Assignment - 1
Numerical Meathods of Algebra

1) radius measured = 12.65 cm
actual length = 12.5 cm

i) Absolute error = 12.65 - 12.5
= 0.15 cm

ii) Relative error = $\frac{0.15}{12.5}$ = 0.012

iii) Percentage error = 0.012 × 100
= 1.2 %

Q2) b) $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$

b) $\frac{5 - 4.5}{4.5 - 5.5} = \frac{y - 0.6853}{0.653 - 0.740}$

∴ y = 0.6967866

a) $\frac{5 - 4}{4 - 6} = \frac{y - 0.60206}{0.60206 - 0.7781513}$

∴ y = 0.69010565

3) $x^4 - x - 10 = 0$

$a = 1.5$; $b = 2$

$\;\;\;\uparrow x_1 \qquad\qquad \uparrow x_2$

For

x	y
1.5	−6.4375
2	4

$x_3 = \frac{x_1 + x_2}{2} = 1.75$

For

x	y
1.75	−2.371
2	4

$x_4 = \frac{x_3 + x_2}{2} = 1.875$

For

x	y
1.875	0.4846
1.75	−2.371

$x_5 = \frac{x_4 + x_3}{2} = 1.8125$

For

x	y
1.8125	−1.0202
1.875	0.4846

$x_6 = \frac{x_5 + x_4}{2} = 1.84375$

For

x	y
1.84375	−0.2877
1.875	0.4846

$\therefore x_7 = \frac{x_6 + x_4}{2} = 1.8593$

$\therefore$ for x_7, $y = 0.093$

4) $f(x) = x^3 - x - 1 = y$

$f'(x) = 3x - 1 = \cancel{y}$

x	$\cancel{y}$ $f(x)$	$\cancel{y'}$ $f'(x)$
0	-1	0
1	-1	2
2	5	5

$$x_1 = 1 - \frac{(-1)}{2} = 1.5$$

for x_1, $\cancel{y}$ $f(x) = 0.875$

$f'(x) = 3(1.5)^2 - 1$

$= 5.75$

$$x_2 = 1.5 - \left(\frac{0.87}{5.75}\right) = 1.3487$$

$f(x_2) = \cancel{1.046}\ 0.1006$

$f'(x_2) = 4.45$

$$x_3 = 1.3487 - \left(\frac{0.1006}{4.45}\right) = \cancel{0.9765}\ 1.3252$$

$f(x_3) = \cancel{1.0.9765}\ \cancel{1.045}\ 0.0019$

$f'(x_3) = \cancel{1.8607}\ 4.2684$

$$x_4 = 1.3252 - \frac{0.0019}{4.2684} = 1.347$$

$f(x_4) = -0.000075.$

5) $A^2 = A$

$7A - (I + A)^3 = 7A - (I^3 + 3I^2A + 3AI^2 + A^3)$

$\because (A^2 = A)$ also ($I^3 = I$, since I is identity matrix).

$= 7A - I \mp 3A + - 3A \cdot A \cdot A$

$= A - I - A^2$

$= A - I - A$

$7A - (I + A)^3 = -I$

6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$ $= 1(-6) + 1(-4-0) + 2(4)$

$= -2$

$M_{11} = 6$, $M_{12} = -4$, $M_{13} = 4$

$M_{21} = -1$, $M_{22} = -1$, $M_{23} = 1$

$M_{31} = -6$, $M_{33} = -2$, $M_{33} = 4$

$C = \begin{bmatrix} -6 & -4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix} \rightarrow C^T = \begin{bmatrix} -6 & -1 & -6 \\ 4 & -1 & -2 \\ 4 & 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -3 & 1/2 & 3 \\ 2 & -1/2 & 1 \\ 2 & 1/2 & 2 \end{bmatrix}$

7) $A = 8\hat{i} - 9\hat{j}$

$B = 7\hat{i} - 9\hat{j}$

$|A| = \sqrt{8^2 + 9^2} = \sqrt{145} =$

$|B| = \sqrt{7^2 + 9^2} = \sqrt{130}$

$A \cdot B = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$

$= 8(7) + (-9)(-9)$

$= 56 + 81$

$A \cdot B = 137$

$$\cos\theta = \frac{A \cdot B}{|A|\,|B|}$$

$$= \frac{137}{\sqrt{145} \times \sqrt{130}}$$

$\cos\theta = 0.998$

$\theta = \cos^{-1}(0.998)$

$\theta = 3.75$

8) Magnitude $= \sqrt{(75)^2 + (25)^2}$

$= \sqrt{5625 + 625}$

$= \sqrt{6250}$

$= 79.057$

9) $a = 101$

$a_n = 998$

$d = 3$

$\because a_n = a + (n-1)d$

$998 = 101 + (n-1)(3)$

$\therefore \dfrac{998-101}{3} = n-1$

$\therefore n = 299 + 1 = 300$

$\therefore S_{300} = \dfrac{300}{2}(101 + 998)$

$= 164850$

10) $t_6 = ar^5 = 32$

$t_8 = ar^7 = 128$

$\dfrac{t_8}{t_6} = \dfrac{ar^7}{ar^5} = \dfrac{128}{32}$

$\therefore r^2 = 4$

$\therefore r = 2$

11) $(4 + 3x)^5$

$= {}^5C_0\, 4^5 + {}^5C_1\, 4^4(3x) + {}^5C_2\, 4^3(3x)^2 + {}^5C_3\, 4^2(3x)^3$
$+ {}^5C_4\, 4^8(3x)^4 + {}^5C_5\, (3x)^5$

