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FM Assignment 1.

1. $d^{(4)} = 8\%$ $= 0.08$

i) Equivalent force of int. = ?

$\rightarrow$: $\frac{d^{(4)}}{4} = \frac{8}{4} = 2\%$

$$d = [1 - (1-2\%)^4]$$

$$\therefore d = 0.07763184 = 7.763184\%$$

$$\delta = \ln \frac{1}{1-d} = \ln \frac{1}{1-d} = 0.0808108292$$

$$\boxed{\delta = 8.08108\%}$$

ii) Eq. effective rate of interest : ?

$$i = \frac{d}{1-d} = \frac{0.07763184}{1-0.07763184}$$

$$i = 0.08416578$$

$$\boxed{i\% = 8.416578\%}$$

iii) Eq. nominal rate of disc. per annum convertible monthly : $d^{(12)} = ?$

$$d^{(12)} = 12\left[1-(1-(0.07763184)^{1/12}\right]$$

$$\boxed{d^{(12)} = 7.9125832\%}$$

2. force of int $= \delta(t) = 0.004t + 0.0002t^2$

i) $PV = c \cdot e^{-\int \delta t} = 1000 \cdot e^{-\int_0^{10} 0.004t + 0.0002t^2}$

$= 1000 \cdot e^{-0.26667}$

$\therefore PV = ₹\,765.9257853$

ii) $e^{-0.266667} = \left(\frac{i}{1+i}\right)^{10}$

$0.973685\,(1+i) = i$

$0.973685 + 0.973685i = i$

$\boxed{i = 2.631\%}$

Q3. (i) $d = iv$

Interest earned on an investment of 1 paid at the beginning of the period is 'd'. Interest earning on an investment at the end of the period is 'i'. Therefore, if we discount 'i' from the end of the period to the beginning of the periods ~~to the beginning of the period~~ with discounting factor v; we obtain 'd'.

(ii) a) $\frac{d^{(4)}}{4} = 2.25\%$

$d = \left[1 - \left(1 - \frac{d^{(P)}}{P}\right)^P\right]$

$d = \left[1 - \left(1 - \frac{2.25}{100}\right)^4\right]$

$d = 8.70078\%$

$\therefore d^{(2)} = P\left[1 - (1-d)^{1/P}\right]$

$= 2\left[1 - (1 - 0.0870078)^{1/2}\right]$

$\boxed{d^{(2)} = 8.89874\%}$

Convertible every 2 years,

b) $d^{(0.5)} = 5\%$

$$d = \left[1 - \left(1 - \frac{d^{(p)}}{p}\right)^{p}\right]$$

$= 0.0513167$

$= 5.13167\%$

$d^{(2)} = 2\left(1 - (1 - 0.0513167)^{1/2}\right)$

$d^{(2)} = 0.051992505$

$\boxed{d^{(2)} = 5.1992505\%}$

iii) $i = 5\%$ p.a.

$$1 - \frac{91}{365} \times d = \left(\frac{i}{(1+i)}\right)^{91/365}$$

$$1 - 0.2493 \times d = \left(\frac{0.05}{1+0.05}\right)^{91/365} = 0.0381998$$

$\boxed{d = 3.81998\%}$

Q4)

i) same as Q3 (i)

