

Financial maths

1) $d^4 = 8\%$ $\Leftrightarrow e^{-8\%} = (1-d)^4$ $-8 = \ln(1-d)$
 $-8 = \ln\left(1 - \frac{d^4}{4}\right)$ $\Leftrightarrow \delta = 8.081082927\%$

$e^{\delta} = 1+i$ $i = 8.416978473\%$
 $1-d = \left(1 - \frac{d^4}{4}\right)^4 = \left(1 - \frac{d^{12}}{12}\right)^{12}$ $d^{12} = 8.053933945\%$

2) $S(t) = 0.004t + 0.0002t^2$

i) $Pv = 1000 e^{\int_0^{10} 0.004t + 0.0002t^2 dt} = 1000 \left[0.002t^2 + 0.00006667t^3 \right]_0^{10}$
 $= 1000 \exp[0.2 + 0.0667] = 765.903 \text{ (Am)}$

ii) $(1+i)^{10} = e^{0.2667}$ $i = 2.702882732\%$

$$3) i) d = \frac{i}{1+i} \quad v = \frac{1}{1+i} \quad \boxed{d = iv}$$

$$c) ii) \text{ find } d^2 \quad \frac{d^4}{4} = 2.25 \quad d^4 = 9\%$$

$$\left(\frac{1-d^2}{2} \right)^2 = \left(1 - \frac{d^4}{4} \right) \quad \boxed{d^2 = 8.89875\%}$$

$$b) d^{0.5} = 5\% \quad \left(\frac{1-d^2}{2} \right)^2 = \left(1 - \frac{d \cdot 5}{0.5} \right)^{0.5} \quad \boxed{d^2 = 5.199250715\%}$$

$$i) ii) i = 0.08 \quad 1.08 = \left(1 - \frac{d^{12}}{12} \right)^{-12} \quad \boxed{d^{12} = 7.671422614\%}$$

$$1.08 = \left(1 + \frac{i^{365}}{365} \right)^{365} \quad \boxed{i^{365} = 7.696915541\%}$$

$$2 \quad 1.08 = e^S \quad S = \ln(1.08) = 7.696104114\%$$

$$1.08 = \left(1 + \frac{0.5}{0.5} \right)^{0.5} \quad \boxed{i^{0.5} = 8.32\%}$$

$$5) i) a) d^4 = 6\% \quad i^2 \quad \left(1 + \frac{i^2}{2} \right)^2 = \left(1 - \frac{d^4}{4} \right)^{-4} \quad \boxed{i^2 = 6.137751552\%}$$

$$b) \left(\frac{1-d^{12}}{12} \right)^{12} = \left(1 - \frac{d^4}{4} \right)^4 \quad \boxed{d^{12} = 6.030252528\%}$$

$$1.024113689$$

$$ii) \quad \begin{array}{ccc} 2 \text{ lakh} & & 2.2 \text{ lakh} \\ | & & | \\ 15/11/05 & 17/9/05 & 17/07/05 \end{array}$$

$$220000 = 200000 (1+i)^{10} \quad \boxed{i = 10\% = 0.1}$$

$$\left(1 + \frac{i^6}{6} \right)^6 = 1.1 \quad i^6 = 9.607120664$$

$$\left(1 + \frac{i^4}{4} \right)^4 = 1.1 \quad i^4 = 9.645475634\%$$

$$AV = 200000$$

$$\boxed{AV' = 204822.7378}$$

7) $P_v = 20000$ $A_v = 26000$ $n = 12 \text{ months} = 1 \text{ hr } 5 \text{ months}$
 $= 1 \frac{5}{12} = 1 \frac{17}{12} \text{ yr}$

