1)

(i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$n = 10$

$$\text{mean} = \frac{\Sigma x_i}{n} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$\therefore \text{mean} = 13.4$$

median $= \left(\frac{n+1}{2}\right)^{th}$ term $= (5.5)^{th}$ term

$$\therefore \text{median} = 5^{th}\text{ term} + 0.5\,(6^{th}\text{ term} - 5^{th}\text{ term})$$

$$= 11 + \frac{1}{2}(15-11)$$

$$= 11 + \frac{1}{2}(4) = 11 + 2 = 13$$

$$\therefore \text{median} = 13$$

mode = 18 $\therefore$ (as it has highest frequency)

Standard deviation, $\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$

$$\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

$$\sigma = \sqrt{E(X^2) - (E(X))^2}$$

where $E(x)$ is expected value.

$E(x) = mean = 13.4$

$$E(x^2) = \frac{\Sigma x_i^2}{n}$$

$$= \frac{16+49+81+81+121+225+324+324+324+625}{10}$$

$$E(x^2) = \frac{2170}{10} = 217$$

$$\sigma = \sqrt{E(x^2) - E(x)^2}$$

$$= \sqrt{217 - (13.4)^2} = \sqrt{217 - 179.56}$$

$$\sigma = \sqrt{37.44} = 6.118823416$$

$$\sigma = 6.118823416$$

let Q_1 and Q_3 be first and third quartiles respectively

so ~~Q_3 is 75%~~

so $Q_3 = (75\% \text{ of } (n+1))^{th}$ term

$Q_3 = \left(\frac{3}{4} \times 11\right)^{th}$ term

$Q_3 = (8.25)^{th}$ term

$Q_3 = 8^{th}$ term $+ 0.25(9^{th}$ term $- 8^{th}$ term$)$

$= 18 + 0.25(18 - 18)$

$\underline{Q_3 = 18}$

now $Q_1 = (25\% \text{ of } (n+1))^{th}$ term

$= \left(\frac{1}{4} \times 11\right)^{th}$ term

$= (2.75)^{th}$ term

$= 2^{nd}$ term $+ 0.75(3^{rd}$ term $- 2^{nd}$ term$)$

$= 7 + 0.75(9 - 7)$

$= 7 + 0.75 \times 2$

$= 7 + 1.5$

$\underline{Q_1 = 8.5}$

~~inter quat~~ inter quartile range $= Q_3 - Q_1$

$= 18 - 8.5$

$= 9.5$

inter quartile range $= 9.5$

mean $= 13.4$

median $= 13$

$\sigma = 6.118823416$.

mode $= 18$

b)

(ii) Let number of household be x_i

Let number of people be f_i

x_i	f_i	$x_i f_i$	cf	$x_i^2 f_i$
0	5	0	5	0
1	11	11	16	11
2	26	52	42	104
3	15	45	57	135
4	3	12	60	48

$n = 60 = \Sigma f_i$

mean $(\bar{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{120}{60} = 2$

$$\boxed{\therefore \text{mean} = 2 \dots (a)}$$

median $= \left(\dfrac{\Sigma f_i + 1}{2}\right)^{th}$ term

$= (30.5)^{th}$ term

$= 30^{th}$ term $+ \frac{1}{2}(31^{st}$ term $- 30^{th}$ term$)$

$= 2 + \frac{1}{2}(2-2)$

$= 2$

$$\boxed{\therefore \text{median} = 2.}$$

mode = 2 ... (highest no. of frequency)

$\sigma = \sqrt{E(X^2) - E(X)^2} = \sqrt{\frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2}$

$\therefore \frac{\sum f_i x_i^2}{n} = \frac{0 + 11 + 104 + 135 + 48}{60} = 4.96667$

