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NMA Assignment

Q1) Measured = 12.65 cm

Actual = 12.5 cm

(i) Absolute Error :- Measured − Actual

= 12.65 − 12.50

= <u>0.15 cm</u>

(ii) Relative Error :- $\frac{|\text{measured} - \text{real}|}{\text{real}}$

$= \frac{0.15}{12.5} = \underline{0.012 \text{ cm}}$

(iii) Percentage error :- Relative Error × 100 %

= <u>1.2 %</u>

Q2)

x	$f(x) = \log x$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

(a) log(5) using the points $x = 4$ and $x = 6$.

→

	x	$\log x$	
x_1	4	0.60206	y_1
x_2	5	$\bar{x}$	y_2
x_3	6	0.7781513	y_2

~~Using Regula falsi method~~

~~$\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1} = \frac{4(0.7781513) - 6(0.60206)}{0.7781513 - 0.60206}$~~

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$

$\frac{4-5}{6-5} = \frac{0.60206 - \bar{x}}{0.7781513 - \bar{x}}$

$-1 = \frac{0.60206 - \bar{x}}{0.778153 - \bar{x}}$

$$\bar{x} - 0.778153 = 0.60206 - \bar{x}$$
$$2\bar{x} = 1.380213$$
$$\therefore \underline{\bar{x} = 0.6901065}$$

(b) log(5) using points $x = 4.5$ and $x = 5.5$.

→

	x	$\log x$	
x_1	4.5	0.6532125	y_1
x_2	5.0	$\bar{x}$	y_2
x_3	5.5	0.7403627	y_3

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\therefore \frac{4.5 - 5}{5.5 - 5} = \frac{0.6532125 - \bar{x}}{0.7403627 - \bar{x}}$$

$$\frac{\cancel{-0.5}}{\cancel{0.5}} = \frac{0.6532 - \bar{x}}{0.7403 - \bar{x}}$$

$$-0.7403627 + \bar{x} = 0.6532125 - \bar{x}$$
$$2\bar{x} = 1.3935752$$
$$\underline{\bar{x} = 0.6967876}$$

Q3) root of $x^4 - x - 10 = 0$

$a = 1.5$; $b = 2$.

→

	x	y	
x_1	1.5	-6.4375	y_1
x_2	2.0	4	y_2

$$x_3 = \frac{x_1 + x_2}{2} = 1.75 \quad \therefore y_3 = -2.37109375$$

$x_3 = 1.75$ $\quad y_3 = -2.37109375$

$x_4 = 2.0$ $\quad y_4 = 4$

$x_5 = 1.875$; $y_5 = 0.484619141$

$x_5 = 1.75$ $\quad y_5 = -2.37109375$

$x_6 = 1.875$ $\quad y_6 = 0.484619141$

$x_7 = 1.8125$; $y_7 = -1.020248413$

$x_7 = 1.8125$ $\quad y_7 = -1.020248413$

$x_8 = 1.8750$ $\quad y_8 = 0.484619141$

$x_9 = 1.84375$ $\quad y_9 = -0.287734032$

$x_9 = 1.84375$ $\quad y_9 = -0.287734032$

$x_{10} = 1.87500$ $\quad y_{10} = 0.484619141$

$x_{11} = 1.859375$; $y_{11} = 0.093378127$

$\therefore$ After 5 iterations, using Bisection Method,

root of $x^4 - x - 10 = 0$ is $\approx \underline{1.859375}$

since $y_{11} = 0.093378127 \approx 0$

Q5) $A^2 = A$

$7A - (I+A)^3 \rightarrow$ Find value.

$\longrightarrow$ we have $7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2I + 3AI^2)$

$= 7A - (I + A \cdot A^2 + 3A^2 + 3A)$

$\cdots (I = I)$

$= 7A - (I + A \cdot A + 3A + 3A)$

$\cdots (A^2 = A)$

$= 7A - (I + A^2 + 6A)$

$= 7A - (I + A + 6A)$

$= 7A - (I + 7A)$

$= \cancel{7A} - I - \cancel{7A}$

$= \underline{-I}$

$\therefore$ $\underline{7A - (I+A)^3 = -I}$

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Q4) $f(x) = x^3 - x - 1$ using Newton Raphson method.