ii) $i = 0.08$

a) $d^{(12)} = ?$

$$\therefore d = \frac{i}{1+i} = \frac{0.08}{1+0.08} = 7.4074\%$$

$d^{(12)} = 12\left[1 - (1 - 0.074074)^{1/12}\right]$

$\boxed{d^{(12)} = 7.671469\%}$

b) $i^{(365)} = 365\left[(1+0.08)^{1/365} - 1\right]$

$= 365\,(0.000210874)$

$= 0.0769691554$

$= 7.69691554\%$

c) $\delta = ?$

$\delta = \ln(1+i) = \ln(1.08)$

$\delta = 0.07696104$

$\boxed{\delta = 7.696104\%}$

d) $i^{(0.5)} = ?$

$= 0.5\left[(1+0.08)^{1/0.5} - 1\right]$

$= 0.0832$

$= 8.32\%$

Q5. a) $d^{(4)} = 6\%$

$d = \left[1 - \left(1 - \frac{d^4}{4}\right)^4\right]$

$d = \left[1 - \left(1 - \frac{0.06}{4}\right)^4\right]$

$d = 0.0586634$

$\boxed{d = 5.86634\%}$

$\therefore i = \frac{0.0586634}{1 - 0.0586634} = \frac{0.0586634}{0.94413366}$

$i = 0.06231926 = 6.231926\%$

$\therefore i^{(2)}$ i.e. convertible half yearly

$= 2\left[(1+0.06231926)^{1/2} - 1\right]$

$i^{(2)} = 0.06137746 = 6.137746\%$

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b) The equivalent rate of disc. per annum conv. monthly $= d^{(12)} = 12\left[1-(1-0.0586634)^{1/12}\right]$

$d^{(12)} = 6.030247\%$

ii) $n = 93$ days

remaining term $= 272$ days

MI

$\therefore$ Rebate amount allowed $= \frac{272}{365} \times 2,00,000$

$= ₹14904.1096$

$\therefore$ Sum paid to settle the loan $= 2,20,000 - 14904.1096 = ₹205095.891$

MII

$2,20,000 = 2,00,000\,(1+i)$

$\frac{22}{20} = (1+i)$

$1.1 = 1+i$

$i = 0.1 = 10\%$

$\therefore AV = 2,00,000\,(1+10\%)^{93/365}$

$AV = ₹2,04,916.3564$

Q7.

i) $26,000 = 20,000(1+i)^{17/12}$

$(1.3)^{12/17} = (1+i) = 0.2034570672$

$i = 20.3457\%$

$\therefore i^{(4)} = 4\left[(1+0.203457)^{1/4}-1\right]$

$i^{(4)} = 4\,(4.7388125)$

$i^{(4)} = 0.1895525$

$= 18.95525\%$

ii) $d = \frac{i}{1+i} = \frac{0.203457}{1.203457}$

$= 0.1690604$

$= \underline{16.90604\%}$

$d^{(2)} = 2[1(1-0.1690604)^{1/2}]$

$d^{(2)} = 0.1768828$

$d^{(2)} = 17.68828\%$

iii) Simple ROI = ?

$26,000 = 20,000\left(1+\frac{17i}{12}\right)$

$1.3 = 1 + \frac{17i}{12}$

$i = 0.2117641$

$\boxed{i = 21.17641\%}$

iv) The simple interest is comparitively more than the interest recieved in compound interest.

Q8. Let the retail price of the toy be ₹100

a) Cash payment = 30% below the recommended r.p

Cash payment = 30% of 100 = 30

→ 100 − 30 = ₹70

b) Six month credit = 25% below the recommended

Payment after 6 months = 100 − 25% of 100

= ₹75

Therefore, the effective rate of discounts:

$$70 = 75(1-d)^{0.5} \rightarrow \frac{70}{75} = (1-d)^{0.5}$$

sq. on b.s.

$$\left(\frac{70}{75}\right)^2 = (1-d) \quad \therefore d = 0.1283889 = \boxed{12.8889\%}$$

ii) $d^{(12)} = ?$ $\quad (1-0.1288839)^{1/2} = 1 - \frac{d^{(12)}}{12}$ $\quad \boxed{\therefore d^{(12)} = 13.7195}$

$i^{(2)} = ?$ $\quad 1 + \frac{i^{(2)}}{2} = \left(\frac{1}{1-0.128889}\right)^{1/2} \therefore \boxed{i^{(2)} = 14.2857142\%}$

$\therefore \delta = \ln(1+i)$, $\delta = \ln x \left(\frac{1}{1-d}\right)$, $\boxed{\delta = 13.79857437\%}$

Q9. i) Measure of int. at individual moments of time is known as force of interest.

ii) $i^{(2)} = 0.0775$

$$i = \left[1 + \frac{i^{(p)}}{p}\right]^p - 1 = \left(1 + \frac{0.0775}{2}\right)^2 - 1$$

$$i = 0.0790016 = 7.90016\%$$

$$\delta = \cancel{\log}\ \ln(1+i) = \ln(1.0790016)$$

$$\delta = 0.0760361 = 7.60361\%$$

$$d = \frac{i}{1+i} = \frac{0.0760361}{1+0.0760361}$$

$$= 0.0732173$$

$$= 7.32173\%$$

$$d^{(12)} = 12\left[1 - (1-0.0732173)\right)$$

$$d^{(12)} = 7.57958\%$$

$$i^{(0.5)} = 0.5\left[(1+0.0790016)^{1/0.5}\right) - 1\right]$$

$$i^{(0.5)} = 8.21222\%$$

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Q10. $\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 \\ 0.06 & \text{for } 5 \leq t \leq 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$

