Probability & Statistics

Assignment - 1

Ques 1 i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$\Rightarrow$ Mean $= \dfrac{4+7+9+9+11+15+18+18+18+25}{10}$

$\bar{x} = 13.4$

$\Rightarrow$ Median $= \dfrac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$

$= \dfrac{11+15}{2} \Rightarrow 13$

$\Rightarrow$ Mode $= 18$

$\Rightarrow$ IQR $\Rightarrow$ $q_1 = 8$ $\quad q_3 - q_1$

$q_2 = 13 \quad \Rightarrow 18 - 8$

$q_3 = 18 \quad \Rightarrow 10$

ii)

No. of householde	0	1	2	3	4
No. of people	5	11	26	15	3

$Q_1 = \dfrac{15^{th} + 16^{th}}{2} = \dfrac{1+1}{2} = 1$

$Q_2 = 2$ $\quad$ IQR $= Q_3 - Q_1$

$Q_3 = \dfrac{3+3}{2} = 3$ $\quad = 2$

$\Rightarrow \bar{x} = \frac{\sum fx}{\sum f} \Rightarrow \frac{0\times5 + 1\times11 + 2\times26 + 3\times15 + 4\times3}{5+11+26+15+3}$

$\Rightarrow \frac{11+52+45+12}{60}$

$\Rightarrow \frac{120}{60} = 2$ Ans

$\Rightarrow$ Median: $n = 60$

$\Rightarrow \frac{\left(\frac{n}{2}\right)^{th} + \left[\left(\frac{n}{2}\right)+1\right]^{th}}{2}$

$\Rightarrow \frac{(30)^{th} + (31)^{st}}{2} \Rightarrow (30.5)^{th}$ term

$(30.5)^{th}$ term lies in the x_i, 2

Thus, median = 2

$\Rightarrow$ Mode = 2

Standard Deviation

x	f	$\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0	5	2	4	20
1	11	2	1	11
2	26	2	0	0
3	15	2	1	15
4	3	2	4	12

$S = \sqrt{\frac{\sum f(x-\bar{x})^2}{n-1}} \Rightarrow \sqrt{\frac{58}{59}} = 0.99149$

Ques 1(a) Standard Deviation

x	$\bar{x}$	$(x-\bar{x})^2$
4	13.4	88.36
7	13.4	40.96
9	13.4	19.36
9	13.4	19.36
11	13.4	5.76
15	13.4	2.56
18	13.4	21.16
18	13.4	21.16
18	13.4	21.16
25	13.4	134.56

$\Sigma(x_i-\bar{x})^2 = 374.4$

$\sigma^2 = \frac{374.4}{10}$

$= 37.44$

$\sigma = \sqrt{37.44}$

$= 6.12$

Ques 2 $x \sim P(0.5)$ $\lambda = 0.5$

(i) $P(x=0) \Rightarrow \frac{e^{-\lambda}\lambda^{x}}{x!}$

$\Rightarrow \frac{e^{-0.5}\, 0.5^{0}}{0!}$

$= e^{-0.5} = 0.6065$

ii) Let y = no. of years with a claim

then $y \sim$ binomial $(3, 0.3935)$

$P(y=1) \Rightarrow {}^{3}C_{1} \times (0.3935)^{1}(0.6065)^{2}$

$\Rightarrow 0.434$

iii) $X \sim \exp(0.5)$

$$P(x > 2) = \int_2^{\infty} \lambda e^{-\lambda x}\, dx$$

OR

$$\Rightarrow 0.5 \int_2^{\infty} e^{-0.5x} \Rightarrow \frac{e^{-0.5x}}{-0.5} \times 0.5$$

$$\Rightarrow - [e^{-0.5x}]_2^{\infty} \Rightarrow - [e^{-\infty} - e^{-1}]$$

$$\Rightarrow - [0 - \frac{1}{e}]$$

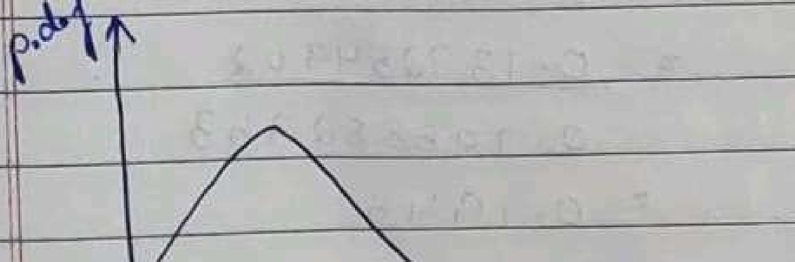
$$\Rightarrow \frac{1}{e} = 0.368$$

Ques 3(i) $X \sim \text{Gamma}(2, 1/2)$ $\boxed{\alpha = 2}$ $\boxed{\lambda = \frac{1}{2}}$

