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Assignment 1

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Subject: Probability And Statistics-1
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① Calculate the mean, median, mode, standard deviation and inter-qautile range of following data:

i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

ii)

No. of household	0	1	2	3	4
No. of people	5	11	26	15	3

Solⁿ

(i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

ⓐ Mean -

$$\bar{x} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$\boxed{\bar{x} = 13.4}$$

ⓑ Median.

$$n = 10 \Rightarrow \frac{n+1}{2} = 5.5$$

$$\boxed{M = 13}$$

ⓒ Mode.

$$\boxed{Z = 18}$$

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(d) Standard deviation:

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{(-9.4)^2+(-6.4)^2+(-4.4)^2+(-2.4)^2+(1.6)^2+(4.6)^2+(4.6)^2+(4.6)^2+(-4.4)^2+(1.6)^2}{10}}$$

$$\sigma = \sqrt{37.44}$$

$$\boxed{\sigma = 6.1188}$$

(e) Inter-Quartile Range:

$Q_1 = \frac{n+1}{4} = 11/4 = 2.75^{th}$ term

$Q_3 = \frac{3(n+1)}{4} \; 33/4 = 8.25^{th}$ term

$Q_1 = 2 + 0.75(9-7)$

$Q_1 = 7 + 1.5$

$Q_1 = 8.5$

$Q_3 = 8 + 0.25(0)$

$Q_3 = 18$

$\boxed{\text{Interquartile Range} = 9.5}$

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(ii)

x	f	xf	cf
0	5	0	5
1	11	11	16
2	26	52	42
3	15	45	57
4	3	12	60
	60	120	

(a) Mean $= \frac{xf}{\Sigma f} = \frac{120}{60} = 2$ $\boxed{\bar{x} = 2}$

(b) ~~Mode~~ Median $= \frac{n}{2} = 30$

$\boxed{M = 2}$

(c) Mode:

$\boxed{Z = 2}$

(d) Standard Deviation:

$$SD = \sqrt{\frac{\cancel{\Sigma}(x-\bar{x})^2}{n}}$$

$$SD = \sqrt{\frac{(0-2)^2+(1-2)^2+(2-2)^2+(3-2)^2+(4-2)^2}{60}}$$

$$SD = \sqrt{\frac{(-2)^2+(-1)^2+0^2+(1)^2+(2)^2}{60}}$$

$$SD = \sqrt{\frac{4+1+0+1+4}{60}}$$

$$SD = \sqrt{\frac{1}{6}}$$

$$\boxed{SD = 0.4082 = \sigma}$$

(e) Interquartile Range:

$$Q_1 = \frac{60+1}{4} = \frac{61}{4} = 15.25$$

$$= 15^{th} + 0.25(1)$$

$$= 1.25$$

$$Q_3 = 3\left(\frac{60+1}{4}\right) = 45.75$$

$$= 45^{th}\text{ term} + 0.75(1)$$

$$= 3.75$$

Interquartile Range = $Q_3 - Q_1$

$$\boxed{\text{Interquarti Range} = 2.5}$$

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(2) It is assumed that claims arising on an industrial policy can be modelled as a Poisson Process at a rate of 0.5 per year.

i) Determine the probability that no claim arise in a single year.

ii) Determine the probability that, in three consecuti years, three is one or more claim in one of the years and no claim in each of other new years.

iii) Suppose a claim has just occured. Determine the probability that more than two years will elapse before the next claim occur.

Soln $\lambda = 0.5$

(i) no claim arise $= x = 0$

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$= \frac{e^{-0.5}(0.5)^0}{0!}$$

$$= e^{-0.5}$$

$$\boxed{P(x=0) = 0.6065}$$

(ii)

here, let x is no of year with a claim

$X \sim \text{binomial}(3, 0.34)$

$P(X=1) = {}^3C_1 p^1 q^2$

$= 3 \times (0.34) \times (0.66)^2$

$\boxed{P(X=1) = 0.444}$

(iii) let A time until next claim

$A \sim \text{Exp}(0.5)$ at $x = 2$

$P(A>2) = 1 - \int_0^2 0.5 e^{-0.5x} \cdot dx$

$= 1 - 0.5 \left[\frac{e^{-0.5x}}{-0.5} \right]_0^2$

$= 1 + [e^{(-1)} - e^{(0)}]$

$= 1 - 0.6321$

$\boxed{P(A>2) = 0.3679}$

③ An analyst is interest in using gamma distribution with parameters $\alpha = 2$ & $\lambda = \frac{1}{2}$. The range of x is defined as $0 < x < \infty$

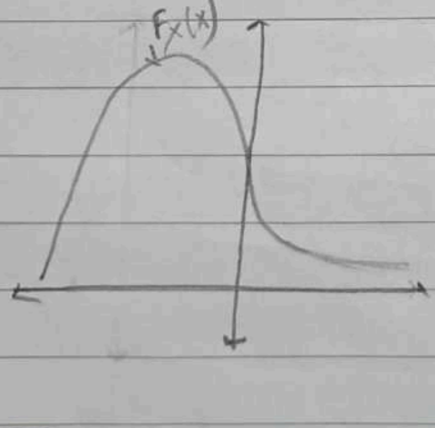
i) State the mean and standard deviation of the distribution
ii) Comment Breifly on it shape
iii) Calculate the cumulative distribution function.

