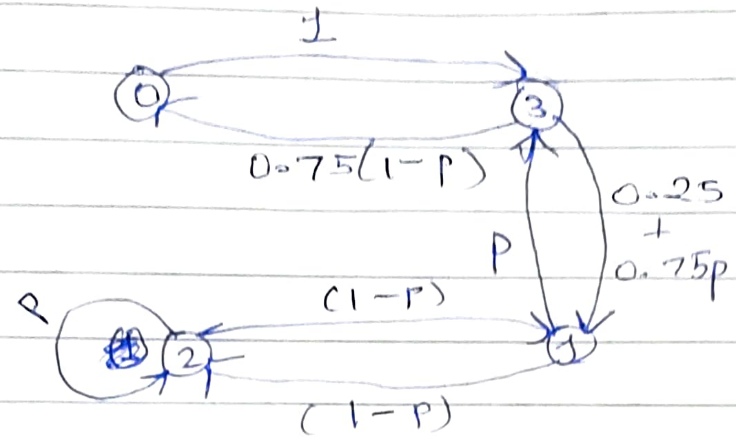
so $5.333x_4 + 3.5x_4 + x_4 = 1$

$x_4 = \frac{6}{59}$ //

$x_3 = \frac{21}{59}$ so $\boxed{35.59\% \text{ is long time proportion}}$

(c) We see in first two years the proportions drop to 37.5% & 36.2% so the steady state of 35.59% will be reached in few years time.

Q10) (i)



$3 \rightarrow 1$

no rain, no good

rain, no good

rain, good.

$= \cancel{0.75}\ 0.25(1-p) + 0.75p + 0.25p$

$$t_m = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & (1-p) & p \\ 0 & (1-p) & p & 0 \\ 0.75(1-p) & \underset{0.25+0.75p}{\cancel{0.75}} & 0 & 0 \end{bmatrix}$$

(ii)

$x_1 = x_4 \times [0.75(1-p)]$ — (1)

$x_2 = x_3(1-p) + x_4[0.25 + 0.75p]$

$x_3 = x_2(1-p) + x_3 p$

$x_4 = x_1 + p x_2$

$x_1 = 0.75 x_4 - 0.75p\, x_4$

$x_4 = 0.75 x_4 - 0.75p\, x_4 + p x_2$

$p x_2 = [0.25 + 0.75p] x_4$ — (2)

$x_2 = x_3(1-p) + p x_2 \Rightarrow x_2 = x_3$ — (3)

$$x_4 + 2 \times \left[\frac{0.25 + 0.75p}{p}\right] x_4 + [0.75 - 0.75p] x_4 = 1$$

$$\left[1 + \left(\frac{0.5 + 1.5p}{p}\right) + 0.75 - 0.75p\right] x_4 = 1$$

$$\left[\frac{p + 0.5 + 1.5p}{p} + \frac{0.75p}{p} - \frac{0.75p^2}{p}\right] x_4 = 1$$

$$\left[\frac{2.75p - 0.75p^2 + 0.5}{p}\right] x_4 = 1$$

$\therefore$

$$x_4 = \frac{p}{2.75p - 0.75p^2 + 0.5} = x_4$$

$$x_1 = \frac{0.75(1-p) \times p}{2.75p - 0.75p^2 + 0.5}$$

$$\Rightarrow \boxed{\frac{0.75p - 0.75p^2}{2.75p + 0.5 - 0.75p^2}}$$

0.15

(iii) When $n=4$, $P(\text{wet}) = 0.07654$

Q11) (i) at 1 $\rightarrow \{ 3$ prob = 1

at 2 $\rightarrow \{ 3$ prob = 1

at 3 $\rightarrow \begin{cases} 1 & 0.25 \\ 2 & 0.25 \\ 4 & 0.25 \\ 5 & 0.25 \end{cases}$ at 5 $\rightarrow \begin{cases} 3 & 0.5 \\ 6 & 0.5 \end{cases}$

at 4 $\rightarrow \begin{cases} 3 & 0.5 \\ 6 & 0.5 \end{cases}$ at 6 $\rightarrow \begin{cases} 4 & 0.5 \\ 5 & 0.5 \end{cases}$

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0.25 & 0.25 & 0 & 0.25 & 0.25 & 0 \\ 4 & 0 & 0 & 0.5 & 0 & \cancel{0.5} & 0.5 \\ 5 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 6 & 0 & 0 & 0 & 0.5 & 0.5 & 0 \end{array}$$

ii) As from the digram we can see that we can reach to any room i from any other room j where i, j ∈ [1, 6] & so the chain is irreducible. To come to room i again starting from room i itself, we would first need to go to different room & then come back so this makes 2 steps, similary more the no of rooms we visit, the steps would inc by factor of 2 as we would have to travel to & fro & so period for every state in this markov chain is 2 & so it is not aperiodic.