$= 1024 + 5(256)(3x) + 10(64)(9x^2) + 10(16)(27x^3)$
$+ 5(4)(81x^4) + 243x^5$

$= 1024 + 340x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\because A - \lambda I = 0$

$\therefore \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$

$\therefore \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = 0$

$\therefore (2-\lambda)\begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} - (-3)\begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} + 0\begin{vmatrix} 2 & -5-\lambda \\ 0 & 0 \end{vmatrix} = 20$

$\therefore (2-\lambda)(-5-\lambda)(3-\lambda) + 3(2)(3-\lambda) = 0$

$\therefore (3-\lambda)(\lambda^2 + 3\lambda - 4) = 0$

$\therefore (3-\lambda)(\lambda+4)(\lambda-1) = 0$

$\therefore \lambda = -4, 1, 3$

Eigen values of A are : $-4, 1, 3$

Eigen vectors of A when $\lambda = -4$:

$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \cancel{\lambda}-4 & 0 & 0 \\ 0 & \cancel{\lambda}-4 & 0 \\ 0 & 0 & \cancel{\lambda}\,-4 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$R_1 \to R_1 - 2R_2$

$\begin{bmatrix} 0 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\therefore \quad 2x_1 - x_2 = 0$

$2x_1 = x_2$ ——— (1)

$7x_3 = 0$

$\therefore \quad x_3 = 0$

$\therefore$ The eigen vector for $\lambda = -4$ are $\begin{bmatrix} x_1 \\ 2x_1 \\ 0 \end{bmatrix}$

When $\lambda = 1$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \quad x_1 - 3x_2 = 0 \quad \& \quad 2x_3 = 0$

$\therefore \quad x_1 = 3x_2 \qquad \therefore \quad x_3 = 0$

The eigen vector for $\lambda = 1$ are $\begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix}$

When $\lambda = 3$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\therefore \quad -x_1 - 3x_2 = 0 \quad$ i.e $\quad x_1 = 3x_2$

$2x_1 - 8x_2 = 0 \quad$ i.e. $\quad x_2 = 0 \rightarrow$ (From 1) on subsituting ② in ①

$x_1 = 0$

$\therefore$ Eigen vectors when $\lambda = 3$ are $\begin{bmatrix} 0 \\ 0 \\ \text{undefined} \end{bmatrix}$

13) $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5 \quad , \quad x_2 = ?$

$f'(x) = 3x^2 - 14x + 8$

$f(x_0) = f(5) = 5^3 - 7(5)^2 + 8(5) - 3 = 9$

$f'(x_0) = f'(5) = 3(5)^2 - 14(5) + 8 = 32$

$\therefore x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$\therefore x_1 = 5 - \left(\dfrac{-13}{13}\right) = 6$

$f(x_1) = f(6) = 6^3 - 7(6)^2 + 8(6) - 3 = 9$

$f'(x_1) = f'(6) = 3(6)^2 - 14(6) + 8 = 32$

$x_2 = 6 - \left(\dfrac{9}{82}\right) = 5.719$

$\therefore x_2 = 5.719$

14) $(1 - x + x^2)^4 = [(x^2 - x) + 1]^4$

By using binomial theorem.

$[(x^2 - x) + 1]^4 = {}^4C_0 (x^2 - x)^4 (1)^0 + {}^4C_1 (x^2 - x)^3 (1)^1 + {}^4C_2 (x^2$
$+ {}^4C_3 (x^2 - x)^1 (1)^3 + {}^4C_4 (x^2 - x)^0 (1)^4$

$= (x^2 - x)^4 + 4(x^2 - x)^3 + 6(x^2 - x)^2 + 4(x^2 - x) + 1$

$= \left[{}^4C_0 (x^2)^4 (-x)^0 + {}^4C_1 (x^2)^3 (-x)^1 + {}^4C_2 (x^2)^2 (-x)^2 + {}^4C_3 \right.$
$\left. + {}^4C_4 (x^2)^0 (-x)^4 \right] +$

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$$+\ 4\left[{}^3C_0(x^2)^3(-x)^0 + {}^3C_1(x^2)^2(-x)^1 + {}^3C_2(x^2)^1(-x)^2 + {}^3C_3(x^2)^0(-x)^3\right]$$
$$+\ 6\left[{}^2C_0(x^2)^2(-x)^0 + {}^2C_1(x^2)^1(-x)^1 + {}^2C_2(x^2)^0(-x)^2\right]$$
$$+\ 4(x^2 - 4) + 1$$

$$= x^8 - 4x^7 + 10x^6 - 16x^5 + 17x^4 - 16x^3 + 10x^2 - 3$$

15) $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

x	1	2
y	0.368	-0.768

$x_1 = 1$; $y_1 = 0.368$
$x_2 = 2$; $y_2 = -0.768$

$x_3 = \dfrac{x_1 + x_2}{2} = \dfrac{1+2}{2} = \dfrac{3}{2} = 1.5$

$\therefore y_3 = e^{-1.5} - 3\log(1.5) = -3.05$

$x_4 = \dfrac{x_1 + x_3}{2} = \dfrac{1 + 1.5}{2} = \dfrac{2.5}{2} = 1.25$

$y_4 = e^{-1.25} - 3\log(1.25) = -0.004$

$\therefore x = x_4 = 1.25.$

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