CALCULUS

ASSIGNMENT - 2

1. $$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

$$= \left[\frac{w e^{2/w}}{2\cancel{w}} \cdot -\frac{1}{w}\right]_{1/50}^{2}$$

$$= \left[\frac{(2)e^{1}}{2} \times -\frac{1}{2}\right] - \left[\frac{(1/50)\, e^{100}}{2}\left(-\cancel{50}\right)\right]$$

$$= -\frac{e}{2} + \frac{e^{100}}{2}$$

$$= \frac{-e+e^{100}}{2}$$

2. $$I = \int \frac{2t^3+1}{(t^4+2t)^3}\, dt$$

Let $t^4+2t = u$

$(4t^3+2)\,dt = du$

$2(2t^3+2)\,dt = du$

$$I = \int \frac{2\,du}{t^3}$$

$$I = -\frac{2}{2t^2} = -\frac{1}{t^2} + c$$

$$I = \frac{-1}{(t^4+2t)^2} + c$$

3. $f'(x) = x^4 + 3x - 9$

$$f(x) = \int f'(x)\,dx$$

$$f(x) = \int (x^4 + 3x - 9)\,dx$$

$$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

4. $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$ find : $\int_{-2}^{3} f(x)\,dx$

$$I = \int_{-2}^{1} f(x) + \int_{1}^{3} f(x)$$

$$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= (1 + 8) + [18 - 6]$$

$$= 9 + 12$$

$$= 21$$

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$$\frac{15(x)^{3/2}}{3/2+1}$$

$$5/2$$

$$\frac{20}{5}(x)^{5/2}$$

5. $I = \int 3(8y-1)\, e^{4y^2-y}\, dy$

Sol: let $4y^2 - y = t$

$(8y-1)\, dy = dt$

$I = 3\int e^t\, dt$

$I = 3e^t + c$

$= \underline{\underline{3e^{4y^2-y} + c}}$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f(1) = -5/4$ find : $f(x)$

$f(4) = 404$

$f(x) = \int \int (15\sqrt{x} + 5x^3 + 6)\, dx\, dx$

$= \int \left(10(x)^{3/2} + \frac{5}{4}x^4 + 6x\right) dx$

$= \underline{\underline{4(x)^{5/2} + \frac{x^5}{4} + 3x^2 + c}}$

0^{-1}

7. $f(x+iy) + g(x-iy)$ P.DE

$z = f(x+iy) + g(x-iy)$

Diff. partially w.r.t x

$$\frac{\partial z}{\partial x} = f'(x+iy) + g'(x-iy)$$

Diff. again partially w.r.t x

$$\frac{\partial^2 z}{\partial x^2} = f''(x+iy) + g''(x-iy)$$

Diff. partially w.r.t. y

$$\frac{\partial z}{\partial y} = f'(x+iy)(i) + g'(x-iy)(-i)$$

θ^{-1} $\frac{\theta^3}{-1}$ $\log\theta$ r^3 $\log\theta$ $-3\log\theta$

Diff. again w.r.t y

$$\frac{\partial^2 z}{\partial y^2} = f''(x+iy)(i^2) + g''(x-iy)[(-1)^2]$$

$$= i^2 [f''(x+iy) + g''(x-iy)]$$

$$\frac{\partial^2 z}{\partial y^2} = i^2 \left[\frac{\partial^2 z}{\partial x^2}\right]$$

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1) = 2$

$$r(\theta) = \int \frac{r^2}{\theta} d\theta$$

$$= r^2$$

$$\frac{\theta}{d\theta} = \frac{r^2}{dr}$$

9. $$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\Rightarrow \frac{dv}{dt} + 0.196v = 9.8$$

$$P(t) = 0.196$$

$$Q(t) = e^{\int 0.196\,dt} = e^{0.196t}$$

$$e^{0.196t}\frac{dv}{dt} + 0.196e^{0.196t}v = 9.8e^{0.196t}$$

$$(e^{0.196t}v)' = 9.8e^{0.196t}$$

Integrating both sides

$$\int (e^{0.196t}v)'\,dt = \int 9.8e^{0.196t}\,dt$$

$$\Rightarrow e^{0.196t}v + k = 50e^{0.196t} + c$$

$$e^{0.196t}v = 50e^{0.196t} + c - k$$

$$e^{0.196t}v = 50e^{0.196t} + c$$

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10. $t y' + 2y = t^2 - t + 1$

Dividing by t

$$y' + \frac{2}{t} y = t - 1 + \frac{1}{t}$$

Let $u(t) = \int \frac{2}{t} \, dt = 2 \ln t$

$$y(t) = e^{2\ln t} \int e^{-2\ln t} \left(t - 1 + \frac{1}{t} \right) dt$$

$$y(t) = t^2 \int \frac{1}{t^2} \left(t - 1 + \frac{1}{t} \right) dt$$

$$y(t) = t^2 \int \left(\frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3} \right) dt$$

$$y(t) = t^2 \left(\ln t + 1/t - 1/2t^2 + c \right)$$

$y(1) = 2$ given

$$1 - \frac{1}{2} + c = 2$$

$$c = \frac{3}{2}$$

$$y(t) = t^2 \left(\ln t + \frac{1}{t} - \frac{1}{2t^2} + \frac{3}{t} \right)$$

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11. $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$$y' - \frac{x}{\sqrt{1+x^2}} y^3 = 0$$

