

$$Q1) \int_{1/50}^2 \frac{e^{2/w}}{w^2} dw$$

$$\text{let } t = e^{2/w}$$

diff w.r.t w

$$\frac{dt}{dn} = -2 e^{2/w} \frac{1}{w^2}$$

$$= \int_{1/50}^2 \frac{t w^2 dt}{-2 w^2 e^{2/w}}$$

$$\frac{1}{2} \int_{1/50}^2 \frac{dt}{t} = \frac{1}{2} \left[\ln t \right]_{1/50}^2$$

$$= \frac{1}{2} (2 - 0.02)$$

$$= \boxed{0.99}$$

$$Q2) \int \frac{2t^3+1}{(t^4+2t)^3} dt$$

$$\text{let } x = (t^4+2t)^3$$

diff w.r.t t

$$\frac{dx}{dt} = 3(t^4+2t)^2 (4t^3+2)$$

$$\int \frac{2t^3+1}{x(3)(t^4+2t)^2(4t^3+2)} dx$$

Multiply and divide by 2

$$\int \frac{4t^3+2}{(x)(3)(t^4+2t)^2(x)^{2/3}} dx$$

$$= \frac{1}{6} \int x^{-5/3} dx$$

$$= \frac{1}{6} \left(\frac{-3}{3} \right) \frac{x^{-5/3+1}}{-5/3+1} + C = \frac{1}{6} \cdot \frac{1}{-2} x^{-2/3} + C$$

$$= \frac{-1}{4} (t^4 + 2t) + C$$

$$= \frac{-1}{4} x^{-2/3} + C$$

Q3) $f'(x) = x^4 + 3x - 9$ $f(x) = \int f'(x) dx$

$$f(x) = \int (x^4 + 3x - 9) dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Q4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^3 f(x) dx = \int_{-2}^1 f(x) dx + \int_1^3 f(x) dx$$

$$\int_{-2}^1 3x^2 dx + \int_1^3 6 dx$$

$$[x^3]_{-2}^1 + [6x]_1^3$$

$$1 + 8 + 18 - 6 = 21$$

$$Q5) \int 3(8y-1) e^{4y^2-y} dy$$

$$x = e^{4y^2-y}$$

$$\int \frac{3(8y-1) e^{4y^2-y} dx}{(e^{4y^2-y})(8y-1)}$$

$$\frac{dx}{dy} = (e^{4y^2-y})(8y-1)$$

$$\frac{3x + C}{3(e^{4y^2-y}) + C}$$

$$Q6) f''(x) = 15\sqrt{x} + 5x^3 + 6$$

$$f'(x) = \int 15\sqrt{x} + 5x^3 + 6 dx$$

$$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k$$

$$f(x) = \int \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k dx$$

$$f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + C$$

$$f(1) = -5/4 = 4 + \frac{1}{4} + 3 + k + C$$

$$k + C = -8.5$$

$$f(4) = 404 = 4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + 4k + C$$

$$4k + C = 164$$

$$4K + C = 164$$

$$-12 + C = -8.5$$

$$3K = 172.5$$

$$K = 57.5$$

$$C = -66$$

$$\therefore f(x) = 4x^{5/2} + \frac{2^4}{4} + 3x^2 + 57.5x - 66$$

Q9) $\frac{dw}{dt} = 9.8 - 0.196v$

$$\int \frac{dw}{9.8 - 0.196v} = \int dt$$

$$\log(9.8 - 0.196v) = -0.196t + C$$

Q12) Taylor series for $f(x) = x^3 - 10x^2 + 6$ $x=3$
 $f(x) = x^3 - 10x^2 + 6$ $f(3) = -57$

$$f'(x) = 3x^2 - 20x \quad f'(3) = -33$$

$$f''(x) = 6x - 20 \quad f''(3) = -2$$

$$f'''(x) = 6 \quad f'''(3) = 6$$

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2}f''(3) + \frac{(x-3)^3}{6}f'''(3) + \dots$$

$$x^3 - 10x^2 + 6 = -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{6(x-3)^3}{6} + \dots$$

Q13) Maclaurian series for $(1+x)^u$

$$f(x) = (1+x)^u \quad f'(x) = u(1+x)^{u-1}$$

$$f''(x) = u(u-1)(1+x)^{u-2}$$

$$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$$

$$f(x) = f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

$$(1+x)^u = 1 + u + \frac{u(u-1)}{2!}x^2 + \frac{u(u-1)(u-2)}{3!}x^3 + \dots$$

Q14) $f(x) = \sqrt{1+x}$

$$f'(x) = \frac{1}{2}(1+x)^{-1/2} \quad f'(0) = 1/2$$

$$f''(x) = -\frac{1}{4}(1+x)^{-3/2} \quad f''(0) = -1/4$$

$$f'''(x) = \frac{3}{8}(1+x)^{-5/2} \quad f'''(0) = -3/8$$

$$\sqrt{1+x} = \sqrt{1+x} + \frac{x^1}{1!} \left(\frac{1}{2}\right) + \frac{x^2}{2!} \left(-\frac{1}{4}\right) + \frac{x^3}{3!} \left(-\frac{3}{8}\right) + \dots$$

Taylor series

Q15)

$$f(x) = e^{-6x}$$

$$x = -4$$

$$f(x) = e^{-6x}$$

$$f(-4) = e^{+24}$$

$$f'(x) = e^{-6x}(-6)$$

$$f'(-4) = e^{24} (6)^1$$

$$f''(x) = (-6)e^{-6x}(-6)$$

$$f''(-4) = e^{24} (6)^2$$

$$f'''(x) = (-6)^3 e^{-6x}$$

$$f'''(-4) = e^{24} (6)^3$$

$$e^{-6x} = e^{+24} + \underbrace{(x+4)(e^{24})(6)}_{L1} + \underbrace{(x+4)^2(e^{24})(6)^2}_{L2} + \underbrace{(x+4)^3(e^{24})(6)^3}_{L3}$$

Q16)

$$f(x) = \ln(3+4x)$$

$$x=0$$

$$\ln(3)$$

$$f'(x) = \frac{4}{3+4x}$$

$$f'(0) = 4/3$$

$$f''(x) = (-4)(4x+3)^{-2}(4)$$

$$f''(0) = (-1)(4)^2(3)^{-2}$$

$$f'''(x) = (+8)(4x+3)^{-3}(4)^2$$

$$f'''(0) = (-1)(3)^{-3}(4)^3$$

$$\ln(3+4x) = \ln(3) + \underbrace{x \cdot 4/3}_{L1} + \underbrace{\frac{x^2}{2}(-1)(4)^2(3)^{-2}}_{L2} + \underbrace{\frac{x^3}{6}(-1)(4)^3(3)^{-3}}_{L3} + \dots$$