Financial Mathematics Assignment 2

Q1)

$\cancel{20000 = 427.9\, a_{\overline{60|}}}$

$\cancel{20000 = 427.9 \times \frac{(1 - v^{60})}{i}}$

$\cancel{\frac{20000 i}{427.9} - 1 + (1+i)^{-60} = 0}$

a) Monthly repayments = ₹427.9

$\therefore$ Total amount repaid = 427.9×60 = ₹25674

$\therefore$ Total interest = $25674 - 20000$

= ₹5674

$\therefore$ $APR = 2 \times \frac{\text{average annual interest}}{\text{original loan}}$

$= 2 \times \frac{5674/5}{20000}$

$= 0.11348$

$= 11.348\%$

5) New monthly payments = ₹274.49

∴ New Total repaid = 12 × n × 274.49

∴ New APR = Old APR

$$\frac{\dfrac{(427.9\times12 + 12\times n + 274.49) - 20000}{n+1}}{10000} = 0.11345$$

$$\frac{(5134.8 + n\times 3293.88) - 20000}{n+1} = 1134.8$$

$$5134.8 + 3293.8n - 20000 = 1134.8n + 1134.8$$

$$(3293.8 - 1134.8)n = 20000 + 1134.8 - 5134.7$$

$$2159 \times n = 16000$$

$$\therefore n = \frac{16000}{2159}$$

$$\therefore n = 7.41 \text{ yrs}$$

6) ∴ New total interest $= 427.9\times12 + 12\times274.49\times7.41$
$= 29542.45$

∴ Interest = ₹29542.45

Q2)

i) (a) Discounted payback period ⇒ The time taken by an investment, by which the cumulative forecasted cashflows becomes equal to the initial investment taken with time value of money.

(b) Payback Period ⇒ This is just the time taken by an investment by which the cumulative forecasted cashflows becomes equal to the initial investment.

ii) Both of them gives us a time period for the cumulative forecasted cashflows to get positive. But, DPP is better as it takes into consideration time value of money.

iii) Project A,

$\therefore NPV = -17 - 2v - 1v^2 + 2v + 2v^2 + 20v^3$

$\therefore NPV = -17 + 1v^2 + 20v^3$

$\therefore$ Project B,

$\therefore NPV = -20 + 14v + 14v^2 + 14v^3 + 14v^5 + 14v^6 + 14v^4 + 20v^6$

$= -20 + 14v(1+v+v^2+v^3+v^4+v^5) + 20v^6$

$= -20 + 14v\dfrac{(1-v^6)}{1-v} + 20v^6$

$= \cancel{-20 + 14v\dfrac{(1-v^6)}{1-\frac{1}{1+i}}} \quad -20 + \dfrac{14(1-v^6)}{i} + 20v^6$

$\therefore$ IRR for both projects,

$NPV_A = 0$ $\quad\therefore\ 0 = -17 + 1v^2 + 20v^3$

$\therefore\ 0 = -17 + (1+i)^{-2} + 20(1+i)^{-3}$

$\therefore$ For $i = 7\%$, $NPV = 0.1993$

$\therefore$ For $i = 8\%$, $NPV = -0.266016354$

$\therefore$ For $i = 7.4\%$, $NPV = 0.11167921$

$\therefore$ For $i = 7.5\%$, $NPV = -0.35455$

by interpolation,

$IRR = 0.074239537$

$\therefore\ IRR = 7.424\%$

$NPV_B = 0$ $\quad\therefore\ 0 = -20 + \frac{14(1-v^6)}{i} + 20v^6$

$\therefore\ IRR = 70\%$

(b) NPV at $i = 6\%$.

$\therefore\ NPV_A = -17 + (1+0.06)^{-2} + 20(1+0.06)^{-3}$

$= 0.68238210 1 \times 10000$

$= 6823.821007$

$\therefore\ NPV_B = -20 + \frac{14(1-v^6)}{i} + 20v^6 =$

$= 62.94175137 \times 10000$

$= 629417.5137$

Q3)

