Financial Mathematics
Assignment-2

Q1.

$A = ₹20000$

monthly payment $= ₹427.9$

$t = 5$ yrs

(1) $A = 427.9 \times 12 \times a^{(12)}_{\overline{5|}}$ @ $i\%$

$$20000 = 427.9 \times 12 \times \left[\frac{1-(1+i)^{-5}}{((1+i)^{1/12}-1)(12)}\right]$$

$\therefore i = 10.8\%$

$\therefore \boxed{APR = 10.8\%}$

(ii) $PV_1 = 427.9 \times 12 \times a^{(12)}_{\overline{4|}}$ @ 10.8%

$$\therefore PV_1 = 427.9 \times 12 \times \frac{1-(1.108)^{-4}}{12 \times [(1.108)^{1/12} - 1]}$$

$$PV_1 = 427.9 \times \frac{0.336499952}{0.008583006}$$

$PV_1 = ₹16775.97737794$

now -

$$16775.97737794 = 274.49 \times 12 \times a^{(12)}_{\overline{t|}} \text{ @ } 10.8\%$$

$$\therefore \frac{16775.977\cancel{38}}{274.49} = \frac{1-(1.108)^{-t}}{(1.1108)^{1/12}-1}$$

$\therefore (\cancel{1}1.108)^{-t} = 0.5375328065$

$\therefore \boxed{t = 7.2479}$

(III) Total interest on initial loan $= 427.9 \times 48 - 16775.97738$
$= 3763.22262$

Total interest on new loan structure $= 7.2479 \times 274.49 \times 12$
$- 16775.97738$
$= 7097.735314$

They paid <u>₹ 3334.512694</u> more as interest

Q2.
(I) a) <u>Discounted payback period</u> -
It is the number of years after which cumulative discounted cash inflow surpasses the initial investment amount.

b) <u>Payback period</u> -
It is the time taken to recover funds invested. It does not include interest.

(II)

PAGE No.
DATE / /

(III) a) Project A

PV outlay $= -170 - 20v - 10v^2$
PV inlay $= 20v + 20v^2 + 200v^3$

$NPV_A = -170 - 20v - 10v^2 + 20v + 20v^2 + 200v^3$
$NPV_A = 200v^3 + 10v^2 - 170 = 0$

$IRR_A \Rightarrow 200(1+i)^{-3} + 10(1+i)^{-2} - 170 = 0$

$IRR_A = \boxed{i = 7.4\%}$

Project B

PV outlay $= -200$
PV inlay $= 14\, a_{\overline{6}|} + 200v^6$

$NPV_B = -200 + 14\left(\frac{1-(1+i)^{-6}}{i}\right) + 200v^6 = 0$

$IRR_B \Rightarrow 200(1+i)^{-6} + 14\left(\frac{1-(1+i)^{-6}}{i}\right) - 200 = 0$

$IRR_B = \boxed{i = 7\%}$

b) at $d = 6\%$

$NPV_A = 200(1-0.06)^3 + 10(1-0.06)^2 - 170$
$\boxed{NPV_A = 4.9528}$

PAGE No.
DATE / /

$$NPV_B = -200 + 14\left(\frac{1-(0.94)^6}{0.06}\right) \times 0.94 + 200(0.94)^6$$

$$\boxed{NPV_B = 5.9958509}$$

c) For project A to ~~yield~~ yield profits, the interest rate should be below 7.4%.

For Project B to yield profits, interest rates should be below 7%.

Q3. Annuity 1

$$PV = 200\, \ddot{a}_{\overline{22}|} \left(v^{3/12}\right)$$

$$= \frac{(1.08)^{-1/4}(200)(10.2007)(0.08)}{(0.074074074)}$$

PV = £2161.365534

Annuity 2

$$PV = 320\, \ddot{a}^{(4)}_{\overline{16.25}|} \left(v^{2/12}\right)$$

$$= 320 \times \frac{1-(1.08)^{-16.25}}{0.08} \times (1.049519) \times (1.08)^{-1/6}$$

PV = £2996.048870

Annuity 3

$$PV = 15 \times \frac{1-(1.08)^{-\frac{226}{12}}}{0.00639292}$$

PV = £1795.651419

Total PV = ₹6953.065823

New annuity-

$$6953.065823 = C \times \ddot{a}^{(2)}_{\overline{21.5}|} \ @\ d^2$$

$$\boxed{C = ₹649.0135657}$$

Q4.] $PV = 1 + 2v + 3v^2 + 4v^3 + \dots\dots nv^{n-1}$ — (1)

$(1-d)PV = v + \cancel{2v}^2 + 3v^3 + 4v^4 + \dots\dots nv^n$ — (2)

subtracting (2) from (1) -

$\cancel{PV} \, PV - PV(1-d) = 1 + v + v^2 + v^3 + \dots\dots v^{n-1} - nv^n$

$$dPV = \ddot{a}_{\overline{n}|} - nv^n$$

$$\therefore I\ddot{a}_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

Hence proved

Q5]

Q6.] let initial amt. = C.

