Fno - 1 [Calculus] 6/2/2

Lagrange's Interpolation

①

x	7	8	9	10
y	3	1	1	9

→ $y = ?$ at $x = 9.5$

$$= f(9.5) = f(7) \times \frac{1.5 \times 0.5 \times -0.5}{(7-8)\times(7-9)\times(7-10)} \quad (3)$$

$$f(8) \times \frac{2.5 \times 0.5 \times -0.5}{(8-7)\times(8-9)\times(8-10)} \quad (1)$$

$$f(9) \times \frac{2.5 \times 1.5 \times -0.5}{(9-7)\times(9-8)\times(9-10)} \quad (1)$$

$$f(10) \times \frac{2.5 \times 1.5 \times 0.5}{(10-7)\times(10-8)\times(10-9)} \quad (9)$$

$$= \frac{1}{640} + \frac{1}{1264} + \frac{1}{416} + \frac{15}{448}$$

$$= + 0.028724657$$

$$= 0.1875 + -0.3125 + 0.9375 + 2.8125$$

$$=$$

② let the independent variable $= x$ and $f(x) = y$

$\rightarrow \therefore$

x	3	7	9	10
y	168	120	72	63

$=$ $y = f(x) = ?$ at $x = 6$.

$$\therefore f(6) = \frac{f(3)(-1)(-3)(-4)}{(3-7)(3-9)(3-10)} + \frac{f(7)(3)(-3)(-4)}{(7-3)(7-9)(7-10)} + \frac{f(9)(3)(-1)(-4)}{(9-3)(9-7)(9-10)} + \frac{f(10)(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)}$$

(168 → 3, 120 → 7, 72 → 9, 63 → 10)

$$= 12 + 180 + (-72) + 27$$

$$= 147$$

③

x	1	2	7	8
y	4	5	5	4

$f(6) = ?$ or $y = ?$ at $x = 6$

$$\rightarrow f(6) = f(1)\frac{(4)}{-1 \times -6 \times -7} + f(2)\frac{(5)}{1 \times -5 \times -6} + f(7)\frac{(5)}{3 \times 5 \times 1} + f(8)\frac{(4)}{7 \times 6 \times 1}$$

(4 → −ve; 5 → +ve; 5 → +ve; 4 → −ve)

$\therefore f(6) = -0.7619047 + 1.6666667 + 6.6666667 + (-1.904761905)$

$= \underline{\underline{9.476190605}}$

(4) $f(5) = ?$

x	1	2	3	4	7
y	2	4	8	16	128

$\rightarrow f(5) = \frac{f(1) \times 5 \times 2 \times 1 \times -2}{-1 \times -2 \times -3 \times -6} + \frac{f(2) \times 4 \times 2 \times 1 \times -2}{1 \times 1 \times -2 \times -3}$

$+ \frac{f(3) \times 4 \times 3 \times 1 \times -2}{2 \times 1 \times 1 \times 2} + \frac{f(4) \times 4 \times 3 \times 2 \times 2}{3 \times 2 \times 1 \times 3}$

$+ \frac{f(7) \times (4 \times 3 \times 2 \times 1)}{6 \times 5 \times 4 \times 3}$

$= f(5) = -\frac{2}{3} + \frac{32}{5} + \left(\frac{-8 \times 3}{1}\right) + \left(\frac{128}{3}\right)$

$+ \frac{128}{15}$

$= 32.933333$

(5) x 10 20 14 16

$f(x)$ 0.25 0.2 0.15 0.1

→ $f(15) = ?$

$$f(15) = 0.25 \times \frac{-5 \times -1 \times +1}{-10 \times +4 \times +6} + 0.2 \times \frac{-5 \times -1 \times -1}{-10 \times 6 \times 4} + 0.15 \times \frac{5 \times +5 \times +1}{4 \times +6 \times +2} + 0.1 \times \frac{5 \times +5 \times 1}{6 \times +4 \times 2}$$

(-ve, -ve, +ve)

$$= -\frac{0.25}{48} + -\frac{0.1}{24} + \frac{0.15 \times 25}{48} + \frac{0.1 \times 25}{48}$$

$$= 0.130208 - \{0.035416\} \; 0.009375$$

$$= 0.094791 \; 0.120833$$

HW - 2 [Regula Falsi Method]