Annuity 1 $= 200 \times v^{(1/4)} \times \ddot{a}_{\overline{22|}}$ at 8%

$= 200 \times v^{(1/4)} \times a_{\overline{22|}} \times \frac{1}{d}$

$= 200 \times 0.980944 \times 10.2007 \times \frac{0.08}{0.074074}$

$= 2161.366301$

Annuity 2 $= 320(1+i)^{1/12} \times a^{(4)}_{\overline{16.25|}}$ at 8%

$= 320(1.006434) \times \frac{1-(0.925926)^{16.25}}{0.0877706}$

$= 320 \times 1.006434 \times 9.184257$

$= 2957.870081$

Annuity 3 $= 180 \times a^{(12)}_{\overline{18.75|}}$

$= 180 \times \frac{1-(0.925926)^{18.75}}{0.077208}$

$= 180 \times 9.892578702$

$= 1780.664166$

Let the revised annuity be A,
Value of revised annuity is,

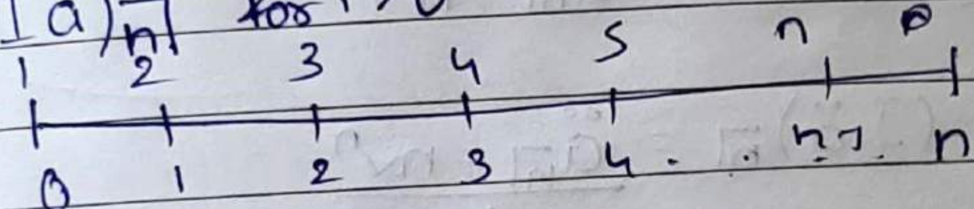
$$TPV = x \cdot a^{(2)}_{\overline{21.5}|} \cdot v^{1/4}$$

$$6889.9 - 180 = x \cdot \frac{(1 - v^{21.5})}{i^{(2)}} \cdot v^{1/4}$$

$$\therefore 6719.9 = x \frac{(1 - (0.925926)^{21.5}}{i^{(2)}} \times v^{1/4}$$

$$\therefore x = 656.56$$

Q4) $(I\ddot{a})_{\overline{n}|}$ for $i > 0$



$$(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + 4v^3 \ldots \ldots nv^{n-1} \quad (1)$$

multiplying both sides by v

$$\therefore v(I\ddot{a})_{\overline{n}|} = v + 2v^2 + 3v^3 + 4v^4 \ldots \ldots nv^n \quad (2)$$

$$\therefore (1) - (2)$$

$$(1-v)(I\ddot{a})_{\overline{n}|} = (1 + v + v^2 + v^3 + \ldots v^{n-1}) - nv^n$$

$$= \left(\frac{1 - v^n}{1 - v}\right) - nv^n$$

$$\frac{i}{1+i}(I\ddot{a})_{\overline{n}|} = \frac{(1 - v^n)(1+i)}{i} - nv^n$$

$$\cancel{(I\ddot{a})_{\overline{n}|} = \frac{1+i}{i} \times \left[\frac{(1-v^n)\times(1+i)}{i} - nv^n \right]}$$

$$\cancel{(I\ddot{a})_{\overline{n}|} = \frac{(1-v^n)(1+i)^2}{i^2} - \frac{(1+i)nv^n}{i}}$$

$$\cancel{= \frac{(1-v^n)(1+i^2) - (i+i)nv^n}{i^2}}$$

$$d(I\ddot{a})_{\overline{n}|} = \frac{(1-v^n)}{d} - nv^n$$

$$d(I\ddot{a})_{\overline{n}|} = \ddot{a}_{\overline{n}|} - nv^n$$

$$\therefore (I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

Q5> TWRR for each fund for the year 2014)

P.F $\quad \frac{16.4}{12.4} - 1 = 0.3226$ or 32.26%

E.F $\quad \frac{15.5}{12.1} - 1 = 0.2810$ or 28.1%

Page No.
Date

(3) Let X be annual repayment

$160000 = X a_{\overline{10}|} @ 8\%$

$X = \dfrac{160000}{a_{\overline{10}|} @ 8\%} = ₹\,23,844.65$

The loan outstanding immediately after 4th payment,

$23,844.65\, a_{\overline{6}|} @ 8\% = 23,844.65 \times 4.6229$
$= 110231.442$

Let Y be the revised annual investment.