(i) $160000 = C \cdot a_{\overline{10|}}$ @ 8%.

$\therefore \boxed{C = £23844.65201}$

(ii) $PV_4 = 23844.65201\ a_{\overline{6|}}$ @ 8%.

$PV_4 = £110231.4422$

let new amt. = X

$110231.4422 = X\ a_{\overline{6|}}$ @ 10%.

$\therefore \boxed{X = £25309.72428}$

(iii) $PV_7 = 25309.72428\ a_{\overline{3|}}$ @ 10%.

$PV_7 = 62942.75331$

let new amt = A.

$62942.75331 = A \cdot a_{\overline{3|}}$ @ 9%.

$\therefore \boxed{A = £24865.78174}$

Q8.] (i) PV outlay $= 2.7 + 0.2\, a_{\overline{3}|}$

PV inlay $= \bar{a}_{\overline{10}|}\left(0.25v + 0.2v^2 + 0.1v^3\right) + 0.1v^3$

$$NPV = \frac{1 - v^{10}}{-\ln(v)}\left(0.25v + 0.2v^2 + 0.1v^3\right) + 0.1v^3 - 2.7 - 0.2 \cdot \frac{(1 - v^3)}{(v^{-1} - 1)}$$

$\therefore \boxed{i = 9.3\%}$

(ii) $NPV = 0.1v^3 + 0.85\, \bar{a}_{\overline{10}|}\, v^3 - 2.7 - 0.2\, a_{\overline{3}|}$ @ 10%.

$\boxed{NPV = 1.00899}$

Q9.]

Q10. (i) $PV = 180000\, a_{\overline{15}|} + 20000\, (Ia)_{\overline{15}|}$ @ 12%.

$\boxed{PV = ₹2040588}$

$\therefore$ Total cost $= \dfrac{2040588}{0.8} = \boxed{₹2550735}$

(ii) $PV_9 = 340000\, a_{\overline{7}|} + 20000\, (Ia)_{\overline{7}|}$ @ 12%.

$PV_9 = ₹1875850.33$

Yr	loan	Repayment	interest	Capital Repayment
9	1875850.33	360000	225102.04	134897.96
10	1740952.37	360000	208914.28	171085.72
11	1569866.65			

(iii) let amt. $= c$

$1569866.65 = c\, a_{\overline{5}|}$ @ 10%.

$\therefore \boxed{c = 414126.87}$

Total instalments outstanding at end of 10^{th} yr $= ₹630133.35$

Extra payment $\Rightarrow (414126.87 \times 5) + (0.01 \times 1569866.65) - 1569866.65$

$\boxed{\text{Extra payment} = ₹516466.35}$

PAGE No.
DATE / /

Q12.

(I) ~~It is the~~ DPP is the time after which the ~~ca~~ cumulative discounted cash inflow ~~sur~~ passes investment amount.

(II) a.) $-800(1+i)^t - 50(1+i)^{t-1} + 100\, \bar{s}_{\overline{t-2}|} = 0$ @ 7%

~~Solution~~

$$-800(1.07)^t - 50(1.07)^{t-1} + 100\left[\frac{(1.07)^{t-2} - 1}{\ln(1.07)}\right] = 0$$

$\therefore \boxed{t = 17.767}$

b.) $AV = 100\, \bar{s}_{\overline{4.233}|}$ @ 6%

$$AV = 100\left[\frac{(1.06)^{4.233} - 1}{\ln(1.06)}\right]$$

AV = £480

(III)

Q13] let amt. = C

(i) $988000 = C \times 12 \times a^{(12)}_{\overline{25}|}$ @ 7%

$988000 = C \times \cancel{12} \times \dfrac{\left[1-(1.07)^{-25}\right]}{\cancel{12} \times \left[(1.07)^{1/12}-1\right]}$

$\boxed{C = ₹6848.04170724}$

(ii) $C\, a^{(12)}_{\overline{34/3}|} = 6848.0417\, a^{12}_{\overline{34/3}|}$ @ 7%

$= 648600$

(iii) After 10th september -

$PV = C\, a^{(12)}_{\overline{83/6}|}$ @ 7%

After 10th october -

$PV = C \cdot a^{12}_{\overline{13.75}|}$ @ 7%

$\text{Capital Repaid} = C\left[\dfrac{v^{13.75} - v^{83/6}}{i^{12}}\right]$ @ 7%

$\boxed{\text{Capital Repaid} = ₹26.86}$

(iv) Capital repayment in 12 instalments -

$C\left[a^{(12)}_{\overline{22/3}|} - a^{(22)}_{\overline{19/3}|}\right]$ @ 7%

$= ₹516.2$