(iii) $SD = [x_1, x_2, x_3, x_4, x_5, x_6]$.

so $x_1 = 0.25x_3$, $x_2 = 0.25x_3$, $x_3 = x_1 + x_2 + 0.5x_4 + 0.5x_5$

$x_4 = 0.25x_3 + 0.5x_6$, $x_5 = 0.25x_3 + 0.5x_6$,

$x_6 = 0.5x_5 + 0.5x_4$

$\cancel{x_5 = x_4 + 0.5x_4 \Rightarrow x_5 = 1.5x_4}$

$x_4 = x_5$, $x_1 = x_2 = 0.25x_3$

$\cancel{x_4 = x_1 + 0.5x_6 = x_2 + 0.5x_6}$

$x_3 = x_1 + x_2 + x_6$

$x_6 = 0.5x_3$

$x_4 = 0.5x_3 = x_5$.

so $0.25x_3 + 0.25x_3 + x_3 + 0.5x_3 + 0.5x_3 + 0.5x_3 = 1$

$3x_3 = 1$, $x_3 = \frac{1}{3} = \frac{4}{12}$

so $x_1 = x_2 = \frac{1}{12}$, $x_4 = x_5 = \frac{1}{6} = \frac{2}{12}$ $x_6 = 0.5x_3 = \frac{1}{6} = \frac{2}{12}$

so $SD = [\frac{1}{12}, \frac{1}{12}, \frac{4}{12}, \frac{2}{12}, \frac{2}{12}, \frac{2}{12}]$

IV $E[\text{reaching state } 1, X_0 = 1] = 1 + E(3)$
$= 1 + 0.25E(1) + 0.25E(2) + 0.25E(4) + 0.25E(5)$

$E(2) = 1$, $E(4) = 0.25E(3) + 0.25E(6)$
$E(5) = E(4)$ -

12 wm

Q 12) ~~$= 1 + E(3)$~~

~~$= 1 + \left[\frac{1}{3}E(4) + \frac{2}{3}E(2)\right]$~~

~~$= 1 + \frac{1}{3}E(2)\ (1 + E(2))\ (1 + \frac{2}{3}E(2))$~~

~~$= 1 + \frac{2}{3} + \frac{2}{3} =$~~

Q 12) (i) $m_{i4} = \begin{cases} 0 & \text{if } i = 4 \\ 1 + \sum_{j \neq 4} p_{ij} \cdot m_{j2} & \text{if } i \neq 4 \end{cases}$

~~$m_{44} = 0$~~.

~~$m_{24} = 1 + \frac{1}{3}m_{34} + \frac{2}{3}m_{44}$~~

~~$E[T_A \mid X_0 = 2]$~~

$=$ ~~$1 + \sum E[T_A \mid X_0 = 3]$~~ $_{24}$

$m_{24} = 1 + m_{34}$

$= 1 + \frac{2}{3}m_{24} + \frac{1}{3}m_{44}$.

$= 1 + \frac{2}{3}m_{24} + 0$.

$m_{24} = 1 + \frac{2}{3}m_{24}$

$\frac{1}{3}m_{24} = 1$

$\boxed{m_{24} = 3}$

(ii) ~~x~~ $t_m =$

$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 1/15 & 3/5 & 0 & 0 & 1/5 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 2/3 & 0 & 1/3 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 7 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

$1 = [\,{}^{3}/_{20},\ \frac{1}{6}+\frac{9}{2}$

Q13) i)(a) Counts.

R → R	R → S	S → R	S → S
卌 |	卌 |||	卌 |||	卌 ||
6	8	8	7

$$\begin{array}{c|cc} & R & S \\ \hline R & 6/\cancel{29}\,14 & 8/\cancel{29}\,14 \\ S & 8/\cancel{29}\,15 & 7/\cancel{29}\,15 \end{array}$$

(b)

$$X_1 = \begin{bmatrix} \overset{R}{1} & \overset{S}{0} \end{bmatrix} \begin{bmatrix} 6/14 & 8/14 \\ 8/15 & 7/15 \end{bmatrix} = \begin{bmatrix} 6/29 & 8/29 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 6/\cancel{29}\,14, & 8/14 \end{bmatrix} \begin{bmatrix} 6/14 & 8/14 \\ 8/15 & 7/15 \end{bmatrix} = \begin{bmatrix} \frac{359}{735}, & \frac{376}{735} \end{bmatrix}$$

$$X_3 = \begin{bmatrix} \frac{359}{735}, & \frac{376}{735} \end{bmatrix} \begin{bmatrix} 6/14 & 8/14 \\ 8/15 & 7/15 \end{bmatrix} = \begin{bmatrix} 0.482, & 0.517 \end{bmatrix}$$

$$\boxed{P(\text{rain on } 3^{rd} \text{ July}) \approx 0.482}$$

(ii) (a) $1 - P(X=0)$

$= 1 - e^{-3} = 0.9502$

(b) $P(2 \leq X < 5)$

$= P(X = 2, 3, 4)$

$= e^{-3}\left[\frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right] = 0.61611$

(c) $= \frac{e^{-3/5} \times 3/5}{1} = 0.329286$