$$E(x) = \alpha/\lambda \Rightarrow 2 \times \frac{2}{1} = 4$$

$$V[x] = \frac{\alpha}{\lambda^2} \Rightarrow 2 \times \frac{1}{1/4} = 8$$

ii)



$$\Rightarrow \int_0^x \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha - 1}}{\Gamma\alpha} = \text{P.d.f}$$

increases in the beginning and then decreases when $\alpha = 2$, $\lambda = \frac{1}{2}$

(ii) CDf of Gamma distribution

$$F(x) = \begin{cases} 0 & , x < 0 \\ \int_0^x \frac{\lambda^{\alpha} x^{\alpha-1}}{\Gamma\alpha} \times e^{-\lambda x} & , x > 0 \end{cases}$$

Ques 4 P(choosing 2 members from the group)

$\Rightarrow$ ${}^{18}C_2 = P(A)$

P(atleast one female) = 1 - P(both are male)

$$P(F) = 1 - \frac{{}^{10}C_2}{{}^{18}C_2}$$

$$= 0.705882353$$

$$P(2 \text{ qualified females}) = \frac{{}^{7}C_2}{{}^{18}C_2}$$

$$P(F \cap Q) = 0.137254902$$

$$P(Q/F) = \frac{P(F \cap Q)}{P(F)}$$

$$= \frac{0.137254902}{0.705882353}$$

$$= 0.1945$$

Ques 5 P(Delivering via level 1) = $P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

P(Arriving late using level 1) = $0.2 = P(L/L_1)$

$P(L/L_2) = 0.4$

$P(L/L_3) = 0.1$

$$P(L_2/L) = \frac{P(L_2)\,P(L/L_2)}{P(L_1)\,P(L/L_1) + P(L_2)\,P(L/L_2) + P(L_3)\,P(L/L_3)}$$

$$= \frac{0.5 \times 0.4}{(0.4 \times 0.2) + (0.5 \times 0.4) + (0.1 \times 0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= \frac{0.2}{0.29}$$

$$= 0.69$$

Ques 6. $x \sim U(1, 2)$

$$f_x X = \frac{1}{\beta - \alpha} = \frac{1}{2 - 1}$$

$$= 1$$

$$Y = \frac{3}{6 \times x} \Rightarrow Y(6 - x) = 3$$

$$6 - x = \frac{3}{Y}$$

$$x = 6 - \frac{3}{Y}$$

$$g^{-1}(y) = 6 - \frac{3}{Y}$$

$$\frac{d}{dy} g^{-1}(y) = -3\left(\frac{-1}{y^2}\right)$$

$$= \frac{3}{Y^2}$$

$f_x\, g^{-1}(y) = 1$ {As R.V X can still only take value b/w 0 & 1}

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$$f_Y(y) = f_X\, g^{-1}(y) \left| \frac{d}{dy} g^{-1}(y) \right|$$

$$= 1 \times \frac{3}{y^2} = 3y^{-2}$$

Qus 7 (i) $F(3) = P(X \leq 3)$

$\Rightarrow 0.05 + 0.15 + 0.35$

$\Rightarrow 0.55$

(ii) $E(x) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05)$

$\Rightarrow 3.25$

iii) $E(x^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05)$

$\Rightarrow 11.45$

$V(x) = E(x^2)^2 - (E(x))^2$

$\Rightarrow 11.45 - (3.25)^2$

$\Rightarrow 0.8875$

(iv) cofficient of skewness $\Rightarrow E(x - \bar{x})^3$

Ques 8 $X \sim B(15, 0.2)$

$P(X<3) \Rightarrow P(X=0) + P(X=1) + P(X=2)$

$\Rightarrow {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$\Rightarrow 1 \times (0.8)^{15} + 15 \times (0.2)(0.8)^{14} + 105 (0.2)^2 (0.8)^{13}$