$v(0,5) = e^{-\int_0^5 \delta\, t dt} = e^{-\int_0^5 0.08} = e^{-0.4}$

$v(5,10) = e^{-\int_5^{10} \delta dt} = e^{-\int_5^{10} 0.06} = e^{-0.3}$

$v(10,t) = e^{-\int_{10}^{t} \delta \cdot t dt} = e^{-\int_{10}^{t} 0.04} = e^{-(0.04t - 0.4)}$

$\therefore v(0,t) = e^{-0.4} \cdot e^{-0.3} \cdot e^{0.4 - 0.04t}$
$= e^{-(0.3 + 0.04t)}$

Eq. $\therefore \boxed{v(t) = e^{-(0.3+0.04t)}}$

Q11.) a) $PV = 50{,}000 \left[\frac{1}{1-0.18}\right]^{9/12}$

$PV = ₹43{,}085.426$

b) i) $\delta = 6\%$

$\therefore d = 1 - e^{-\delta} = 1 - e^{-0.06}$
$\therefore d = 5.82354\%$

$d^{(12)} = 12\left[1 - (1 - 0.0582354)^{1/12}\right]$
$d^{(12)} = [0.05985018]$
$d^{(12)} = 5.985018\%$

ii) $d^{(4)} = 6\%$

$d = \left\{1 - \left[1 - \frac{d^{(4)}}{4}\right]^{p}\right\}$

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$d = [1 - (1 - 0.015)^4]$

$d = 0.05866345$

$d = 5.8866345\%$

$i = \dfrac{d}{1-d} = \dfrac{0.05866345}{(1 - 0.05866345)}$

$i^{(4)} = 4[(1 + 0.062319)^{1/4} - 1]$

$i^{(4)} = 6.09137\%$

$i^{(12)} = 12[1 + 0.062319]^{1/12} - 1]$

$i^{(12)} = 6.060679\%$

Q12.

7,049 — $d^{(12)} = 6\%$

0 — $i^{(12)} = 6\%$ — 2y — $i = 1.5\%$ — 4y 6m — 5y

$C = ₹7{,}049$

AV after 2yrs ; $i^{(12)} = 6\%$

$\rightarrow \left[1 + \dfrac{0.06}{12}\right]^{12} = 1 + i$

$i = 0.061677812$

$AV_2 = 7049\,(1 + 0.061677812)$

$= ₹7945.349262$

$i = 1.5\%$ for 10 quarters

Acc. Fact $= (1 + ni)$

$= 1.15$

Acc. val. after $4\frac{1}{2}$ years $= 7945.3493$

$= ₹9137.151651$

$d^{(12)} = 6\%$ for $1\frac{1}{2}$ yrs.

$\rightarrow$

$$d^{(12)} = \left[1 - \frac{d^{(12)}}{12}\right]^{12} = (1-d)$$

$d = 0.058377193$

$$AF = \frac{1}{(1-d)^n} = \underline{1.094421325}$$

Acc. val. after 6 years =

$= 9137.157651 \times (1.094421)$

$= ₹\ \underline{9999.893617}$

Q13. i) Let the C be x

$\therefore AV = 2x$

$AV = C \cdot (1+0.1)^t$

$2x = x\,(1.1)^t$

$2 = (1.1)^t \rightarrow$ taking ln on b.s.

$\ln(2) = t \cdot \ln(1.1)$

$$t = \frac{\ln(2)}{\ln(1.1)}$$

$\therefore t = 7.2725$ yrs.

ii) nominal $d = 10\% = 0.1 \rightarrow \frac{d^{(12)}}{12}$

$$\left(1 - \frac{0.1}{12}\right)^{-12t} = 2$$

taking ln on b.s.

$\ln 2 = -12t \ln 0.991667$

$$\frac{\ln(2)}{\ln(0.991667)} = -12t$$

$-12t = -82.83060471$

$\boxed{t = 6.902550392 \text{ years}}$