→ Let $f(x) = x^3 - x - 1$

∴ Diffr w.r.t x

$f'(x) = 3x^2 - 1$

x	$y\ [f(x)]$
0	-1
1	-1
2	5
3	23

∴ $x_0 = 1$; $f(x_0) = -1$

∴ $f'(x_0) = 2$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(-1)}{2} = 1 + \frac{1}{2}$$

$$= 1.5$$

∴ $x_1 = 1.5$; $f(x_1) = 0.875$

; $f'(x_1) = 5.75$

$$\therefore x_2 = 1.5 - \frac{0.875}{5.75} = 1.3478$$

∴ $x_2 = 1.3478$; $f(x_2) = 0.100566$

$f'(x_2) = 4.449694$

$$\therefore x_3 = 1.3478 - \frac{0.100566}{4.449694}$$

$$= 1.325199$$

∴ $x_3 = 1.325199$; $f(x_3) = 0.0020523391$

$f'(x_3) = 4.268457$

$$\therefore x_4 = 1.325199 - \frac{f(x_3)}{f'(x_3)}$$

∴ $x_4 = 1.324718$

Since $x_3 \approx x_4$ we can say $\underline{\sqrt{x^3 - x - 1} \approx 1.324718}$

Q6] $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}^{-1} = ?$ Let given matrix be A.

$\rightarrow$ $\therefore |A| = 1(0-6) + 1(-4-0) + 2(4-0)$

$= -6 - 4 + 8$

$= \underline{-2}$

& $M_{11} = -6$	$M_{21} = -1$	$M_{31} = -6$
$M_{12} = -4$	$M_{22} = -1$	$M_{32} = -2$
$M_{13} = 4$	$M_{23} = 1$	$M_{33} = 4$

$$\therefore A^{-1} = \frac{1}{|A|} \begin{bmatrix} +M_{11} & -M_{12} & +M_{13} \\ -M_{21} & +M_{22} & -M_{23} \\ +M_{31} & -M_{32} & +M_{33} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{-1}{2} \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$$

Q7] a) $A = 8i - 9j$ and $B = 7i - 9j$

$\rightarrow$ Angle betn 2 vectors :- $\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right]$

$$\therefore \theta = \cos^{-1}\left[\frac{(8\hat{i} - 9\hat{j})\cdot(7\hat{i} - 9\hat{j})}{|(8\hat{i} - 9\hat{j})||(7\hat{i} - 9\hat{j})|}\right]$$

$$= \cos^{-1}\left[\frac{(8\times 7) + (-9\times -9)}{\sqrt{8^2 + (-9)^2}\cdot\sqrt{7^2 + (-9)^2}}\right]$$

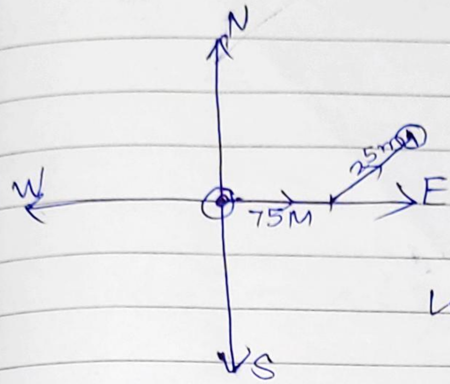
$$= \cos^{-1}\left[\frac{56 + 81}{\sqrt{145}\cdot\sqrt{130}}\right]$$

$$= \cos^{-1}\left[\frac{137}{5\sqrt{754}}\right] = \cos^{-1}(0.997849146)$$

$$= 3.758555685$$

$\therefore \theta = 3.76^\circ$

Q8)



Let $75M = A$

$25M = B$ & $\theta = 30^\circ$

∴ Using resultant vector magnitude formula,

Magnitude of sabrina's displacement

$$\text{vector} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{75^2 + 25^2 + 2(75)(25)\cos 30^\circ}$$

$$= \sqrt{9497.595264}$$

$$= 97.4556 \text{ metres}$$

∴ Magnitude of sabrina's displacement vector is 97.46 metres.

Q9) First 3 digit no. that leaves 2, when divided by 3 is 101 and last is 998 and there are 300 such nos.

$$\therefore S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{300}{2}[101 + 998]$$

$$= \frac{300 \times 1099}{2}$$

$$= 164850$$

∴ Sum of all nos. is 164,850.

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Q10] $t_6 = 32$; $t_8 = 128$.

$r = ?$

$\therefore 32 = a \cdot r^5$

$128 = a \cdot r^7$

$$\therefore \frac{t_8}{t_6} = \frac{\cancel{a}r^7}{\cancel{a}r^5} = \frac{128}{32}$$

$r^2 = 4$

$\therefore r = 2$

$\therefore$ The common ratio of the G.P. is 2.

Q11] $(4+3x)^5$

$\rightarrow$ $(a+b)^n = a^n + {}^nC_1 a^{n-1} \cdot b + {}^nC_2 a^{n-2} \cdot b^2 + \dots + b^n$

$\therefore (4+3x)^5 = 4^5 + {}^5C_1(4)^4(3x) + {}^5C_2(4)^3(3x)^2 + {}^5C_3(4)^2(3x)^3 + {}^5C_4(4)^1(3x)^4 + {}^5C_5(3x)^5$

$= 1024 + (768 \times 5)x + (10 \times 576)x^2 + (10 \times 16 \times 27)x^3 + (5 \times 4 \times 81)x^4 + (1 \times 243)x^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

$\therefore (4+3x)^5 = 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$.

