Calculus

Assignment - 2.

Q.1]

$$I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2} \cdot dw$$

put $2/w = t$

$\therefore \frac{-2}{w^2} = \frac{dt}{dw}$

When $w = 1/50$ $t = 100$

$w = 2$ $t = 1$

$\therefore 1/w^2 \cdot dw = -1/2 \cdot dt$

$$\therefore I = \int_{\cancel{1/50}\,100}^{\cancel{2}\,1} e^t \cdot \frac{-dt}{2}$$

$$= \frac{1}{2} \int_{\cancel{1/50}\,1}^{\cancel{2}\,100} e^t \cdot dt$$

$$= \frac{1}{2} \left[e^t \right]_1^{100}$$

$$= \frac{1}{2} (e^{100} - e^1)$$

$$= \frac{1}{2} (2.68811714 - 2.718281828 4)$$

$$= 1.34405857.$$

Q.2]

$$I = \int \frac{2t^3+1}{(t^4+2t)^3} \cdot dt$$

put $t^4 + 2t = x$

$\therefore$ Differentiate w.r.t t

$\therefore 4t^3 + 2 = \frac{dx}{dt}$

$\therefore 2(2t^3+1) \cdot dt = dx$

$\therefore 2t^3+1 \cdot dt = dx/2$

Page No.
Date

$$\therefore I = \int \frac{1}{x^3} \cdot \frac{dx}{2}$$

$$= \frac{1}{2} \int x^{-3} \cdot dx$$

$$= \frac{1}{2} \frac{x^{-2}}{-2} + C$$

$$= \frac{-1}{4} \cdot \frac{1}{x^2} + C$$

$$= \frac{-1}{4x^2} + C.$$

Q.3.] $f'(x) = x^4 + 3x - 9$

Integrating on both sides,

$$\therefore \int f'(x) \cdot dx = \int x^4 + 3x - 9 \cdot dx$$

$$\therefore f(x) = 4x^3 + 3 + C.$$

Q.4.] $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$I = \int_{-2}^{3} f(x) \cdot dx$$

$$= \int_{-2}^{1} f(x) \cdot dx \cdot \int_{1}^{3} f(x) \cdot dx$$

$$= 6 \cdot \int_{1}^{3} 3x^2 \cdot dx$$

$$= 6 \left[6x \right]_1^3$$

$$= 6 \left[18 - 6 \right]$$

$$= 6 \times 12$$

$$= 72.$$

Q.5]

$$I = \int 3(8y-1) \cdot e^{4y^2-y} \, dy$$

put $4y^2 - y = t$

Differentiating on both sides w.r.t y

$$8y - 1 = \frac{dt}{dy}$$

$$\therefore 8y - 1 dy = dt$$

$$\therefore I = \int 3e^t \cdot dt$$

$$= 3e^t + c$$

$$= 3e^{4y^2-y} + c.$$

Q.6] $f''(x) = 15\sqrt{x} + 5x^3 + 6.$ $f(1) = -5/4$ $f(4) = 404$

Integrating on both sides,

$$\therefore \int f''(x) \cdot dx = \int 15 \cdot x^{1/2} + 5x^3 + 6 \cdot dx$$

$$f'(x) = \frac{15x^{3/2}}{3/2} + \cancel{15x} \frac{15x^4}{4} + 6x + C_1$$

$$= 10x^{3/2} + 15/4 x^4 + 6x + C_1$$

Integrating on both sides,

$$\therefore \int f'(x) \cdot dx = \int 10x^{3/2} + \frac{15}{4}x^4 + 6x + C_1 \cdot dx$$

$$\therefore f(x) = \cancel{10x^{3/7}} + \cancel{\frac{15}{4}x^4} + \cancel{6/2}$$

$$= \frac{10x^{5/2}}{5/2} + \frac{15}{4} \cdot \frac{x^5}{5} + \frac{6x^2}{2} + C_1x + C_2$$

$$\therefore f(x) = 4x^{5/2} + \frac{3}{4}x^5 + 3x^2 + C_1 x + C_2$$

$$f(1) = \frac{-5}{4} \quad - \text{given}$$

$$f(1) = 4(1)^{5/2} + \frac{3}{4}(1)^5 + 3(1)^2 + C_1(1) + C_2$$

$$\therefore \frac{-5}{4} = 4 + \frac{3}{4} + 3 + C_1 + C_2$$

$$\therefore \frac{-5}{4} = \frac{16+3+12}{4} + C_1 + C_2$$

$$\therefore \frac{-5}{4} = \frac{31}{4} + C_1 + C_2$$

$$\therefore \frac{-5}{4} \cancel{\times 4} - \frac{31}{4} = C_1 + C_2$$

$$\therefore C_1 + C_2 = \frac{-36}{4} \quad \text{--- (1)}$$

$$\text{i.e. } C_1 + C_2 = -9$$

$$f(4) = 404 \quad - \text{given.}$$

$$\therefore f(4) = 4 \times (4)^{5/2} + \frac{3}{4} \times (4)^5 + 3(4)^2 + C_1(4) + C_2$$

$$\therefore 404 = 128 + 768 + 48 + 4C_1 + C_2$$

$$404 = 944 + 4C_1 + C_2$$

$$\therefore 4C_1 + C_2 = -540 \quad \text{--- (2)}$$

Subtracting eq. (1) from eq. (2)

