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Calculus - Assignment - 2

1. Let $I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2+1} dw$

let $\frac{2}{w} = t$ | when $w = \frac{1}{50}$

$-\frac{2}{w^2} dw = dt$ | $t = 2/w = 2(50) = 100$

$\frac{dw}{w^2} = -\frac{1}{2} dt$ | when $w = 2$, $t = \frac{2}{2} = 1$

$I = -\frac{1}{2} \int_{100}^{1} e^t dt$

$= -\frac{1}{2} \left[e^t \right]_{100}^{1}$

$= -\frac{1}{2} \left(e^1 - e^{100} \right)$

$\boxed{\approx (1.3440586)(10)^{43}}$

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2. $I = \int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

let $x = t^4 + 2t$

$dx = (4t^3 + 2)\, dt$

$2(2t^3+1)\, dt = dx$

$(2t^3+1)\, dt = \frac{1}{2} dx$

$I = \frac{1}{2}\int \frac{1}{x^3}\, dx = \frac{1}{2}\int x^{-3}\, dx$

$= \frac{1}{2}\left[\frac{x^{-2}}{-2}\right]$

$= \frac{-1}{4x^2} + c$

$= \boxed{\frac{-1}{4(t^4+2t)^2} + c}$

3. $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)$

$= \int (x^4 + 3x - 9)\, dx$

$= \boxed{f(x) = \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + c}$

4. $f(x) = \begin{cases} 6 & , x > 1 \\ 3x^2 & , x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \frac{\cancel{3}}{\cancel{3}}\left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= \left((1)^3 - (-2)^3\right) + \left[18 - 6\right]$$

$$= 1 + 8 + 12 = \boxed{21}$$

5. $I = \int 3(8y - 1)e^{4y^2 - y}\,dy$

let $4y^2 - y = t$

$(8y - 1)\,dy = dt$

$I = 3\int e^t\,dt$

$= 3e^t$

$= \boxed{3e^{4y^2 - y} + c}$

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6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int f''(x)\, dx$

$f'(x) = \int (15\sqrt{x} + 5x^3 + 6)\, dx$

$= \frac{15x^{3/2}}{3}(2) + \frac{5x^4}{4} + 6x + C_1$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$

now,

$f(x) = \int f'(x)$

$f(x) = \int (10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1)\, dx$

$= \frac{10x^{5/2}}{5}(2) + \frac{5}{4}\frac{x^5}{(5)} + \frac{6x^2}{2} + C_1x + C_2$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$ — 1

given $f(1) = -5/4$

substituting $x = 1$ in 1

$f(1) = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + C_1(1) + C_2$

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$$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + c_1 + c_2$$

$$-\frac{5}{4} = \frac{28+1}{4} + c_1 + c_2$$

$$-\frac{5}{4} - \frac{29}{4} = c_1 + c_2$$

$$c_1 + c_2 = -\frac{34}{4} \quad \text{--- (2)}$$

given $f(4) = 404$

substituting $x = 4$ in (1)

$$f(4) = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + c_1(4) + c_2$$

$$404 = 128 + 256 + 48 + 4c_1 + c_2$$

$$-28 = 4c_1 + c_2$$

$$-4(7 + c_1) = c_2 \quad \text{--- (3)}$$

substituting c_2 in (2)

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$$-\frac{34}{8} = c_1 - 4(7+c_1)$$

$$-\frac{34}{8} = c_1 - 28 - 4c_1$$

$$-\frac{34}{8} + 28 = -3c_1$$

$$\frac{190}{8} = -3c_1$$

$$c_1 \approx -7.916667$$

$$c_2 = -4(7+c_1)$$

$$\therefore c_2 \approx 3.667$$

$$\boxed{\Rightarrow f(x) = 4x^2\sqrt{x} + \frac{x^5}{4} + 3x^2 + (-7.91667)c_1 + 3.667.}$$

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7. let

$$z = f(x+iy) + g(x-iy) \quad \text{—} ①$$

diff eq ① partially w.r.t x

$$\frac{\partial z}{\partial x} = f'(x+iy) + g'(x-iy) \quad \text{—} ②$$

diff ② again partially wrt x

$$\frac{\partial^2 z}{\partial x^2} = f''(x+iy) + g''(x-iy)$$

diff ① partially wrt y

$$\frac{\partial z}{\partial y} = f'(x+iy)(i) + g'(x-iy)(-i) \quad ③$$

diff ③ partially wrt y

$$\frac{\partial^2 z}{\partial y^2} = f''(x+iy)(i)^2 + g''(x-iy)(-i)^2$$

$$\frac{\partial^2 z}{\partial y^2} = i^2 \left[f''(x+iy) + g''(x-iy)\right]$$

$$\boxed{\frac{\partial^2 z}{\partial y^2} = i^2 \left[\frac{\partial^2 z}{\partial x^2}\right]}$$ (as solved above)

required PDE

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8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $r(1) = 2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

integrating :

$\int \frac{1}{r^2} dr = \int \frac{1}{\theta} d\theta$

$= \frac{r^{-1}}{-1} + c = \log \theta$

$c = \log \theta + \frac{1}{r}$ — (1)

$r(1) = 2$

~~put~~ $\theta = 1$ & $r = 2$

$c = \log 1 + 1/2$

$= 0 + 1/2$

$c = 1/2$

$$\boxed{\log \theta + \frac{1}{r} = \frac{1}{2}}$$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\frac{dv}{0.196v} = 9.8\,dt$

integrating :

