Numerical Methods & Algebra

Assignment - 1

Q.1 Actual length $= r_1 = 12.5\ cm$

Actual area $= \pi r_1^2$

$= \pi (12.5)^2$

$= 490.87385\ cm^2$

Approximate radius $= 12.65$

Approximate area $= \pi r^2$

$= \pi (12.65)^2$

$= 502.725 51\ cm^2$

1] Absolute error = Approx area - Actual area

$= 502.72551 - 490.87385$

$= 11.58166\ cm^2$

2] Relative error $= \dfrac{\text{Absolute error}}{\text{Actual area}}$

$= \dfrac{11.58166}{490.87385}$

$= 0.024144$

3] ~~Relative~~ Percentage error = Relative error $\times 100$

$= 0.024144 \times 100$

$= 2.4144\ \%$

Q.2 1] Using linear interpolation

$x_1 = 4$ $\quad y_1 = 0.60206$

$x_2 = 6$ $\quad y_2 = 0.7781513$

$(6-4) = (0.7781513 - 0.60206)$

$2 = 0.1760913$

1 cause a change of $\frac{0.1760913}{2} \times 1$

$\therefore 1 = 0.08804565$

$\therefore x = 5$

$y = 0.60206 + 0.08804565$

$= 0.69010565$

2] $x_1 = 4.5$ $\quad y_1 = 0.6532125$

$x_2 = 5.5$ $\quad y_2 = 0.7403627$

$(5.5 - 4.5) = 0.7403627 - 0.6532125$

$1 = 0.0871502$

$\therefore$ change caused by $0.5 = 0.0435751$

$\therefore x = 5$

$y = 0.6532125 + 0.0435751$

$= 0.6967876$

Q.3 $x^4 - x - 10 = 0$

$x_0 = 1.5$ $y_1 = -6.4375$

$x_1 = 2$ $y_2 = 4$

$\rightarrow x_2 = \frac{x_0 + x_1}{2}$

$= 1.75$

$y_2 = -2.37109$

$\rightarrow x_3 = \frac{x_1 + x_2}{2}$

$= 1.875$

$y_3 = 0.48462$

$\rightarrow x_4 = \frac{x_2 + x_3}{2}$

$= 1.8125$

$y_4 = -1.02025$

$\rightarrow x_5 = \frac{x_4 + x_3}{2}$

$= 1.84375$

$y_5 = -0.28773$

$\rightarrow x_6 = \frac{x_3 + x_5}{2}$

$= 1.859375$

$y_6 = 0.93378$

Q.4 $x^3 - x - 1 = 0$

$f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{-1}{2}$$

$$= 1.5$$

$y_1 = 0.875$

$f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \left(\frac{0.875}{5.75}\right)$$

$$= 1.34783$$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44994$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3252$$

$y_3 = 0.00205$

$f'(x_3) = 4.26846$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.32472$$

$y_4 = 0.000008712$

Q.5 $A^2 = \cancel{A}\ A$

$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - (I + A^2(A) + 3A^2 \cancel{I} + 3A\cancel{I})$

$= 7A - (I + A^2 + 6A)$

$= 7A - I - \cancel{6A}\ 7A$

$= -I$

Q.6 $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1 \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} - (-1) \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1(0-6) + 1(-4-0) + 2(4-0)$

$= -6 + -4 + 8$

$= -2$

$adj(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj(A)$

$= \frac{1}{-2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$

Q.7 $\vec{a} = 8\hat{i} - 9\hat{j}$

$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$\vec{a} \cdot \vec{b} = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$

$= 56 + 81$

$= 137$

$a = \sqrt{64 + 81}$

$= \sqrt{145}$

$b = \sqrt{49 + 81}$

$= \sqrt{130}$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$\cos\theta = \frac{\vec{a}\,\vec{b}}{ab}$