$$26000 = 20000 (1+i)^{17/12}$$

$$i = 20.34570672\%$$

$$26000 = 20000 \left(1 + \frac{i^4}{4}\right)^{68/12}$$

$$i^4 = 18.95529456\%$$

$$d^2 \quad 26000 = 20000 \left(1 - \frac{d^2}{2}\right)^{-17/6}$$

$$d^{12} = 17.68823531\%$$

$$d^2 = 17.68823531\%$$

$$26000 = 20000 \left(1 + \frac{17\%}{12}\right)$$

$$i = 21.17647059\%$$

9) ii) $i^2 = 0.0775$

$$1+i = \left(1 + \frac{i^2}{2}\right)^2$$

$$i = 7.90015625\%$$

$$e^{\delta} = 1+i \quad \delta = \ln(1.079001563) = 7.604\%$$

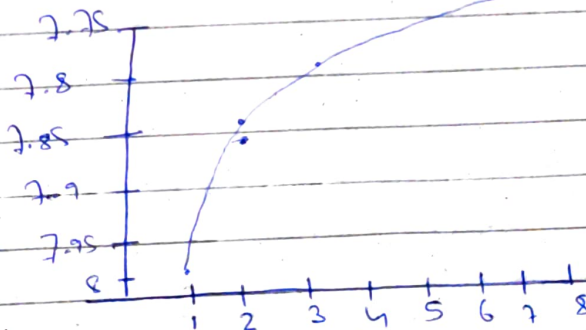
$$\left(1 + \frac{i^2}{2}\right)^2 = \left(1 - \frac{d^{12}}{12}\right)^{-12}$$

$$d^{12} = 7.57957468\%$$

$$\left(1 + \frac{0.5}{0.5}\right)^{0.5} = \left(1 + \frac{i^2}{2}\right)^2$$

$$i^{0.5} = 8.212218594\%$$

6) 8) $i = 8\%$



$$10) P_v(0 \leq t \leq 5) = 1 - \exp\left[-\int_0^5 0.088 ds\right] = \exp\left[-0.088 \cdot 5\right] = e^{-0.44} = e^{-(0.088 \cdot 5)} = e^{-(0.5) - t}$$

$$P_v(5 \leq t \leq 10) = 1 - \left(e^{-0.088 \cdot 5}\right) \exp\left[-\int_5^{10} 0.06 ds\right] = e^{-0.44} \left(\exp\left[-0.06(10) - 0.06(5)\right]\right)$$

$$= e^{-0.44} \left(\exp\left[-0.6 - 0.3\right]\right) = e^{-0.44} \cdot e^{-0.9} = e^{-1.34} = e^{-(0.7) - t}$$

$$e^{-0.44} \left(\exp\left[-0.06t - 0.3\right]\right) = e^{-0.44} \cdot e^{-0.3} \cdot e^{-0.06t} = e^{-(0.06t + 0.7)} \quad (t = 0, 10)$$

$$P_v(t \geq 10) = e^{-0.7} \exp\left[-\int_{10}^t 0.04 dt\right] = e^{-0.7} \exp\left[-0.04t - 0.4\right]$$

$$= e^{-0.7} \cdot e^{-0.4} \cdot e^{-0.04t} = e^{-(0.04t + 1.1)} \quad t \geq 10$$

11) $n = 3\frac{1}{4}$ year $Av = 50000$ $d_{3d} = 18\%$

$$Pv = Av \times DF = 50000 \left(1 - \frac{3}{4} \times 18\right) = \boxed{43250}$$

b) $i = 6\%$ find d^{12}

$$e^{-s} = 1 - d = \left(1 - \frac{d^{12}}{12}\right)^{12}$$

$$d^{12} = 5.985024169\%$$

ii) $d^4 = 6\%$ find i^{12}

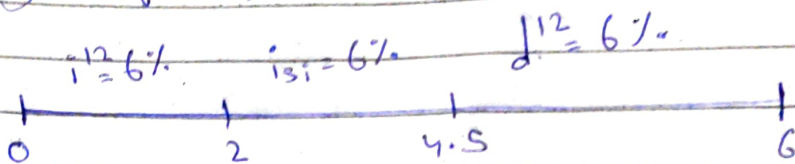
$$\left(1 + \frac{i^{12}}{12}\right)^{12} = \left(1 - \frac{d^4}{4}\right)^{-4}$$

$$i^{12} = 6.060708865\%$$

$$\left(1 + \frac{i^4}{4}\right)^4 = \left(1 - \frac{d^4}{4}\right)^{-4}$$

$$i^4 = 6.091370558\%$$

12)



$$Av_0 = 7049 \left(1 + \frac{0.06}{12}\right)^{24} \left(1 + \frac{5(0.06)}{2}\right) \left(1 - \frac{0.06}{12}\right)^{-18} (1.5)^{-12}$$

$$= 7049 (1.005)^{24} (1.15) (0.995)^{-18} = \boxed{13108.3799}$$

9999.89

13) $Av = 2$ $Pv = 1$ $i = 10\%$ find n

$$2 = 1 (1+i)^n = (1.1)^n \quad \log 2 = n \log 1.1$$

$$\boxed{n = 7.2725 \text{ yr}}$$

ii) $Av = 2$ $Pv = 1$ $d^{12} = 10\%$

$$2 = 1 \left(1 - \frac{d^{12}}{12}\right)^{-12n} \quad \log 2 = -12n \log 0.99167$$

$$\boxed{n = 6.9053 \text{ yr}}$$