~~Introduction~~

# Assignment - 2

Q 1] ~~Dont~~ Why crude mortality Rate are Graduated :-

Mortality rate are belived to vary smoothly with the age, as the crude mortality rate to carry the information of the mortality rate of adjacent age

By smoothing ~~the~~, the data of adjacent age can be use to ~~estimate the each age~~. improve the estimate ~~of the each age~~. at the each age.

This will Remove the sampling error

- Graduation will Not be able to remove any biases in any data. arising from data collection.

Q 4] a) cumulative test has been performed to check the overall biases in dataset.

$H_0$ = Standard mortality rate are true mortality rate for reinsurance.

$$z = \frac{\sum (O_x - E_x)^2}{\sqrt{E_x}}$$

$$z_1 = \sum \frac{(D_x - E_x \times \text{graduated rate})^2}{\sum E_x^2 \times q_x}$$

| $x$ | $d_x$ | $E_x \cdot q^S_x$ | $d_x - E_x q^S_x$ |
|---|---|---|---|
| 40 | 87 | 102.65 | −15.65 |
| 41 | 84 | 109.11 | −25.11 |
| 42 | 101 | 114.11 | −13.11 |
| 43 | 112 | 114.75 | −2.75 |
| 44 | 123 | 123.48 | −0.48 |
| 45 | 110 | 125.64 | −15.64 |
| 46 | 108 | 128.07 | −20.17 |
| 47 | 122 | 134.97 | −12.97 |
| 48 | 150 | 136.20 | 13.80 |
| 49 | 139 | 136.85 | 2.15 |
| 50 | 151 | 137.39 | 13.61 |
| Total | | 1363.32 | −76.32 |

$$N(0,1) \sim \frac{\sum (O_x - E_x q^S_x)}{\sqrt{E_x q^S_x}}$$

$$N(0,1) \sim \frac{-76.32}{\sqrt{1363.32}} = -2.066$$

As, z score value $|-2.066| > 1.96$ two sided test, we have enough evidence to reject the null hypothesis,

Conclude that standard mortality rate does not adequately reflect the true mortality rate for term assurance policyholder.

~~9.~~ i] a] ~~conflit~~ conflict arises ~~may be due to~~ default ~~made while~~ collecting the data!

i] a] Conflict arises because Sometime ~~Crude~~ mortality rate shows the ~~high mortality~~

~~9.5~~

i] a] Conflict arises, because Sometime ~~crude~~ crude rate of mortality shows that mortality for ~~eldest~~ young people higher than ~~eldest~~ elder ones

b] This can be resolve by smoothing of the data set, graduated rate, adjacent age data can be use to estimate mortality rate of each individ

i]

| x | No. of death | $q_x$ | $p_x$ |
|---|---|---|---|
| 30 | 245 | -7.014 | 249755 |
| 31 | 276 | -6.8971 | 249474 |
| 32 | 313 | -6.7816 | 249166 |
| 33 | 323 | -6.6654 | 248843 |
| 34 | 339 | -6.5492 | 248504 |
| | $\Sigma$ | | |

$$\mu = \frac{\Sigma d_x}{\Sigma l_x} = \frac{1496}{2488504}$$

$$= 60.1164 \times 10^{-4}$$

2) Aim of graduation are:-

1) To produce a smooth set of rates that are suitable for a particular purpose.

2) To Remove random sampling error

3) To use the information available from adjacent age.

Q.3 i) As we can see from data, the actual death & expected death are ~~dicrea~~ decreasing after age 80.