$\therefore \sigma = \sqrt{4.96667 - (2)^2} = \sqrt{0.96667}$

$\therefore \sigma = 0.98319208$

let Q_1 and Q_3 be first and third quartile respectively

$Q_3 = (75\% \text{ of } (n+1))^{th} \text{ term} = \frac{3}{4} \times (\sum f_i + 1)$

$Q_3 = \left(\frac{3}{4} \times 61\right)^{th} \text{ term} = (45.75)^{th} \text{ term}$

$Q_3 = 45^{th} \text{ term} + 0.75\,(46^{th} - 45^{th} \text{ term})$

$= 3 + 0.75\,(3 - 3)$

$Q_3 = 3$

$Q_1 = \left(25\% \text{ of } (\sum f_i + 1)\right)^{th} \text{ term}$

$= \left(\frac{1}{4} \times 61\right)^{th} = 15.25^{th} \text{ term}$

$Q_1 = 15^{th} + 0.25\,(16^{th} - 15^{th}) = 1$

interquartile range $= Q_3 - Q_1$

$= 3 - 1$

$= 2$

2) rate $(\lambda) = 0.5$

(i) ~~claims $P(X=0)$~~

let claims arising on an industrial policy be X

Poison distribution is used as $Poi(\lambda)$

$P(\text{no claim arrising}) = P(X=0)$

$$P(X=0) = \frac{e^{-\lambda} \times \lambda^{x}}{x!}$$

$$= \frac{e^{-0.5} \times (0.5)^{0}}{0!} = e^{-0.5}$$

$\therefore P(X=0) = 0.60653066$.

(ii) $P(\text{one or more claims}) = P(X>0)$

$P(X>0) = 1 - P(X=0)$

$= 0.39346934$.

now we have to use binomial distribution

thus $n = 3$, $p = 0.39346934$

where p is probability of success

$X \sim Bin(n, p)$

$\therefore X \sim Bin\ (3, 0.39346934)$

$\therefore P(X=1) = {}^nC_x\, p^x (1-p)^{n-x}$

$= {}^3C_1 (0.39346934)^1 (0.60653066)^2$

$P(X=1) = 0.434247843.$

(iii) Since, it contains time; as we are waiting for 1 claims and we need to calculate the waiting period for the next claim, we will use the exponential distribution.

Time is a continuous variable and follows exponential distribution.

$P(X > 2) = 1 - P(X \leq 2)$

$= 1 - F(2)$

$= 1 - (1 - e^{-\lambda x})$

$= e^{-\lambda x} = e^{-0.5 \times 2} = e^{-1}$

$P(X > 2) = 0.36787944$

3) We are using gamma distribution

$\alpha = 2$, $\lambda = 1/2$, and ~~x~~ range of $x \in (0, \infty)$

(i) mean = Expected value = $E(x) = \frac{\alpha}{\lambda}$

$\therefore E(x) = \frac{2}{1/2} = 4$

$\therefore$ variance $= \frac{\alpha}{\lambda^2} = \frac{2}{1/4} = 8$

as we know that $\sigma^2 =$ variance

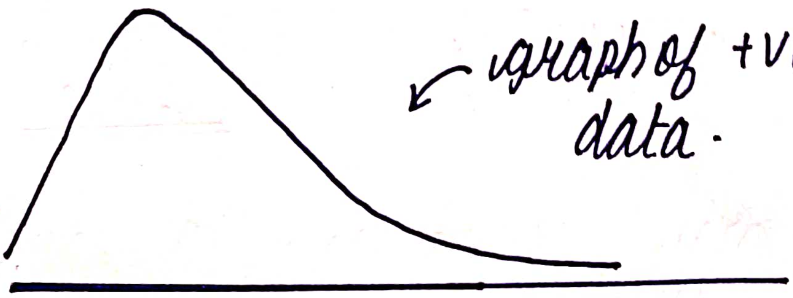
$\therefore \sigma = \sqrt{8} = 2.828427125$

(ii) ~~St~~ In order to comment about the shape, we are going to calculate its skewness.

coefficient of skewness $= \frac{2}{\sqrt{\alpha}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

coefficient of skewness $= 1.414213562$.

the data is rightly skewed or +ve skewed



graph of +ve skewed data.

