Q1]

$$\lim_{h \to 0} \frac{2(9 - 6h + h^2) - 18}{h}$$

$$\lim_{h \to 0} \cancel{18 - 6h} \quad \frac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$$

$$\lim_{h \to 0} \cancel{2-h(} \quad \frac{\cancel{h}(2h - 12)}{\cancel{h}}$$

$$\therefore = 2(0) - 12 = -12$$

Q2]

a)

$$f(4) = 2 \times (4) = 8$$

$$\cancel{f} \lim_{x \to 4^-} f(x) = 2 \times 4 = 8$$

$$\lim_{x \to 4^+} f(x) = 2 \times (4) = 8$$

$$\therefore \text{As } f(4) = \lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = 8$$

x is continuous at 4

b) $f(6) = 6 - 1 = 5$

$$\lim_{x \to 6^-} f(x) \cancel{2x} = 2 \times 6 = 12$$

$$\lim_{x \to 6^+} f(x) = 6 - 1 = 5$$

$\therefore$ ~~f(6) =~~ As $\lim_{x \to 6^-} f(x) \neq \lim_{x \to 6^+} f(x)$

Hence

$f(x)$ is discontinous at $x = 6$

Q3] $$\lim_{x \to 9} \frac{x-9}{-(\sqrt{x}-3)} \times \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

$$\cancel{lx} \lim_{x \to 9} = \frac{-(x-9)(\sqrt{x}+3)}{(\sqrt{x})^2-(3)^2}$$

$$\lim_{x \to 9} \frac{-\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}}$$

$$\cancel{l} = -(\sqrt{9}+3)$$

$$= -(3+3) = -6$$

Q4] a) $$f(-1) = \frac{4(-1)+5}{9-3(-1)}$$

$$= \frac{1}{9+3} = 1/12$$

As $f(x)$ excsists and is same as $\lim_{x \to -1} f(x)$ it is continous at ~~$x=$~~ $x = -1$

b) $f(0) = \dfrac{4(0)+5}{9-3(0)}$

$= 5/9$

$\therefore$ $f(x)$ exsists and is same as $\lim\limits_{x\to 0} f(x)$ it is continous at $x = 0$

c) ~~f(x)~~ $f(3) = \dfrac{4(3)+5}{9-3(3)}$

$= \dfrac{17}{0} \rightarrow$ undefinied

as $f(x)$ doesn't exsist it is discontinous at $x = 3$

Q5] $\lim\limits_{x\to\infty} \dfrac{\cancel{e^{4x}}(6 - e^{-6x})}{\cancel{e^{4x}}(8 - e^{-2x} + 3e^{-5x})}$

$\cancel{\lim\limits_{x\to\infty}} \dfrac{(6 - 1/e^{6(\infty)})}{(8 - 1/e^{2(\infty)} + 3/e^{5(\infty)})}$

$\dfrac{(6-0)}{(8-0+0)} = 6/8 = 3/4$

Q6] a) $\lim\limits_{y\to 6} g(y)$

$\lim\limits_{y\to 6} 1-3y$

$= 1-3(6)$

$= -17$

b) $\lim\limits_{x\to -2^-} y^2+5$

$\cancel{\lim\limits_{x\to -2^-}} (-2)^2+5 = 9$

$\lim\limits_{x\to -2^+} 1-3y$

$1-3(-2) = 7$

as $\lim\limits_{x\to -2^-} g(y) \neq \lim\limits_{x\to -2^+} g(y)$. limit doesn't exsist.

Q7] $\dfrac{d\,f(t)}{dt} = (4t^2 - t).(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t)$

$= 12t^4 - 3t^3 - 64t^3 + 16t^2 + 8t^4 - 64t^3 + 96t$

$= 20t^4 - 131t^3 + 16t^2 + 96t$

Q8] $\dfrac{dR(w)}{dw} = \dfrac{(2w^2+1).(3+4w^3) - (3w + w^4).(4w)}{(2w^2+1)^2}$