$Y a_{\overline{6}|} = 110231.442$

$Y = \dfrac{110231.442}{a_{\overline{6}|}} = \dfrac{110231.442}{4.353} = 25309.72$

The loan outstanding immediately after 7th payment

$25309.72\, a_{\overline{7}|} = 25309.72 \times 2.4869$
$= 62942.74$

Let Z be the revised annual installment

$Z a_{\overline{3}|} = 62942.74 @ 9\%$

$Z = \dfrac{62942.74}{2.5313} = 24865.77$

IRR,

$\cancel{160000 - 1465.07 a_{\overline{4|}} - 443.95 a_{\overline{7|}} - 24865.77 a_{\overline{10|}} = 0}$

$\therefore \cancel{160000 - 1465.07\frac{(1-v^4)}{i} - 443.95\frac{(1-v^7)}{i}}$

$\cancel{- 24865.77\frac{(1-v^{10})}{i}}$

$\therefore 23844.65\times 4 + 25309.72\times 3 + 24865.77\times 3$
$= 245905.07$

$\therefore 245905.07 = 160000(1+i)^{10}$

$\therefore i = \left(\frac{245905.07}{160000}\right)^{1/10} - 1$

$\therefore i = 4.3914067\%$

Q7)

i) The return on property is lower than that in government security.
It is difficult to invest in property.
Less flexible because of the large unit size.
The market ability for property is poor.
Large buying and selling expenses.

Advantages → They will know the price that the investor will pay at outset so it will know the cost of borrowing money. It will be administratively less difficult by lender.

Disadvantages → Investors may be willing to pay more for the bond than the set price and so the money could be borrowed more cheaply if the tender is used. Insufficient investor may be prepared to pay the set price causing any part of the offer to be sold and government may not meet the financial requirement.

Q8) ~~The~~ P.V of outlay $= 2.7 + 0.2a_{\overline{3}|}$

PV of inlay $= \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$

$NPV = 0 = \bar{a}_{\overline{10}|}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2a_{\overline{3}|}$

$0 = \frac{1-v^{10}}{-\ln(v)}(0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2\left(\frac{1-v^3}{}\right.$

$i = 9.3\%$

$NPV = 0.1v^3 + 0.85\bar{a}_{\overline{10}|}v^3 - 2.7 - 0.2a_{\overline{3}|}$ @ 10%

$= 0.1(1.1)^{-3} + 0.85(1.1)^{-3}\left(\frac{1-(1.1)^{-10}}{\ln(1.1)}\right) - 2.7 - 0.2\left(\frac{1-(1.1)^{-3}}{0.1}\right)$

$= 1.0089$

Q9) TWRR,

$$(1+i)^3 = \frac{33}{30.5} \times \frac{41.05-4.5}{33+6} \times \frac{45.6}{41.05} \times \frac{47}{45.6-2.50}$$

$$(1+i)^3 = 1.0819713 \times 0.9371794872 \times 1.11084438 \times 1.090487$$

$$(1+i)^3 = 1.2281327$$

$\cancel{(1+i)}$ ∴

$$i = \sqrt[3]{1.2281327} - 1$$

$$i = 0.070951281$$

TWRR = 7.0951281%

$$NPV = 30.5(1+i)^3 + 6(1+i)^{2.26} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} - 47$$