(iii) $f_X(x) = \dfrac{\lambda^{\alpha}}{\Gamma(\alpha)} \times x^{\alpha-1} \times e^{-\lambda x}$

$$f_X(x) = \frac{\left(\frac{1}{2}\right)^2}{(2-1)!} \times x^{2-1} \times e^{-x/2}$$

$$= \frac{xe^{-x/2}}{4}$$

$$f_X(x) = \frac{xe^{-x/2}}{4}$$

$$F_X(x) = \int_0^x f_X(x)\,dx$$

$$= \int_0^x \frac{xe^{-x/2}}{4}\,dx$$

$$= \frac{1}{4} e^{-x/2}\left(\frac{x}{-1/2} - \frac{1}{1/4}\right)\Bigg|_0^x$$

$$= \frac{1}{4} e^{-x/2}\left(-2x - 4\right)\Bigg|_0^x$$

$$= \frac{1}{4}\left[e^{-x/2}(-2x-4) - e^{0}(0-4)\right]$$

$$= \frac{-1}{4}\left(2xe^{-x/2} - 4e^{-x/2} + 4\right)$$

$$F_X(x) = -\frac{1}{2}\left(xe^{-x/2} - 2e^{-x/2} + 2\right)$$

4) let S= chosing 2 members

F'= chosing atleast one female.

QA= chosing qualified acutariey.

M = chosing ~~m~~ both male.

~~P(S)~~ 1. ~~n(S)= 18C2~~ , ~~P(F')~~ =

4) let S= chosing 2 members

F'= chosing atleast one female.

M= chosing both male.

QA= chosing 2 qualified acutary.

$n(S) = {}^{18}C_2$

$P(F') = 1 - P(M)$

$= 1 - \frac{{}^{10}C_2}{{}^{18}C_2}$

$P(F') = \frac{{}^{18}C_2 - {}^{10}C_2}{{}^{18}C_2}$

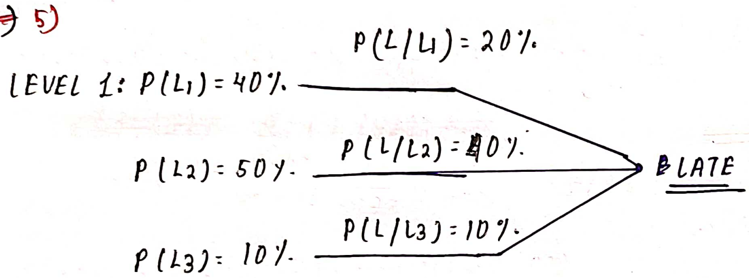
$P(QA \cap F') = \frac{{}^{7}C_2}{{}^{18}C_2}$

now probability of finding a team of two workers consist of two qualified females given that it contains atleast one female is →

$P(QA/F') = \frac{P(QA \cap F')}{P(F')} = \frac{{}^{7}C_2}{{}^{18}C_2 - {}^{10}C_2} = \frac{42}{216}$

$\boxed{\therefore P(QA/F') = 42/216}$

~~5)~~ 5)



$P(L_1)$, $P(L_2)$, $P(L_3)$ are probability ~~of~~ of the packages delivered ~~by~~ via level 1, level 2, and level 3 respectively.

$P(L/L_1)$ is probability of package arriving late via level 1.

$P(L/L_2)$ and $P(L/L_3)$ are probability of package arriving late via level 2 and level 3 respectively.