Soln

(i) $\bar{x} = 2\alpha$
$\boxed{\bar{x} = 4}$

$V(x) = 2\bar{x} = 8$
$\boxed{SD = 2.8284}$

(ii)



The shape is above at the high peak at the left hand side.

(iii) $f_X(x) \Rightarrow x \sim \text{gamma}(2, \frac{1}{2})$
$\Rightarrow \chi^2$
$\boxed{F_X(x) = P(0 < \chi^2 < \infty)}$

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(4) In an acturial office there are 10 males & 8 female staff. 6 of the males & 7 of the female are fully qaulified acturies. Calculate the probability of the team of 2 worker randomly selected consist of two qaulified female given that it contains at least one female.

Sol^n Males = 10
female = 8
qaulified males = 6
qaulified females = 7

Probability = $\frac{\text{2 qualified fmale}}{\text{Atleast one female}}$

$$= \frac{{}^{7}C_2}{({}^{10}C_1 \times {}^{8}C_1) + ({}^{8}C_2)}$$

$$= \frac{21}{(80) + 28}$$

$$= \frac{21}{108}$$

$$\boxed{P = \frac{7}{36}}$$

(5) The supply chain service in a country has three levels of service in a country has level 1 (next day), level 2 (2-day delivery) & Level 3 (3-day delivery). 40% of package are delivered via level 1 and 50% are delivered via level 2. The probability of package arriving late through each level is 20%, 40% & 10% respectively. Calculate probability of the late package was sent via level 2 service.

Soln

$P(1) = 40\%$

$P(2) = 50\%$

$P(3) = 10\%$

$P(L|1) = 20\%$

$P(L|2) = 40\%$

$P(L|3) = 10\%$

$$P(2|L) = \frac{P(2/2) \times P(2)}{P(1) \times P(L|1) + P(2) \times P(L|2) + P(3) \times P(L|3)}$$

$$P(2|L) = \frac{0.4 \times 0.5}{0.2 \times 0.4 + 0.4 \times 0.5 + 0.1 \times 0.1}$$

$$P(2|L) = \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$P(2|L) = \frac{0.2}{0.29}$$

$$\boxed{P(2|L) = 0.6896}$$

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(6) If $X \sim U(1,2)$ and $Y = \frac{3}{6-x}$, determine PDF of Y.

Ans $y = \frac{3}{6-x}$

$F_y(y) = P(Y \leq y)$

$F_y(y) = P\left[\frac{3}{6-x} \leq y\right]$

$F_y(y) = P\left[\frac{6-x}{3} \geq y\right]$

$F_y(y) = P[6-x \geq 3y]$

$F_y(y) = P[x \leq 6-3y]$

$F_y(y) = \begin{cases} 0 & , y = 2 \\ 6-3y & , y \neq 2 \end{cases}$

$F_y(y) = \begin{cases} 0 & , y = 2 \\ -3 & , y \neq 2. \end{cases}$

(7) A discrete random variable X has the probability function:

X	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

Calculate:

i) $F(3)$
(ii) $E(X)$
(iii) $V(X)$
(iv) Coeffecient of Skewness.

Solⁿ: i) $F(3) = P(X \leq 3)$

$= P(X=1) + P(X=2) + P(X=3)$

$= 0.05 + 0.15 + 0.35$

$\boxed{F(3) = 0.55}$

(ii) $E(X) = \sum_{i=1}^{5} x \cdot P(X=x)$

$= (1 \times 0.05) + (2 \times 0.15) + (3 \times 0.35) + (4 \times 0.4) + (5 \times 0.05)$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$\boxed{E(X) = 3.25}$

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(iii) $E(x^2) = \sum_{i=1}^{5} x^2 \cdot P(X=x)$

$= (1\times0.05) + (4\times0.15) + (9\times0.35) + (16\times0.4) + (25\times0.05)$

$= 0.05 + 0.6 + 3.15 + 6.4 + 1.25$

$E(x^2) = 11.45$

$V(x) = E(x^2) - [E(x)]^2$

$= 11.45 - (3.25)^2$

$\boxed{V(x) = 0.8875}$

(iv) $\text{Skewness} = \dfrac{\Sigma(x_i - \bar{x})^3}{(N-1)\times\sigma^3}$

$= \dfrac{(1-3.25)^3 + (2-3.25)^3 + (3-3.25)^3 + (4-3.25)^3 + (5-3.25)^3}{4\times(0.9421)^3}$

$= \cancel{0.9421}$

$= \dfrac{-11.3906 - 1.9531 - 0.0156 + 0.4219 + 5.3594}{4\times(0.9421)^3}$

$= \dfrac{-7.578}{4\times(0.9421)^3}$

$\boxed{\text{Coeffecient of Skewness} = -2.2657}$

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(8) Insurance claim amount are independent and exceed 1000 with probability 0.2. In a sample of 15, find the probability that fewer than 3 of them exceed 1000.