$\frac{1}{0.196}\int \frac{dv}{v} = 9.8\int dt.$

$\frac{1}{0.196}\log v = 9.8t + c$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\Rightarrow \frac{dv}{dt} + 0.196v = 9.8$

$IF = e^{\int 0.196\,dt} = e^{0.196t}$

solution is:

$v e^{0.196t} = 9.8\int e^{0.196t}\,dt$

$ve^{0.196t} = \frac{9.8}{0.196} e^{0.196t} + c$

$ve^{0.196t} = 50e^{0.196t} + c$

$$\boxed{v = 50 + \frac{c}{e^{0.196t}}}$$

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~~let~~ $\cancel{c_1 = \frac{c}{e^{0.196}}}$

10. IVP :

$$ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2$$

divide by t on both sides :-

$$y' + \frac{2y}{t} = t - 1 + t^{-1}$$

$$\Rightarrow P = \frac{2}{t} \quad \& \quad Q = t - 1 + t^{-1}$$

$$\text{If} = e^{\int \frac{2}{t} dt}$$

$$= \cancel{e^{2\log t}} = e^{\log t^2}$$

$$= \cancel{2}$$

$$= t^2$$

~~solution is~~ :

$$\cancel{2y = \int}$$

solution is :-

$$yt^2 = \int (t^3 - t^2 + t) \, dt$$

$$yt^{-2} = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c \quad -①$$

put $t = 1$ & $y = 1/2$

$$\frac{1}{2}(1)^2 = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + c$$

$$C = \frac{1}{12} \quad -②$$

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put ② in ① :-

$$\boxed{yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}}$$

↓

solution

11. $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$

$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

integrating :-

$\int \frac{dy}{y^3} = \int \frac{dx}{\sqrt{1+x^2}}$

$-\frac{1}{2y^2} = \ln\left|\sqrt{x^2+1} + x\right| + C$

put $x = 0$ & $y = -1$

$\Rightarrow \frac{1}{2} = \ln(1) + C$

$$\boxed{C = \frac{1}{2}}$$

particular solution :

$$\boxed{\frac{-1}{2y^2} = \ln\left|\sqrt{x^2+1} + x\right| + \frac{1}{2}}$$

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12. $f(x) = x^3 - 10x^2 + 6$ about $x = 3$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6$

$x = a = 3$

$$f(x) = f(a) + (x-a)\frac{f'(x)}{1!} + \frac{f''(x)}{2!}(x-a)^2$$

$$+ \frac{f'''(x)}{3!}(x-a)^3 + \cdots$$

$$= -57 + (x-3)(-33) + (x-3)^2(-1) + (x-3)^3 \times 1$$

$$+ \cdots$$

$$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3 + \cdots$$

13. Maclaurian series

$f(x) = (1+x)^{u}$

$f'(x) = u(1+x)^{u-1}$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

let $x = a = 0$

$f(x) = 1^{u} = 1$

$f'(x) = u$

$f''(x) = u(u-1)$

$f'''(x) = u(u-1)(u-2)$

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$$f(x) = (1+x)^{u} = 1 + ux + \frac{u(u-1)x^2}{2!} + \frac{u(u-1)(u-2)}{3!}.x^3 + \ldots.$$

14. $f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/3}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{2} \times \frac{-1}{2} (1+x)^{-3/2}$ $\qquad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{-1}{4}\left(\frac{-3}{2}\right)(1+x)^{-5/2}$ $\qquad f'''(0) = \frac{3}{8}$

Maclaurian series.

$$f(x) = 1 + \frac{1}{2}(x-0)^1 + \frac{(x-0)^2}{2!}\left(\frac{-1}{4}\right) + -$$

$$f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots\ldots$$

15. $f(x) = e^{-6x}$ $\qquad$ about $x = -4$

$f'(x) = -6e^{-6x}$

$f''(x) = 36e^{-36}$

$f'''(x) = -216e^{-6x}$

put $x = a = -4$

$f(a) = e^{24}$

$f'(a) = -6e^{24}$

$f''(a) = 36e^{24}$

$f'''(a) = -216e^{24}$

By Taylor series:-

$$e^{-6x} = e^{24} + (x+4)\left(\frac{-6e^{24}}{1}\right) + (x+4)^2\left(\frac{36e^{24}}{2}\right) + \ldots (x+4)^3\left(\frac{-216e^{24}}{6}\right)$$

$$e^{-6x} = e^{24}\left(1 - 6(x+4) + 18(x+4)^2 - 36(x+4)^3 + \ldots\right)$$

16. Taylor series:

$f(x) = \ln(3+4x)$ about $x = 0$

$$f'(x) = \frac{4}{3+4x}$$

$$f''(x) = 4(3+4x)^{-2}(4)$$
$$= -16(3+4x)^{-2}$$

$$f'''(x) = 128(3+4x)^{-3}$$

$x = a = 0$

$f(a) = \ln(3)$

$\cancel{f''(a)}$

$$f'(a) = \frac{4}{3}$$

$$f''(a) = \frac{-16}{9}$$

$$\boxed{f(x) = \ln 3 + \frac{4x}{3} - \frac{8}{9}x^2 + \frac{64}{81}x^3 + \ldots}$$