$\Rightarrow 0.39802$

Ques 9 So, he won two prizes in 6 weeks

So, $P(x=7) = {}^6C_2 (0.01)^3 (0.99)^4$

$\Rightarrow 0.00001409$

Ques 10 It will follow a Poisson distribution

$\lambda = \frac{2}{5} = 0.4$

$P(x > 2) = 1 - P(x \le 2)$

$P(x=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.4)^0 e^{-0.4}}{0!}$

$= e^{-0.4}$

$= 0.670320046$

$P(x=1) = \frac{(0.4)^1 e^{-0.4}}{1!}$

$= 0.268128018$

$P(x=2) = \frac{(0.4)^2 e^{-0.4}}{2!} = 0.053625604$

$P(x \le 2) = 0.992073668$

$P(x > 2) = 1 - P(x \le 2)$

$= 0.007926332$

Ques 11 $P(b) = P(g)$

$P(b) + P(g) = 1$

$2P(b) = 1$

$P(b) = \frac{1}{2}$

$P(g) = \frac{1}{2}$

Let E be the event of having 3 boys

$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$P(E) = \frac{1}{8}$

Ques 12 It is following exponential dist^n as exp(10)

$\lambda = 10$

$\alpha = 1$

$t = 0.1$ days

$P(T < 0.1) = F(0.1)$

$= 1 - e^{-\lambda t}$

$= 1 - e^{-1}$

$= 0.632120559$

Ques13 $X \sim U(75, 120)$

$$f_X(x) = \frac{1}{120-75}$$

$$= \frac{1}{45}$$

$$P(80 < x < 100) = \frac{100-80}{45}$$

$$= \frac{20}{45}$$

$$= 0.444$$

Ques14. $X \sim \text{Gamma}(5, 0.4)$

$$\alpha = 5$$

$$\lambda = 0.4$$

$X \sim \text{Gamma}(\alpha, \lambda)$

then $2\lambda x \sim \chi^2_{2\alpha = \lambda}$

$$2 \times 0.4x \sim \chi^2_{10}$$

$P(x < 22.89)$

$= P[2\lambda x < 2\lambda(22.89)]$

$= P(\chi^2_{10} < 22.89 \times 0.8)$

$= P(\chi^2_{10} < 18.312)$

$$18.0 \rightarrow 0.945$$

$$18.5 \rightarrow 0.9529$$

$0.5 \longrightarrow 0.0079$

$1 \rightarrow \dfrac{0.0079}{0.5}$

$0.31 \rightarrow 0.0079 \times 0.31$

$18.31 \rightarrow 0.945 + \dfrac{0.0079}{0.5} \times 0.31$

$18.31 \rightarrow 0.949898$

$P(\chi^2_{10} < 18.312) = 0.949898$

Ques 15 $f_x(x) = \dfrac{k}{(x+5)^4}, x > 0$

(i) $\int_0^\infty f_x(x) = 1$

$$\left.\begin{aligned} &k\int_0^\infty \frac{1}{(x+5)^4}\,dx = 1 \\ &\text{Let } (x+5) = t \\ &\quad dx = dt \end{aligned}\right\}$$

$$k\int_5^\infty t^{-4}\,dt = 1$$

$$k\left[\frac{-t^{-3}}{-3}\right]_5^\infty = 1$$

$$k\left[\frac{-1}{3t^3}\right]^\infty = 1$$

$$k\left[0 - \left(\frac{-1}{3(125)}\right)\right] = 1$$

$$\frac{k}{375} = 1$$

$$k = 375$$

ii) $F(10) = P(X \leq 10)$

$$= \int_0^{10} \frac{k}{(x+3)^4}$$

$$= 375 \int_0^{10} \frac{1}{(x+5)^4}$$

Let, $(x+5) = t$

$dx = dt$

$$= 375 \int_5^{15} t^{(-4)}$$

$$= 375 \left[\frac{-1}{3t^3} \right]_5^{15}$$

$$= 375 \left[\frac{1}{375} - \frac{1}{10125} \right]$$

$$= -1 - \frac{375}{10125}$$

$$= 0.963$$

iii) $E(x) = \int_0^\infty x \, f_x(x) \, dx$

$$= \int_0^\infty \frac{xk}{(x+5)^4}$$

$$= 375 \int_0^\infty \frac{x}{(x+5)^4}$$

Let, $(x+5) = t$

$dx = dt$

$$= 375 \int_5^\infty \frac{t-5}{t^4}$$

$$= 375 \int_5^\infty \frac{1}{t^3} - \frac{5}{t^4}$$

$$= 375 \left[\frac{-1}{2t^2} + \frac{5}{3t^3} \right]_5^\infty$$

$$= 375 \left[\frac{5}{3t^3} - \frac{1}{2t^2} \right]_5^\infty$$

$$= 375 \left[0 - 0 - \frac{5}{375} + \frac{1}{50} \right]$$

$$= 375 \left[\frac{1}{50} - \frac{5}{375} \right]$$

$$= \frac{375}{50} - 5$$

$$= 7.5 - 5$$

$$E(x) = 2.5$$