$$y' + P(x)y = Q(x)$$

$$e^{\int P(x)dx} = e^{\int x/\sqrt{1+x^2}\,dx}$$

$$= e^{\sqrt{1+x^2}}$$

Multiplying the diff. eq^n with $\sqrt{1+x^2}$

$$\frac{dy}{dx} - \frac{x}{\sqrt{1+x^2}} y^3 = 0$$

$$\sqrt{1+x^2}\ \frac{dy}{dx} \cdot \frac{x}{\sqrt{1+x^2}} (\sqrt{1+x^2})\, y^3 = 0$$

$$\frac{dy}{dx}(\sqrt{1+x^2}\ y) - x \cdot y^3 = 0$$

Integrating both sides

$$\int \frac{dy}{dx}(\sqrt{1+x^2}\ y) - x \cdot y^3 = \int 0\, dx$$

$$\sqrt{1+x^2} \cdot y = c$$

$$y = \frac{1}{\sqrt{1+x^2}} c$$

$y(0) = -1$ given

$$-1 = \frac{1}{\sqrt{1}} c$$

$$C = -1$$

$$y = \frac{-1}{\sqrt{1+x^2}}$$

12. $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$$f(x) = f(k) + (x-k)f'(k) + \frac{(x-k)^2}{2!}f''(k) + \frac{(x-k)^3}{3!}f'''(k)$$
$$+ \frac{(x-k)^4}{4!}f''''(k) \dots$$

$f(x) = x^3 - 10x^2 + 6$ $\quad$ $f(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\quad$ $f'(3) = 27 - 60 = -33$

$f''(x) = 6x - 20$ $\quad$ $f''(3) = 18 - 20 = -2$

$f'''(x) = 6$ $\quad$ $f'''(3) = 6$

$f''''(x) = 0$ $\quad$ $f''''(4) = 0$

~~$f(x) =$~~

$$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) + \frac{(x-3)^2}{2!}(-2)$$
$$+ \frac{(x-3)^3}{3!}(6) + \frac{(x-4)^4}{4!}(0) \dots$$

$$x^3 - 10x^2 + 6 = -57 - 33 + 99 + \frac{(x-3)^2}{\cancel{2}} \times -\cancel{2}$$
$$+ \frac{(x-3)^3}{\cancel{3} \times \cancel{2}} \times \cancel{6} + 0 \dots$$

$$x^3 - 10x^2 + 6 = \underline{\underline{9}} + (x-3)^2 + (x-3)^3$$

13. Maclaurian $(1+x)^{\mu}$

$f(x) = (1+x)^{\mu}$ $\quad f(0) = 1$

$f'(x) = \mu(1+x)^{\mu-1}$ $\quad f'(0) = \mu$

$f''(x) = (\mu)(\mu-1)(1+x)^{\mu-2}$ $\quad f''(0) = \mu(\mu-1)$

$f'''(x) = (\mu)(\mu-1)(\mu-2)(1+x)^{\mu-3}$

$f'''(0) = (\mu)(\mu-1)(\mu-2)$

$$f(x) = f(0) + (x)f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4}f''''(0)$$

$$(1+x)^{\mu} = 1 + \mu + \frac{x^2}{2!}(\mu)(\mu-1) + \frac{x^3}{3!}(\mu)(\mu-1)(\mu-2) \cdots$$

$$(1+x)^{\mu} = 1 + \sum_{i=1}^{n} \frac{x^n \mu(\mu-n)}{n!}$$

14. $f(x) = \sqrt{1+x}$ maclaurian

$f(x) = \sqrt{1+x}$ $\quad f(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4\sqrt[3]{1+x}}$ $\quad f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{3}{8(1+x)^{5/2}}$ $\quad f'''(0) = \frac{3}{8}$

$$f(x) = f(0) + xf'0 + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{16} + \cdots$$

15. $f(x) = e^{-6x}$ $\quad x = -4$ $\quad$ taylor

$f(x) = e^{-6x}$ $\quad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\quad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad f''(-4) = 36e^{24}$

$f''(x) = -216e^{-6x}$ $\quad f'''(-4) = -216e^{24}$

$$f(x) = f(k) + (x-k)f'(k) + \frac{(x-k)^2}{2!}f''(k) + \frac{(x-k)^3}{3!}f'''(k)$$

$$e^{-6x} = e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2}(36e^{24}) + \frac{(x+4)^3}{6}(216e^{24})$$

$$e^{-26} = e^{24} - (6e^{24})(x+4) + (18e^{24})(x+4)^2 + (36e^{24})(x+4)^3$$

16. $f(x) = \ln(3+4x)$ $\quad x = 0$

$f(x) = \ln(3+4x)$ $\quad f(0) = \ln(3)$

$f'(x) = \frac{1}{3+4x}$ $\quad f'(0) = \frac{1}{3}$

$f''(x) = -\frac{4}{(3+4x)^2}$ $\quad f''(0) = -\frac{4}{9}$

$f'''(x) = \frac{8}{(3+4x)^3}$ $\quad f'''(0) = \frac{8}{27}$

$$f(x) = f(k) + (x-k)f'(k) + \frac{(x-k)^2}{2!}f''(k) + \frac{(x-k)^3}{3!}f'''(k)$$

$$\ln(3+4x) = \ln(3) + (x)\frac{1}{3} + \frac{x^2}{2}\left(\frac{-4}{9}\right) + \frac{x^3}{6}\left(\frac{8}{27}\right)$$

$$\ln(3+4x) = \ln 3 + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81} \ldots$$