So, To check any baisness in the collected data company is performing a graduation.

ii) graduation with parametric formula
graduation with standard table
graduation with ~~Spline function~~ Graph (Graphical Graduation)

iii)

| Actual death | expected death | $O_i - E_i$ | $z_i^2$ |
|---|---|---|---|
| 248 | 227.28 | 20.72 | 429.31 |
| 392 | ~~672+3367~~ | 24.45 | 597.80 |
| 680 | 672.13 | 7.87 | 61.936 |
| 987 | 997.13 | -10.13 | 102.61 |
| 1380 | 1360.77 | 19.23 | 369.79 |
| 1625 | 1584.59 | 40.41 | 1632.9 |
| 1564 | 1484.59 | 79.41 | 6305.4 |
| 1564 | 1487.57 | 76.43 | 5841.54 |
| 925 | 891.36 | 43.64 | 1904.49 |
| 130 | 155.41 | -25.41 | 649.23 |
| | 8428.6 | | 17894.3 |

degree of freedom = 1

$\chi^2_8 = |g4 - g|$

for 5% level of significance, is 15.51

So, we failed to reject the Null hypothesis

ii] Sign test

$P(P \leq 2)$

$P \sim Bin(9, 0.5)$

~~$P(P_9 \leq 8) = 0.9902$~~

$P(P \leq 8) = 1.000$

Q.6. a) Age band for crude rate should in nearest b'day term ($l_x$) whereas, given data contain the age band of the last b'day $[l_{x+\frac{1}{2}}]$

ater age interval is $[x, x+1]$.

5) $H_0$:- Graduated rate provide the mortality of the life insurance company.

$$\frac{\Sigma (D_x - E^c_x l_{x+\frac{1}{2}})}{\sqrt{E^c_x \cdot l_{x+\frac{1}{2}}}}$$

| $E^c_x$ | Graduated Rate | $E^c_x \cdot l_{x+\frac{1}{2}}$ | $Z_i^2$ |
|---|---|---|---|
| 1388.90 | 0.0061 | 8.472 | 0.2755 |
| 1188.80 | 0.0131 | 15.57 | 0.1306 |
| 880.50 | 0.0262 | 23.06 | 1.0539 |
| 841.60 | 0.0487 | 40.98 | 1.1907 |
| 402.80 | 0.0839 | 33.79 | 1.5361 |
| 123.90 | 0.1338 | 16.57 | 0.3539 |
| 27.90 | 0.1925 | 5.51 | 0.4057 |
| 10.00 | 0.2706 | 2.706 | 0.031 |
| 7.50 | 0.3455 | 2.591 | 0.00135 |
| | | 149.229 | 4.96576 |

$$\chi^2_{8} \overset{n=9}{=} \frac{4.96576}{\sqrt{149.229}} = 0.40$$

for 5% level of significance.

$0.40 < 15.51$

• We fail to reject the Null hypothesis

Q.9

| expected death | Actual death | A − E | $z_i^2$ |
|---|---|---|---|
| 14.75 | 15 | 0.25 | 0.0062 |
| 17.08 | 16 | −1.08 | 1.1664 |
| 20.29 | 22 | 1.71 | 2.9241 |
| 23.49 | 23 | −0.49 | 0.2401 |
| 27.58 | 27 | −0.58 | 0.3364 |
| 63.26 | 66 | 2.74 | 7.5076 |
| 69.44 | 67 | −2.44 | 5.9536 |
| 89.27 | 88 | −1.27 | 1.6129 |
| 95.94 | 102 | 6.06 | 36.7236 |
| 91.39 | 80 | −11.39 | 129.73 |
| 86.64 | 85 | −1.64 | 2.6896 |
| 599.12 | | | 188.946 |

a) chi-squared test.

$$\frac{\sum (O_i - e_i)^2}{\sqrt{e_i}} \sim \chi^2_{11-1}$$

$$\chi^2_{10} = \frac{188.9468}{\sqrt{599.12}} = 7.719$$

level of significance is 5%.

$$\chi^2_{10} = 18.31$$

$$7.719 \leq 18.31$$

We reject the Null hypothesis, standard mortality rates are not actual mortality of group annuity policyholders.

5] Cumulative deviation test.