∴ for $i = 1$, NPV = −9

$i = 1.1$, NPV = 2.9346

$i = 1.07 \rightarrow -0.867$

$i = 1.08 \rightarrow 0.3782$

$i = 1.072 \rightarrow -0.0047103193$

$i = 1.073 \rightarrow 0.1216951665$

∴ by interpolation,

$i = 7.204\%$

Linked rate of return,

Year 2011,

$30.5(1+i) + 6(1+i)^{0.25} = 38.5$

$30.5(1+i) + 6(1+i)^{0.25} - 38.5 = 0$

$1 \to -2$

$1.1 \to 1.19468$

$1.06 \to -0.081$

$1.07 \to 0.2373$

$1.062 \to -0.018$

$1.063 \to 0.0138$

$1.0625 \to -0.0021204$

$1.0626 \to 0.00107288 2008$

$\therefore i = 0.0626$ or 6.26%

Year 2012,

$38.5(1+i) + 4.5(1+i)^{0.5} = 45$

$38.5(1+i) + 4.5(1+i)^{0.5} - 45 = 0$

$1 \to -2$

$1.1 \to 2.0696$

$1.04 \to -0.37$

$1.05 \to 0.0361$

$1.049 \to 0.004568 4$

$1.0492 \to 0.000357089024$

$\therefore i = 0.0492$ or 4.92%

Page No.
Date

Year 2013,

$45(1+i) - 2.5(1+i)^{0.5} = 47$

$45(1+i) - 2.5(1+i)^{0.5} - 47 = 0$

$1.1 \rightarrow -0.122$

$1.2 \rightarrow 4.2013$

$1.11 \rightarrow 0.316$

$1.102 \rightarrow -0.034404$

$1.103 \rightarrow 0.0094048$

$1.1027 \rightarrow -0.003738$

$1.1028 \rightarrow 0.000642881431$

$\therefore i = 10.279\%$ or 0.10279

LRR,

$(1+i)^3 = (1.0626) \times (1.0492)(1.0279)$

$(1+i)^3 = 1.229478427$

$\therefore i = 0.07289803$ or 7.1289803%

Q10) Let the loan amount be L,

$L = 180000\, a_{\overline{15}|} + 20000\, (Ia)_{\overline{15}|}$

$L = 180000 \times 6.8109 + 20000 \times \left(\frac{\ddot{a}_{\overline{n}|} - 15v^{15}}{i} \right)$

$= 180000 \times 6.8109 \times 20000 \left(\frac{4.887760487}{0.12} \right)$

$= 1225962 + 20000 \times 40.73133739$

$= 2040588$

Page No.
Date

Total cost of house $= \frac{2040588}{0.8} = 2550735$

The loan outstanding at the beginning of the 9^{th} year

$L = 180000v + 180000v^2 \ldots 180000v^7) + [(9 \times 20000v) + (10 \times 20000v^2)]$

$L(9) = 340000\, a_{\overline{7|}} + 20000 (Ia)_{\overline{7|}}$

$L(9) = 340000 \times 4.5638 + 20000 \left(\frac{5.1114 - 7 \times 0.45235}{0.12} \right)$

$L(9) = 1875850.33$

Loan schedule,

Year	Loan outstc...	Loan install.	Interest	Capital Repay
9	1875850.33	360000	225102.04	134897.96
10	1740952.37	360000	208914.28	171085.73
11	1569866.65			

Let the new installment be Rs P.

$1569866.65 = P a_{\overline{5|}}$

$1569866.65 = P \times 3.790786$

$P = 414126.87$

Extra payment by Mr. Sam if the loan continued with the first bank,

Total installment − loan outstanding at the end 10^{th} year.

$= (400000 + 420000 + 440000 + 460000 + 480000) - 1569866.65$

$= 630133.36$

Page No.
Date

Extra payment made by Mr. Sam if the loan transferred to 2nd bank.
Total installment + 1% processing fees − loan o/s at the end of 10th year

$= 5 \times 414126.9 + 0.01 \times 1569866.65 - 1568866.65$
$= 516466.35$

Q11) TWRR,

$$(1+i)^2 = \frac{500}{460} \times \left(\frac{550+50}{500+40}\right) \times \frac{x}{550}$$

$$(1+i)^2 = \frac{600 \times x}{46 \times 540 \times 11}$$

$$(1+i)^2 = 1.44$$

$$\therefore x = \frac{1.44 \times 46 \times 540 \times 11}{600}$$

$$\boxed{x = 655.776}$$

MWRR,

$$460(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.776$$

Let $1+i = x$

$1.1 \rightarrow -106.915622$
$1.2 \rightarrow 1.6575$
$1.19 \rightarrow -9.6356$

$1.198 \rightarrow -0.609$
$1.199 \rightarrow 0.5237$
$1.1985 \rightarrow 0.04278952413$
$1.1986 \rightarrow 0.0704$
$1.19854 \rightarrow 0.00252233$

$\therefore i = 0.19854$ or 19.854%

Page No.
Date

The effective rate of return for year 2015 is given by

$460(1+i) + 40(1+i)^{3/4} = 600$

~~460 x 40~~ $1.2 \rightarrow -2.138$ $1.204 \rightarrow -0.184$

$1.3 \rightarrow 46.698$ $1.205 \rightarrow 0.3044$

$1.21 \rightarrow 2.7475$ $1.2044 \rightarrow 0.01131475$

$1.2043 \rightarrow -0.037$

$1.20438 \rightarrow 0.00154201$

$i = 0.20438$, 20.438%

Effective rate of return for the year 2016,

$(1+i) = \frac{655.78}{550} = 1.92527$

So linked rate of return,

$(1+i)^2 = 1.92527 \times 1.202138$

$i = 0.198338483$ or 19.8338483%

LIRR = 19.8338483%

Q12) The discounted payback period of an investment project is the first time when the NPV of the cash flow from the project is positive.