$\therefore$ The probability that a late package was sent via level 2 is

$$P(L_2/L) = \frac{P(L_2 \cap L)}{P(L)} = \frac{P(L_2) \times P(L/L_2)}{P(L_1) \times P(L/L_1) + P(L_2) \times P(L/L_2) + P(L/L_3) \times P(L_3)} = \frac{50\% \times 40\%}{40\% \times 20\% + 50\% \times 40\% + 10\% \times 10\%}$$

$$\boxed{P(L_2/L) = \frac{20}{8+20+1} = \frac{20}{29} = 0.689655172}$$

6) if $X \sim U(1,2)$ and $Y = \frac{3}{6-x}$

$f_x(x) = \frac{1}{\beta - \alpha} = \frac{1}{2-1} = 1 \cdots (a)$

$$y = \frac{3}{6-x} \Rightarrow \quad x = 6 - \frac{3}{y}$$

$$\therefore \quad g^{-1}(y) = 6 - \frac{3}{y}$$

$$\therefore \quad \cancel{f_x(g^{-1}(y))} = \frac{d}{dy}(g^{-1}(y)) = \frac{3}{y^2}$$

$$f_x(g^{-1}(y)) = 1 \cdots (\text{from } a)$$

$f_y(y) = f_x(g^{-1}(y)) \times \frac{d}{dy} g^{-1}(x)$

$$= 1 \times \frac{3}{y^2} = 3y^{-2}$$

$$\therefore \quad f_y(y) = 3y^{-2}$$

PDF of y is $3y^{-2}$.

7) x	1	2	3	4	5	TOTAL
$P(X=x)$	0.05	0.15	0.35	0.4	0.05	1
$xP(X=x)$	0.05	0.3	1.05	1.6	0.25	3.25
$x^2P(X=x)$	0.05	0.6	3.15	6.4	1.25	11.45

(i) $F(3) = P(X \leq 3) = \cancel{P(1)+P}\ P(X=1)+P(X=2)+P(X=3)$

$= 0.05 + 0.15 + 0.35$

$F(3) = \underline{\underline{0.55}}$

(ii) $E(x) = \sum_{i=1}^{5} x_i P(X=x_i) = \underline{\underline{3.25}}$

(iii) $V(x) = \sum_{i=1}^{5} x_i^2 P(X=x_i) = \underline{\underline{11.45}}$

(iv) coefficient of skewness $= \dfrac{\text{skewness}}{\sigma^3} = \dfrac{\mu_3}{\sigma^3} = \dfrac{E(x-\mu)^3}{\sigma^3}$

$\therefore E(x-\mu)^3 = E\left(x^3 - 3x^2\mu + 3\mu^2 x - \mu^3\right) = E(x^3) - 3\mu\left(E(x^2) - \mu E(x)\right] - \mu^3$

$= E(x^3) - 3\mu \times Vx + 3\mu^2 \times \mu - \mu^3 \ldots \left(\text{as } E(x^2) = V(X),\ E(X) = \mu\right)$

$= E(x^3) - 3\mu\sigma^2 + 2\mu^3$

$= E(x^3) - 3(3.25)(11.45) + 2(3.25)^3$

$= E(x^3) - 111.6375 + 68.65625 = E(x^3) - 42.98125$

$E(x^3) = \sum_{i=1}^{5} x_i^3 \cdot P(X=x_i)$

$= 1(0.05)+8(0.15)+27(0.35)+64(0.4)+125(0.05)$

$= 42.55$

$$\boxed{\text{coeff of skewness} = \frac{E(x-\mu)^3}{\sigma^3} = \frac{42.55 - 42.98125}{(11.45)^{3/2}} = \frac{-0.43125}{38.74433} = -0.011130661}$$

8) Let success be claim amount exceeding 1000:

$P(s) = p = 0.2$, $n = 15$ This is Binomial distribution

$P(f) = 1-p = q = 0.8$, $X \sim Bin(15, 0.2)$

$P(X<3) = P(X=0)+P(X=1)+P(X=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{15} + 15\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{14} + \frac{15 \times 14}{2}\left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{13}$

$= \left(\frac{4}{15}\right)^{13}\left((0.8)^2 + 3 \times \frac{4}{5} + \frac{\cancel{15}^{3} \times \cancel{14}^{7}}{\cancel{2}} \times \frac{1}{\cancel{5}} \times \frac{1}{5}\right)$

$= (0.8)^{13}(0.64 + 2.4 + 4.2)$

$= (0.8)^{13}(7.24)$

$P(X<3) = 0.398023209$

$\therefore$ The probability that fewer than 3 of them exceed 1000 is 0.398023209.