$= \dfrac{6w^2 + 3 + 8w^5 + 4w^3 - 12w^2 + 4w^5}{(2w^2+1)^2}$

$= \dfrac{12w^5 + 4w^3 - 6w^2 + 3}{(2w^2+1)^2}$

Q9] $f(x) = 2e^x - 8^x$

$\dfrac{d f(x)}{dx} = 2.e^x - 8^x \log 8$

Q10] $\dfrac{dy}{dz} = 5z^4 - \left[e^3 . \dfrac{1}{z} + \ln(z) . e^3\right]$

$= 5z^4 - e^3/z + \ln(z)e^3$

Q11] a) $\frac{d f(x)}{dx} = 4 \cdot (6x^2 + 7x)^3 \cdot (12x + 7)$

$= 4(12x+7)(6x^2+7x)^3$

b) $\frac{d f(t)}{dt} = 0 + e^{4t+t^7} \cdot (4 + 7t^6)$

$= (4+7t^6)e^{(4t+t^7)}$

Q12] a) $\cancel{d f6 \; 2\ln(}$ $\frac{dz}{dx} = \frac{1}{(7-x^3)} \cdot (0 - 3x^2)$

$= \frac{-3x^2}{(7-x^3)}$

$\cancel{d f''(x) = (7-x^3(}$

$f''(x) = \frac{(7-x^3)(-6x) - (-3x^2)(0-3x^2)}{(7-x^3)^2}$

$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$

$= \frac{-3x^4 - 42x}{(7-x^3)^2}$

$= \frac{-3x(x^3+14)}{(7-x^3)^2}$

b) $\frac{d\theta(v)}{dv} = 2 \cdot -4 \, (6+2v-v^2)^{-5} \cdot (2-2v)$

$$= \frac{(2)\cdot(-4)(2-2v)}{(6+2v-v^2)^5}$$

$$= \frac{(-8)(2-2v)}{(6+2v-v^2)^5}$$

$$= \frac{(-16+16v)}{(6+2v-v^2)^5}$$

$$\theta''(v) = \frac{(6+2v-v^2)^5(16) - (16v-16)\,\cancel{2}\,(5)\,(6+2v-v^2)^4\,(2-2v)}{(6+2v-v^2)^{5\times 2}}$$

$$= \frac{(6+2v-v^2)^5(16)}{(6+2v-v^2)^{10}} - \frac{(16v-16)(10-10v)}{(6+2v-v^2)^{6}}$$

$$= \frac{16}{(6+2v-v^2)^5} - \frac{\cancel{(16v-16)}\ \cancel{16(v-1)}\ (16v-16)(10-10v)}{(6+2v-v^2)^{6}}$$

$$\boxed{= 16(6+2v-v^2)^{-5} - (16v-16)(10-10v)(6+2v-v^2)^{-6}}$$

c) $\frac{d\,f(t)}{dt} = \frac{1}{(1+t^2)} \cdot (2t)$

$$= \frac{2t}{(1+t^2)}$$

$$\cancel{= \frac{t}{(1+t^2)} + \frac{t}{(1+t^2)}}$$

$$f''(t) = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}$$

$$\cancel{= \frac{2t+2}{t}} = \frac{2+2t^2-4t^2}{1+2t^2+t^4}$$

$$\cancel{f''(t) = 2 + \frac{2(-1)}{t^2}}$$

$$\cancel{= 2 - \frac{2}{t^2}} \qquad \cancel{= \frac{2t^2-2}{t^2}} \qquad \boxed{= \frac{2(1-t^2)}{(1+t^2)^2}}$$