~~Let's code~~

$$-800(1+i)^t - 50(1+i)^{t-1} + 100\bar{s}_{\overline{t-2}|} = 0$$

$$-800(1+i)^t - 50(1+i)^{t-1} + 100 \times \frac{(1+i)^{t-2} - 1}{\ln(1+i)} = 0$$

$$-800(1.07)^t - 50(1.07)^{t-1} + 100\,\frac{(1.07)^{t-2} - 1}{\ln(1.07)} = 0$$

$t \to 17 \to 74.79$

$18 \to 23.4626$	$17.76 \to -0.757$
$17.7 \to -6.741$	$17.77 \to 0.2426$
$17.8 \to 3.2462$	$17.767 \to 0.057$
	$17.768 \to 0.0426$

$\therefore t = 17.767$ yrs.

Accumulated profits after 22 yrs;

$$100\bar{s}_{\overline{4.333}|} @6\% = 100 \times \frac{(1+0.06)^{4.233} - 1}{\ln(0.06)}$$

$$= 480.0743066$$

Acc. Amt $= 480074.3066$

The businessman may accumulate is rental income for each year @6%.

$$100\bar{s}_{\overline{1|}} = 100 \times \frac{(1+0.06)^{1}-1}{\ln(1.06)}$$

$$= 100 \times 1.029708672$$

$$= 102.9708672$$

The PP is,

$$-800(1+i)^{t} - 50(1+i)^{t-1} + 102.9708672\,\bar{s}_{\overline{t-2|}} \text{ @ } 7\% > 0$$

$$-800(1.07)^{t} - 50(1.07)^{t-1} + 102.9708672\,\frac{(1.07)^{t-2}-1}{\ln(0.07)} > 0$$

$t \to 17 \to -87.1$

$t \to 18 \to 9.7866$

$\therefore t = 18$ yrs

The balance in the businessmans account after transtation at 18 years

$$-800(1+i)^{18} - 50(1+i)^{17} + 102.9708672\,\bar{s}_{\overline{16|}}$$

$$= 9.77896$$

Acc value after 22 years is,

$$9.77896(1+i)^{4} + 100\bar{s}_{\overline{4|}} \text{ @ } 6\%$$

$$= 462.801357$$

Acc. amt after 22 yrs 462801.3571

Page No.

Date

Q13) Let $x/12$ be the monthly repayment

$$X a_{\overline{25}|}^{(12)} = 9880 \text{ @ } 7\%.$$

$$X a_{\overline{25}|} \times \frac{i}{i^{(12)}} = 9880$$

$$X \cdot (11.6536) \times \frac{0.07}{0.06785} = 9880$$

$$\therefore X = 821.7669096$$

$$\therefore X/12 = 68.4805758$$

Hence, monthly repayment 6848.05758

Loan o/s immediately after repayment on 10^{th} March 2029.

$$X a_{\overline{34/3}|}^{(12)} = 821.7669096 \frac{\left(1-\left(1/1.07\right)^{34/3}\right)}{0.06785}$$

$$= 6485.745315$$

$$= 6485574.5313$$

The loan o/s just after the repayment on 10^{th} September 202[illegible] is made $X a_{\overline{83/6}|}^{(12)}$ @ 7%.

The loan o/s just after the repayment on 10^{th} October 202[illegible] is made $X a_{\overline{55/4}|}^{(12)}$ @ 7%.

$\longrightarrow X a_{\overline{13.75}|}^{(12)}$ @ 7%

Capital repaid on 10th October 2016 is thus,

$$X\left[a^{(12)}_{\overline{83/6}|} - a^{(12)}_{\overline{55/4}|}\right] @ 7\%$$

$$X\left[\frac{v^{13.75} - v^{83/6}}{i^{(12)}}\right]$$

$$X\left[\frac{(1/1.07)^{13.75} - (1/1.07)^{83/6}}{0.06785}\right]$$

$$821.7669096\,(0.03268447611) - \cancel{20}$$

$$= 26.85902093$$

The capital repayment continued in these 12 installments

$$X\left[a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|}\right] @ 7\%$$

$$821.7669096\left[\frac{v^{19/3} - v^{22/3}}{i^{(12)}}\right]$$

$$= 821.7669096\left[\frac{(1/1.07)^{19/3} - (1/1.07)^{22/3}}{0.06783}\right]$$

$$= 516.1973861$$

$$\therefore \text{Total interest} = 821.7669096 - 516.1973861$$

$$= 305.5695235.$$