$$\therefore 4C_1 + \cancel{C_2} = -540$$

$$C_1 + \cancel{C_2} = -9$$

$$(-) \quad (-) \quad (+)$$

$$3C_1 = 531$$

$$\therefore C_1 = \frac{531}{3} = 177$$

Substituting $C_1 = 177$ in eq. (1)

$$177 + C_2 = -9$$

$$\therefore C_2 = -9 - 177$$

$$\therefore C_2 = -186.$$

$$\boxed{\therefore f(x) = 4x^{5/2} + \frac{3x^5}{4} + 3x^2 - 177x - 186}$$

Q.7.] $f(x+iy) + g(x-iy)$

Q.8]

$$\frac{dr}{d\theta} = \frac{r^2}{\theta} \qquad r(1) = 2$$

$$\therefore \frac{1}{r^2} \cdot dr = \frac{1}{\theta} \cdot d\theta$$

$$\therefore r^{-2} dr = \frac{1}{\theta} \cdot d\theta$$

Integrating on both sides.

$$\therefore \int r^{-2} \cdot dr = \int 1/\theta \cdot d\theta$$

$$\therefore \frac{r^{-1}}{-1} = \ln\theta + c$$

$$\frac{-1}{r} - \ln\theta = c$$

$r(1) = 2$

$\therefore$ 11

Date

Q.9]

$$\frac{dv}{dt} = 9.8 - 0.196v$$

$$0.196v \cdot \frac{dv}{dt} = 9.8$$

$$\therefore 0.196v \cdot dv = 9.8\, dt$$

Integrating on both sides,

$$\therefore \int 0.196v \cdot dv = \int 9.8 \cdot dt$$

$$\therefore 0.196v = 9.8t + c$$

$$0.196v - 9.8t = c,$$

Q.10]

Q.12] $f(x) = x^3 - 10x^2 + 6 \quad , \; x = 3$

$f(x) = x^3 - 10x^2 + 6 \qquad f(3) = -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6 \qquad f'''(3) = 6$

$\therefore x^3 - 10x^2 + 6 = f(a)\,(x-a) + f'(a)\dfrac{(x-a)}{1!} + f''(a)\dfrac{(x-a)}{2!} + f'''(a)\dfrac{(x-a)}{3!}$

$$\therefore x^3 - 10x^2 + 6 = -57(x-3) - 33(x-3) - \frac{\cancel{2}(x-3)}{\cancel{2}\times 1} + \frac{\overset{2}{\cancel{6}}(x-3)}{\cancel{3}\times 2\times 1}$$

$$= -57(x-3) - 33(x-3) - (x-3) + (x-3)$$

$$= -57(x-3) - 33(x-3)$$

Q.15] Taylor Series

$f(x) = e^{-6x}$. $x = -4$

$f(x) = e^{-6x}$

$f'(x) = -6e^{-6x}$

$f''(x) = 36e^{-6x}$

$f'''(x) = -216e^{-6x}$

$f''''(x) = 1296e^{-6x}$

$f(-4) = 26489122,129$

$f'(-4) = -158,934,732,779.$

$f''(-4) = 953,608,396,674$

$f'''(-4) = -5721650380046$

$f''''(-4) = 3432990228027 7$

$$e^{-6x} = f(a)(x-a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)}{2!} + f'''(a)\frac{(x-a)}{3!} + f''''(a)\frac{(x-a)}{4!}$$

$$= 2648 9122129(x+4) - 158934732779(x+4) + 953608396674\frac{(x-4)}{2\times 1}$$

$$+ -5721650380046\frac{(x+4)}{3\times 2\times 1} + 34329902280277\frac{(x+}{4\times 3\times}$$

$$= \cancel{26489122129(x+4) - 158934732779(x+4) + 953608396674(x+4)}$$

$$e^{-6x} = 26489122129(x+4) - 158934732779(x+4) + 476804198337(x+$$

$$- 286082519023\frac{(x+4)}{3} + 1144300760092\frac{(x+4)}{8}$$

Q.16] $f(x) = \ln(3+4x)$ $\quad x = 0$

$f(x) = \ln(3+4x)$

$f'(x) = \frac{1}{3+4x} \times 4 = \frac{4}{3+4x} = 4(3+4x)^{-1}$

$f''(x) = -4(3+4x)^{-2} \times 4 = -16(3+4x)^{-2}$

$f'''(x) = 32(3+4x)^{-3} \times 4 = 128(3+4x)^{-3}$

$f''''(x) = 32 \times -3(3+4x)^{-4} \times 4 = -384(3+4x)^{-4}$

$f(0) = 1.0986122887$

$f'(0) = 1.3333333333$

$f''(0) = -1.77777778$

$f'''(0) = 4.7407407407$

$f''''(0) = -4.74074074$

$$\ln(3+4x) = f(a)(x-a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)}{2!} + f'''(a)\frac{(x-a)}{3!}$$

$$+ f''''(a)\frac{(x-a)}{4!}$$

$$= 1.0986122887(x-0) + 1.33333333(x-0) - 1.77777778\frac{(x-0)}{2}$$

$$+ 4.740740 74\frac{(x-0)}{3\times 2} - 4.740740 74\frac{(x-0)}{4\times 3\times 2\times 1}$$

$$= 1.0986122887(x-0) + 1.33333333(x-0) - 0.8888889(x-0)$$

$$+ 0.790123456(x-0) - 0.1975308641(x-0)$$