Q12] $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\rightarrow$ $|A - \lambda I| = 0$

$$\therefore \left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\therefore \left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$\therefore 2-\lambda[(-5-\lambda)(3-\lambda)] + 3[6-2\lambda] + 0 = 0$

$\therefore 2-\lambda[-15+5\lambda-3\lambda+\lambda^2] + 18 - 6\lambda + 0 = 0$

$\therefore -30+10\lambda-6\lambda+2\lambda^2+15\lambda-5\lambda^2+3\lambda^2-\lambda^3+18-6\lambda = 0$

$-\lambda^3+19\lambda-6\lambda-12=0$

$\therefore -\lambda^3+13\lambda-12=0$

i.e

$-(2-\lambda)(5+\lambda)(3-\lambda)+6(3-\lambda)=0$

$(3-\lambda)[(\lambda-2)(\lambda+5)+6]=0$

$(3-\lambda)(\lambda^2+3\lambda-4)=0$

$(3-\lambda)(\lambda-1)(\lambda+4)=0$

$\therefore \lambda=3$; $\lambda=1$; $\lambda=-4$

are the eigen values.

Q13] $f(x) = x^3-7x^2+8x-3$ if $x_0=5$; $x_2=?$

$\rightarrow$ $\therefore f(x) = x^3-7x^2+8x-3$

$\therefore f'(x) = 3x^2-14x+8$

	x	$f(x)$
x_0	5	-13
	6	9
	7	53

$\therefore x_0 = 5$; $f(x_0) = -13$

$f'(x_0) = 13$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{(-\cancel{13})}{\cancel{13}} = \underline{\underline{6}}$

$\therefore x_1 = 6 \quad ; f(x_1) = 9$

$f'(x_1) = 32$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 6 - \frac{9}{32}$$

$$= 6 - 0.28125$$

$$\therefore x_2 = \underline{5.71875}$$

Q14) $(1-x+x^2)^4$

$\longrightarrow$ $(1-x+x^2)^4 = (\underbrace{1}_{a} + \underbrace{(x^2-x)}_{b})^4$

$\therefore$ Using Binomial theorem,

$$(1+x-x^2)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$= 1 + 4(x^2-x) + 6(x^2-x)^2 + 4(x^2-x)^3 + (x^2-x)^4$$

$$= 1 + 4x^2 - 4x + 6(x^4 - 2x^3 + x^2) + 4(x^6 - 3x + 3x^2 - x^3) + (x^8 - 4x^7 + 6x^6 - 4x^5 + x^4)$$

$$= 1 + 4x^2 - 4x + 6x^4 - 12x^3 + 6x^2 + 4x^6 - 12x + 12x^2 - 4x^3 + x^8 - 4x^7 + 6x^6 - 4x^5 + x^4$$

$$= 1 - 16x + 22x^2 - 16x^3 + 7x^4 - 4x^5 + 10x^6 - 4x^7 + x^8$$

$$\therefore (1-x+x^2)^4 = 1 - 16x + 22x^2 - 16x^3 + 7x^4 - 4x^5 + 10x^6 - 4x^7 + x^8.$$

Q15] $e^{-x} = 3\log(x)$

$\longrightarrow$ $\therefore e^{-x} - 3\log x = 0 \Rightarrow y$

x	y
0	—
1	0.367879441
2	-0.767754704

$\therefore$ By Bisection Method,

$x_1 = 1 \quad | \quad x_2 = 2$

$y_1 = 0.367879 \quad | \quad y_2 = -0.767754$

$\therefore x_3 = \frac{x_1 + x_2}{2} = 1.5 \; ; \; y_3 = -0.305143$

$x_3 = 1.5 \quad | \quad x_4 = 1$

$y_3 = -0.305143 \quad | \quad y_4 = 0.367879$

$\therefore x_5 = 1.25 \; ; \; y_5 = -0.004225$

$x_6 = 1 \quad | \quad x_5 = 1.25$

$y_6 = 0.367879 \quad | \quad y_5 = -0.004225$

$\therefore x_7 = 1.125 \; ; \; y_7 = 0.171194$

$x_7 = 1.125 \quad | \quad x_8 = 1.25$

$y_7 = 0.171194 \quad | \quad y_8 = -0.004225$

$\therefore x_9 = 1.1875 \; ; \; y_9 = 0.081081$

$x_9 = 1.1875 \quad | \quad x_{10} = 1.25$

$y_9 = 0.081081 \quad | \quad y_{10} = -0.004225$

$\therefore x_{11} = 1.21875 \; ; \; y_{11} = 0.037855$

$\therefore$ After 5 iterations, nearest value for $f(x) = e^{-x} - 3\log x$ is $\underline{\underline{1.25}}$.

—— x —— 0 —— x ——