9) Let success be winning the prize

$P(s) = p = 0 \cdot 01$, $P(f) = 1 - p = 0 \cdot 99$, $n = 7$

it is a binomial distribution

$X \sim Bin(n, p)$

$X \sim Bin(7, 0 \cdot 01)$

$P(X=3) = {}^{7}C_{3}\, p^{3} (1-p)^{4}$

$= 0 \cdot 000033621$

The probability that he takes 7 weeks up to and including the week where he wins his third prize is $P(X=3)$ which is $0 \cdot 000033621$

10) It will follow Poison's distribution.

$\lambda = 2/5 = 0.4$

$P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(0.4)^0 e^{-0.4}}{0!} = e^{-0.4} = 0.670320046$

$P(X>2) = 1 - P(X \leq 2)$

$= 1 - P(X=0) - P(X=1) - P(X=2)$

$P(X=1) = \frac{\lambda^1 e^{-\lambda}}{1!} = (0.4) \times e^{-0.4} = 0.268128018.$

$= 1 - 0.99206583$

$= 0.00793417$

$P(X=2) = \frac{\lambda^2 e^{-\lambda}}{2!} = (0.4)(0.2)e^{-0.4} = 0.053625604$

$P(X>2) = 0.00793417.$

11) ~~B~~ let M and F be the events of having a male and female child respectively

$\therefore P(M) = P(F)$

$\therefore P(M) + P(F) = 1$ $\therefore P(M) = P(F) = 1/2$ ---- (i)

Let E be the event of having 3 boys.

$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

The event of having a child is not dependent on previous events.

$P(E) = 1/8$

12) as time is continuous variable, that's why we'll use exponential dist.

$\lambda = 10$, $\alpha = 1$, ~~$t = 0.1$~~

$$P(T < 0.1) = F(0.1)$$
$$= 1 - e^{-\lambda t}$$
$$= 1 - e^{-10 \times 0.1}$$
$$= 1 - e^{-1}$$
$$= 0.632120559$$

---

13) $X \sim U(75, 120)$

$$f_X(x) = \frac{1}{b-a} = \frac{1}{45}$$

$$P(80 < x < 100) = \frac{100 - 80}{45} = \frac{20}{45}$$

$$P(80 < x < 100) = 0.444444$$

14) $X \sim \text{Gamma}(5, 0.4)$ calculate $P(X < 22.89)$

$$P(X<22.89) = F_x(22.89) = \int_0^{22.89} \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\Gamma(\alpha)} dx$$

$$= \int \frac{0.4^5 e^{-0.4x} x^4}{\Gamma(5)} dx$$

$$= \frac{0.4^5}{4!} \times \int x^4 e^{-0.4x} dx$$

$$= \frac{0.4^5}{4!} \left(e^{-0.4x} \left(x^4 - 4x^3 + 12x^2 - 24x + 24\right)\right)_0^{22.89}$$

$$P(X<22.89) = \frac{0.4^5}{4\times3\times2\times1} \left(e^{-0.4\times22.89} \left((22.89)^4 - 4(22.89)^3 + 12(22.89)^2 - 24(22.89) + 24\right) - e^0 \left(0 - 0 + 0 - 0 + 24\right) \right)$$

$$F(22.89) = 0.0004266 \left(e^{-9.156} \begin{pmatrix} 274{,}525.8031 - 47{,}973.05428 + \\ 6287.4252 - 549.36 + 24 \end{pmatrix} - 24 \right)$$