① $x^3 + x - 1 = 0$ let $y = x^3 + x - 1$

x	0	1
y	-1	1

$x_1 = 0 \quad y_1 = -1$

$x_2 = 1 \quad y_2 = 1$

$x_N = 0.5 \quad x_2 = 1$

$y_N = -0.375 \quad y_2 = 1$

$x_N = 0.636 \quad x_2 = 1$

$y_N = -0.1059 \quad y_2 = 1$

$x_N = 0.6708 \quad x_2 = 1$

$y_N = -0.027 \quad y_2 = 1$

$x_N = 0.6794 \quad x_2 = 1$

$y_N = -0.007 \quad y_2 = 1$

$x_N = 0.68160 \quad x_2 =$

$y_N = 0.00170 \quad y_2 = 1$

= $x_N = 0.6821$ $x_2 = 1$

$y_N = -0.00055$ $y_2 = 1$

= $y_N = 0.6822$ $x_2 = 1$

= $y_N = -0.00038$ $y_2 = 1$

= $x_N = 0.6823$ $x_2 = 1$

= $y_N =$ ~~-0.000010~~ 0 $y_2 = 1$

= root = 0.6823

(2) $x^3 - 2x + 0.5 = 0$

x	0	1
y	0.5	-0.5

= $x_1 = 0$ $x_2 = 1$

$y_1 = 0.5$ $y_2 = -0.5$

= $x_N = 0.5$ $x_2 = 1$

$y_N = -0.375$ $y_2 = -0.5$

= $x_N = -1$ $x_2 = 1$

$y_N = 1.5$ $y_2 = -0.5$

= $x_N = 0.5$ $x_2 = 1$

$y_N = -0.375$ $y_2 = -0.5$

$[N =$ ~~0.000552~~

(8) $x^3+2x-20=0$ b/w 2 & 2.5

→

x	0	1	2	3
y	-20	-17	-8	13

$x_1 = 2$ $x_2 = 3$

$y_1 = -8$ $y_2 = 13$

$x_N = 2.3809$ $x_2 = 3$

$y_N = -1.7406$ $y_2 = 13$

$x_N = 2.454$ $x_2 = 3$

$y_N = -0.313$ $y_2 = 13$

$x_N = 2.46698$ $x_2 = 3$

$y_N = -0.052021$ $y_2 = 13$

$x_N = 2.4691$ $x_2 = 3$

$y_N = -0.00904$ $y_2 = 13$

$x_N = 2.46946$ $x_2 = 3$

$y_N = -0.00155$ $y_2 = 13$

$x_N = 2.4695$

$y_N = -0.00042$

root = 2.4695

④ $3x - \cos x - 1 = 0$ b/w 0 & 1

→

x	0	1
y	−2	1.000152

=

$x_1 = 0$ $x_2 = 1$

$y_1 = -2$ $y_2 = 1.000152$

=

$x_N = 0.66663289$

$y_N = -0.0000336$

∴ Root = 0.66663289

⑤ $x^3 - 3x - 5 = 0$ b/w 2 & 2.5

→

x	2	2.5	3
y	−3	3.125	13

=

$x_1 = 2$ $x_2 = 3$

$y_1 = -3$ $y_2 = 13$

=

$x_N = 2.1875$ $x_2 = 3$

$y_N = -1.09497$ $y_2 = 13$

=

$x_N = 2.2506$ $x_2 = 3$

$y_N = -0.352$ $y_2 = 13$

$x_N = 2.2703$ $x_2 = 3$

$y_N = -0.108$ $y_2 = 13$

$x_N = 2.2763$ $x_2 = 3$

$y_N = -0.0338$ $y_2 = 13$

$x_N = 2.278$ $x_2 = 3$

$y_N = -0.013$ $y_2 = 13$

$x_N = 2.28$ $x_2 = 3$

$y_N = 0.012352$ $y_2 = 13$

$\therefore$ root = 2.28

---

(6) $x \log_{10} x - 1.2 = 0$

x	1	2	3
y	-1.2	-0.59794	0.231363

$x_N = 2.72401$ $x_2 = 3$

$y_N = -0.017083$ $y = 0.231363$

$x_N = 2.740206$

$y_N = -0.0003$

Root = 2.740206

(9) $x^3 - 4x + 1 = 0$

→

	①	②
x	1	2
y	−2	1

$\therefore x_N = \dfrac{1-(-4)}{1-(-2)} = \dfrac{5}{3} = 1.66667$ $\quad x_2 = 2$

$y_N = -1.037022$ $\quad y_2 = 1$

$x_N = 1.83636$ $\quad x_2 = 2$

$y_N = -0.1528$ $\quad y_2 = 1$

$x_N = 1.8530$ $\quad x_2 = 2$

$y_N = -0.0175$ $\quad y_2 = 1$

$x_N = 1.86$ $\quad x_2 = 2$

$y_N = -0.00694$ $\quad y_2 = 1$

$x_N = 1.861$

$y_N = +0.00124$

$\therefore$ Root $= 1.861$

$=$

(8) $x^3 + 2x^2 - 8 = 0$

→

x	0	1	2
y	-8	-5	8

$x_1 = 1 \quad x_2 = 2$

$y_1 = -5 \quad y_2 = 8$

$$x_N = \frac{8 + 10}{13} = \frac{18}{13} = 1.3841$$

$x_2 = 2$

$y_2 = 8$

$x_N = -1.5118$

$x_N = 1.482$

$y_N = -0.346$

$x_2 = 2$

$y_2 = 8$

$x_N = 1.5035$

$y_N = -0.08$

$x_2 = 2$

$y_2 = 8$

$x_N = 1.5084$

$y_N = -0.017$

$x_2 = 2$

$y_2 = 8$

$x_N = 1.5095$

$y_N = -0.0032$

$x_2 = 2$

$y_2 = 8$

$x_N = 1.51$

$y_N = -0.0031$

→ root

∴ Root = 1.51

(9) $x^3 - 9x + 1 = 0$

→

x	0	1
y	1	-7

$\therefore x_1 = 0 \quad | x_2 = 1$

$y_1 = 1 \quad | y_2 = -7$

$x_N = 0.125 \quad x_2 = 1$

$y_N = -0.123 \quad y_2 = -7$

$x_N = 0.10938$

$y_N = 0.017605$

root = 0.1093

(10) $x^3 - x - 4 = 0$

x	0	1	2
y	-4	-4	2

$x_1 = 1 \quad x_2 = 2$

$y_1 = -4 \quad y_2 = 2$

$x_N = 1.66667$ $x_2 = 2$

$y_N = -1.0370$ $y_2 = 2$

$x_N = 1.78053147$ $x_2 = 2$

$y_N = -0.135$ $y_2 = 2$

$x_N = 1.794$ $x_2 = 2$

$y_N = -0.02$ $y_2 = 2$

$x_N = 1.796$ $x_2 = 2$

$y_N = -0.0028$ $y_2 = 2$

$x_N = 1.7963$

$y_N = -0.00019$

$\therefore$ Root $= \underline{1.7963}$