$= \frac{137}{\sqrt{145} \cdot \sqrt{130}}$

$= 0.997849146$

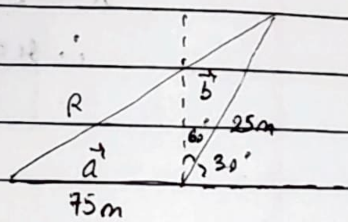
$\theta = 3.75856^\circ$

Q.8 Displacement vector $= \vec{R}$

$\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30^\circ$



$R^2 = 75^2 + 25^2 + 2(75)(25)\cos 30^\circ$

$R^2 = 9497.595264$

$R = \underline{97.4556}$ metres

Q.9 Common difference = $d = 3$

$AP = 101, 104, 107, \dots, 998$

$a = 101$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$897 = (n-1)3$

$n = 300$

$$S_n = \frac{n}{2}(a + a_n)$$

$$= \frac{300}{2}(101 + 998)$$

$$= 164850$$

Q.10 $a_6 = 32$ & $a_8 = 128$

$ar^5 = 32$ $ar^7 = 128$

$$\frac{ar^7}{ar^5} = \frac{128}{32}$$

$r^2 = 4$

$r = 2$ or -2 (-2 rejected as it gives (-ve) value)

$\therefore r = 2$

Q.11 $(3x+4)^5$

Using Pascal's Triangle →

1	a
1 1	$a+b$
1 2 1	$(a+b)^2$
1 3 3 1	$(a+b)^3$
1 4 6 4 1	$(a+b)^4$
1 5 10 10 5 1	$(a+b)^5$

$$(3x+4)^5 = 3x^5 + (5\times(3x)^4\times 4) + (10\times(3x)^3\times 4^2) + (10\times(3x)^2\times 4^3) + (5\times 3x\times 4^4) + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3480x + 1024$$

~~(3x+5)~~

$$(3x+4)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3480x + 1024$$

Q.12 $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$|A - \lambda I|$

Expanding using 3rd row

$$|A - \lambda I| = 0 - 0 + (3-\lambda)[(2-\lambda)(-5-\lambda) + 6]$$

$$= (3-\lambda)[-10 - 2\lambda + 5\lambda + \lambda^2 + 6]$$

$$= (3-\lambda)[\lambda^2 + 3\lambda - 4]$$

$$= 3\lambda^2 + 9\lambda - 12 - \lambda^3 - 3\lambda^2 + 4\lambda$$

$$= -\lambda^3 + 13\lambda - 12$$

$$-\lambda^3 + 13\lambda - 12 = 0$$
$$\lambda^3 - 13\lambda + 12 = 0$$

Comparing coefficient,

$$a + b + c + d = 1 - 13 + 12$$
$$= 0$$

$\therefore$ only one root $= 1$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -13 & -12 \\ & & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$$\lambda^2 + \lambda - 12 = 0$$
$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$
$$\lambda(\lambda + 4) - 3(\lambda + 4) = 0$$
$$(\lambda - 3)(\lambda + 4) = 0$$
$$\lambda = 3 \text{ or } -4$$

$\therefore$ Eigen values $= -4, 3 \,\&\, 1$

Q.14 $(x^2 - x + 1)^4 = ((x-1)^2 + x)^4$

Using Pascal's triangle

1	a
1 1	$a+b$
1 2 1	$(a+b)^2$
1 3 3 1	$(a+b)^3$
1 4 6 4 1	$(a+b)^4$

$$((x-1)^2 + x)^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$
$$(x^2 - x + 1)^4 = (x-1)^8 + 4(x-1)^6 x + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$$

Q.13 $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13$

$f'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \frac{-13}{13}$$

$$= 6$$

$f(x_1) = 9$

$f'(x_1) = 32$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 6 - \frac{9}{32}$$

$$= 5.71875$$

$f(x_2) = 0.84787$

Q.15 $e^{-x} = 3\log(x)$

$3\log x + e^{-x} = 0$

x	y
0.1	-2.09516
0.2	-1.27818
0.3	-0.82782
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	0.03188

$x_0 = 0.6 \qquad y_0 = -0.11673$

$x_1 = 0.7 \qquad y_1 = 0.03188$

$\rightarrow x_2 = \frac{x_0 + x_1}{2}$

$= 0.65$

$y_2 = -0.03921$

$\rightarrow x_3 = \frac{x_2 + x_1}{2}$

$= 0.675$

$y_3 = -0.00293$

$\rightarrow x_4 = \frac{x_3 + x_1}{2}$

$= 0.6875$

$y_4 = 0.01465$

$\rightarrow x_5 = \frac{x_4 + x_3}{2} = 0.68125$

$y_5 = 0.0059$