$$= 0.0004266 \left(0.000105 (232{,}314.8183) - 24 \right)$$

$F(\cancel{F}) = 0.0004266 \quad (24.5288183 - 24)$

$\cancel{F(2)} = 0.000225594$

~~F(2) = P(X<~~

$F(22.89) = P(X < 22.89) = 0.000225594$

---

14) $X \sim Gamma(5, 0.4)$

$2\lambda x \sim \chi^2_{2\alpha}$ $\quad \alpha = 5$, $\lambda = 0.4$

$0.8x \sim \chi^2_{10}$

$18.5 \rightarrow 0.9529$

$18 \rightarrow 0.9450$

$P(X < 22.89) = P(0.8x < 22.89 \times 0.8)$

$= P(0.8x < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$= \cancel{1 - P(\chi^2_{10} > 18.312)}$

$= 0.9499296$

<u>using table.</u>

$P(\chi^2_{10} < 18.5) = 0.9529$

$P(\chi^2_{10} < 18) = 0.9450$

$P(\chi^2_{10} < 18.312) = \frac{0.312}{18.5 - 18} \times (0.9529 - 0.9450) + 0.9450$

$= \frac{0.312 \times 0.0079}{0.5} + P(\chi^2_{10} < 18)$

$= \underline{0.9499296}$

$P(X < 22.89) = P(\chi^2_{10} < 18.312) = \underline{\underline{0.9499296}}.$

15) $f_X(x) = \frac{k}{(x+5)^4}, \; x>0$

(i) $\int_0^\infty f_X(x)\,dx = 1$

$\therefore$ LHS, $\int_0^\infty \frac{k}{(x+5)^4}\,dx = k\,\frac{(x+5)^{-4+1}}{-4+1}\Big|_0^\infty$

$= \frac{k}{3}\left(\frac{1}{(x+5)^3}\right)_\infty^0$

$= \frac{k}{3}\left(\frac{1}{5^3} - \frac{1}{\infty}\right)$

$= \frac{k}{3\times 125} = \frac{k}{375}$

as LHS = RHS

$\therefore \frac{k}{375} = 1$

$\therefore k = 375$

(ii) $F(10) = P(X \leq 10)$

$$= \int_0^{10} \frac{375}{(x+5)^4}\, dx$$

$$= \frac{375}{3}\left(\frac{1}{(x+5)^3}\right)_{10}^{0}$$

$$= \cancel{3}\,125\left(\frac{1}{125} - \frac{1}{3375}\right)$$

$$= \cancel{125}\left(\frac{3250}{\cancel{125} \times 3375}\right)$$

$$= \frac{3250}{3375}$$

$F(10) = 0.962962963$

(iii) $$E(x) = \int_0^{\infty} x\, f_x(x)\, dx$$

$$= \int_0^{\infty} \frac{375x}{(x+5)^4}\, dx \quad \ldots\ldots \left(\begin{array}{ll} \text{let } t = x+5 & x=0,\ t=5 \\ \quad dt = dx & x=\infty,\ t=\infty \end{array}\right)$$

$$= \int_5^{\infty} \frac{375(t-5)}{t^4}\, dt$$

$$E(x) = 375 \int_5^{\infty} \frac{1}{t^3} - \frac{5}{t^4}\, dt$$

$$= 375 \left[\frac{-1}{2t^2} + \frac{5}{3t^3} \right]_5^{\infty}$$

$$= 375 \left[-0 + 0 + \frac{1}{2 \times 25} - \frac{5}{3 \times 125} \right]$$

$$= \cancel{125} \times \cancel{3} \left(\frac{15 - 10}{2 \times \cancel{25} \times \cancel{5} \times \cancel{3}} \right)$$

$$= \frac{5}{2}$$

$$E(x) = 2 \cdot 5$$