Q13] $f(x) = x^3 - 3x^2 - 9x + 12$

Diff w.r.t. x

$f'(x) = 3x^2 - 6x - 9$

Eq. to 0

$3x^2 - 6x - 9 = 0$

$(x^2 - 2x - 3) = 0$

$x^2 + x - 3x - 3 = 0$

$x(x+1) - 3(x+1) = 0$

$x = 3$ or -1

$f''(x) = 6x - 6$

$f''(3) = 6(3) - 6$

$= 18 - 6$

$= 12 \leftarrow$ minima

$f''(-1) = 6(-1) - 6$

$= -12 \leftarrow$ maxima

$\therefore f(x)$ is maximum at $x = -1$

is minimum at $x = 3$

Q14] $f'(x) = 12x^2 - 36x + 24$

Eq. to 0

$12(x^2 - 3x + 2) = 0$

$x^2 - x - 2x + 2 = 0$

$x(x-1) - 2(x-1) = 0$

$x = 2$ or 1

$f''(x) = 24x - 36$

$f''(1) = 24(1) - 36$

$= -12 \leftarrow$ maxima

$f''(2) = 24(2) - 36$

$= 12 \leftarrow$ minima

$\therefore f(x)$ is maximum at $x = 1$

is minimum at $x = 2$

Q15] $f'(x) = x^2(1) + (x+3)(2x)$

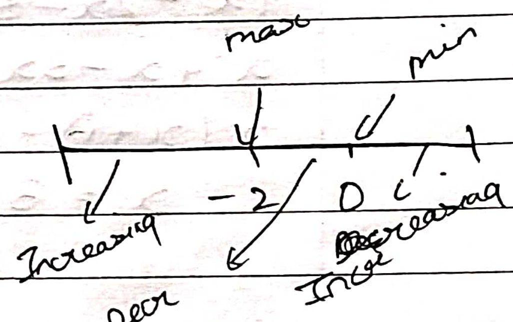
$= x^2 + 2x^2 + 6x$

$= 3x^2 + 6x$

Eq to zero

$3x(x+2) = 0$

$\therefore x = -2$ or 0



$f''(x) = 6x + 6$

$f''(-2) = 6(-2) + 6$

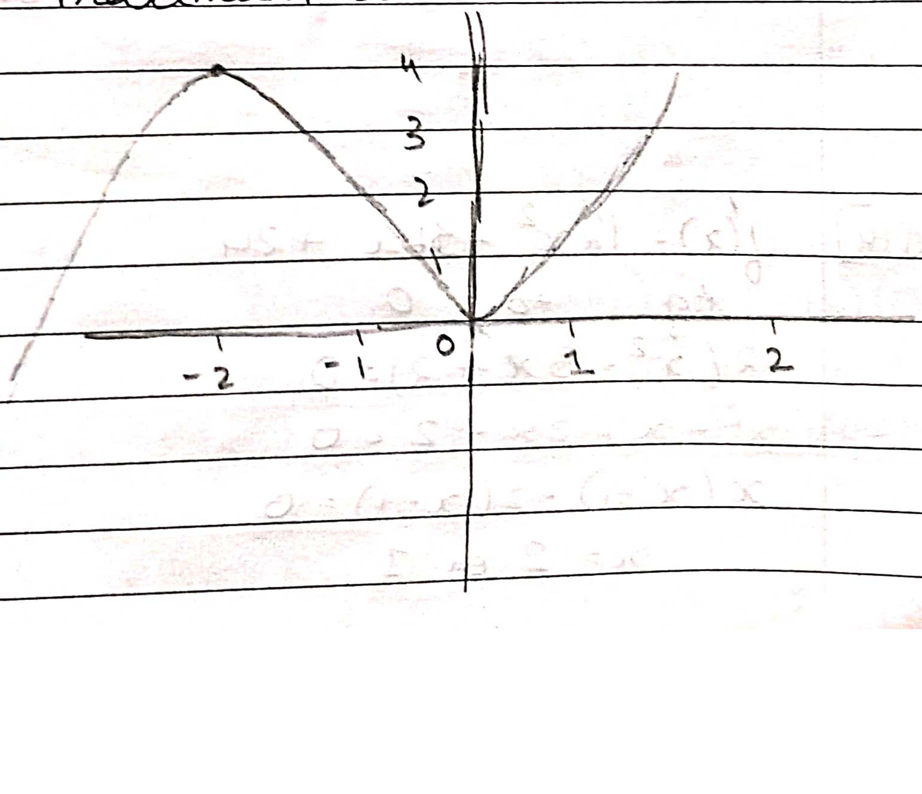
$= -6 \leftarrow$ maxima

minimum at $x = 0$

$f(x)$ is maximum at $x = -2$

$\therefore f(-2) = 4$

$f(0) = 0$



Q16

$f'(x) = 6x - 6$

Eq to 0

$6(x-1) = 0$

$x = \underline{1}$

$\therefore f''(x) = 6$

$f''(1) = 6 \leftarrow$ minima

$\therefore f(1) = -3$

$f(0) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 2$ or 0