Sol^n $p = 0.2$

$q = 0.8$

let A be event the probability that fewe the 3 of them exceed 1000.

$P(A) = {}^{15}C_0 p^0 q^{15} + {}^{15}C_1 p^1 q^{14} + {}^{15}C_2 p^2 q^{13}$

$= 1(0.2)^0 (0.8)^{15} + 15(0.2)(0.8)^{14} + 105(0.2)^2(0.8)^{13}$

$= 0.0352 + 0.1319 + 0.2308$

$P(A) = 0.3979$

$\boxed{P(A) \approx 0.4}$

Thus, The probability that fewer than 3 of them exceed 1000 is almost equal to 0.4.

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(9) A man enters a raffle each week where the probability of winning a prize is 0.01. Calculate the probability that he takes seven weeks up to and including the week where he wins his third price.

Sol^n $p = 0.01$

$q = 0.99$

$P(x=3) = {}^6C_2 p^2 q^4$

$= \frac{6!}{2! \times 4!} \times (0.01)^2 \times (0.99)^4$

$= \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}} \times (0.01)^2 \times (0.99)^4$

$\boxed{P(x=3) = 0.00001441}$

(10) The number of claims submitted to an insurance office average two per working week. Calculate the probability that more than two claim arrive in one day?

Sol^n Average claim per week = 2

days in a week = 5

Thus rate $(\lambda) = 2/5 = 0.4$

So to find the probability on one day.

$\lambda = 0.4$ at $x = 2$

$$P(X) = \frac{e^{-\lambda} \cdot (\lambda)^{x}}{x!}$$

$$= \frac{e^{(-0.4)} \times (0.4)^2}{2!}$$

$$= \frac{e^{(-0.4)} \times (0.4)^2}{2}$$

$$\boxed{P(X) = 0.0536}$$

Thus, the probability of more than 2 claim arrive in day is 0.0536.

(11) If the probability of having a male and female child is equal, and a women has 5 boys and 2 girls. Calculate the probability that her next 3 child are boys.

Soln The probability of having male = 0.5

The probability of having female = 0.5

Probability of having 3 next boys are,

$P = {}^3C_3 (0.5)^3 (0.5)^0$

$P = 1 \times (0.125) \times 1$

$\boxed{P = 0.125}$

→ Thus, the probability of having next 3 boys child are 0.125.

(12) Claim follow a Poisson process with the rate 10 per day. Calculate the probability that the time between two claim is less than 0.1 days.

<u>Soln</u> $\lambda = 10/\text{day}$

$x = 0.1$

let A be time less than 0.1 day.

$A \sim \exp(10)$ at $x = 0.1$

$$P(A < 0.1) = 1 - \int_0^{0.1} \lambda e^{-\lambda x}\, dx$$

$$= 1 - 10 \int_0^{0.1} e^{-10x}\, dx$$

$$= 1 - 10 \left[\frac{e^{-10x}}{-10} \right]_0^{0.1}$$

$$= 1 + \left[e^{-1} - e^{0} \right]$$

$$\boxed{P(A < 0.1) = 0.3678}$$

(13) If X is following uniform distribution with parameters $a = 75$ and $b = 120$, Calculate the probability ~~that more than two claim arrive in one day.~~ that X lies between 80 and 100.

Sol^n

$P_x(80, 100) = ?$

$a = 75$

$b = 120$

$$f_x(x) = \frac{1}{20}$$

$$P(80,100) = \frac{1/45}{1/20} = \frac{20}{45}$$

$$\boxed{P(80,100) = 0.45}$$

(14) If $X \sim \text{Gamma}(5, 0.4)$ Calculate $P(X < 22.89)$

Sol^n $\alpha = 5$

$\lambda = 0.4$

$$(2\lambda)x \sim \chi^2_{2\alpha} \Rightarrow [0.8x \sim \chi^2_{10}]$$

$$[0.8x < 22.89] \Rightarrow [\chi^2_{10} < 22.89]$$

$\nu = 10$

$x = 22.89$

$$+1 \begin{bmatrix} \cancel{22} \to 0.9849 \\ 23 \to 0.9893 \end{bmatrix} 0.0044$$

$$P(X < 22.89) = P(22 + 0.89) = [0.9849 + 0.003916)$$

$$\boxed{P(X<22.89) = 0.9880}$$

(15) A continuous random variable X has the following probability density function:

$$\frac{K}{(x+5)^4}, \quad x > 0$$

Calculate.

i) K
ii) F(10)
iii) E(X)

Soln PDF $= \frac{K}{(x+5)^4}, \quad x > 0$

(i) K

$$\int_0^\infty K/(x+5)^4 \, dx \geq 0$$

$$5K > 0$$

$$K > 0$$

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(ii) $F(10) = \int_0^{10} \frac{K}{(x+5)^4} \, dx$

$= \left[\frac{5K}{(x+5)^4} \right]_0^{10}$

$= \left[\frac{50}{(15)^4} - 0 \right]$

$= \frac{50}{(15)^4}$

$\boxed{F(10) = 0.00098}$

(iii) $E(X) = \int_0^{x} x^2 \cdot \frac{K}{(x+5)^4} \cdot dx$