$H_0$ :- No bias  $H_1$ = bias

$$\frac{(O_i - E^s_x \times q^s_x)}{\sqrt{E^s_x \times q^s_x}} \sim N(0,1)$$

| $E^s_x q^s_x$ | Actual death | $Z_i^2$ |
|---|---|---|
| 14.752 | 15 | 0.061504 |
| 17.0775 | 16 | 1.1449 |
| 20.2875 | 22 | 2.9326 |
| 23.49 | 23 | 0.2401 |
| 27.579 | 27 | 0.33524 |
| 63.262 | 66 | 7.496604 |
| 69.4375 | 67 | 5.941406 |
| 89.271 | 88 | 1.615441 |
| 95.94 | 102 | 36.7236 |
| 91.392 | 80 | 129.77 |
| 86.636 | 85 | 2.6764 |
| 599.1295 | 591.00 | |

$$\frac{591 - 599.1295}{\sqrt{599.1295}} = -0.33$$

$|-0.33|$

Since value is less than 1.96 critical level we fail to reject the Null hypothesis Their is no biasess in data.

Q.11 H0 :- Mortality Rate of young professionals engaged in IT company is same as standard Rate.

| Age | $O_i$ | $E^c_x \cdot q_x$ | $O_i - E^c_x q_x$ | $z_i^2$ |
|---|---|---|---|---|
| 20 | 12 | 13.25 | -1.25 | 1.5625 |
| 21 | 14 | 14.75 | -0.75 | 0.5625 |
| 22 | 16 | 16.214 | -0.214 | 0.045796 |
| 23 | 9 | 7.14 | 1.86 | 3.4596 |
| 24 | 11 | 9.7125 | 1.2875 | 1.6576 |
| 25 | 15 | 13.015 | 1.985 | 3.940225 |
| 26 | 10 | 10.143 | -0.143 | 0.02044 |
| 27 | 16 | 17.653 | -1.653 | 2.732 |
| 28 | 17 | 13.176 | 3.824 | 14.6229 |
| 29 | 14 | 17.161 | -3.161 | 9.991 |
| | | 132.4645 | | |

$n = 10$

degree of freedom = 9.

$$\chi^2_9 = \frac{9.991}{\sqrt{132.4645}} = 0.86807$$

$3.325 > 0.86807$

We failed to reject the Null hypothesis.

Q[4] 1] Undergraduation :- It occurs when too much emphasis is given to goodness of fit, undergraduated rates adhere closely to the crude rate, but the resulting rates do not show a smooth progression from age to age.

2] Overgraduation :- occurs when too much emphasis given to smoothness overgraduated rates show a smooth progression from age to age, but resulting rates dont adhere closely to crude rates.

11] Chi-squared test.

$$Z_i = \frac{(O_x - E_x q_x')}{\sqrt{E_x q_x' (1 - q_x}}$$

| Average age | expected death. | $Z_x$ | $Z_x^2$ |
|---|---|---|---|
| 23 | 1.98 | 0.01421 | 0.00020 |
| 28 | 2.88 | 0.65997 | 0.43556 |
| 33 | 3.38 | 0.88116 | 0.77645 |
| 38 | 5.40 | 0.68853 | 0.47407 |
| 43 | 5.94 | 0.845223 | 0.71441 |
| 48 | 7.20 | 0.67092 | 0.45020 |
| 53 | 9.75 | -0.24019 | 0.05769 |
| 58 | 8.05 | -1.07498 | 1.1555 |

$\sum z_x^2 = 4.0639?$

The test statistic has a chi-squared distribution with degree of freedom given by No. of age group less 1 for parametric fit and further reduction for using standard table.

The critical value of chi-squared distribution with 6 degree fredom at 5% level is 12.59

Since $4.06397 < 12.59$, there is No evidence to reject the Null hypothesis that the graduated rates are the true rates underlying the crude rates.

ii) Sign test:-

1) It looks for overall biase,
2) If the Null hypothesis is true. positive sign is distributed binomial $(8, 0.5)$

$P(P \leq 6) = 0.965$

this is greater than 0.025

we failed to reject the Null hypothesis conclude that graduated rates are not systematically higher or lower than crude rate.

* Grouping of sign test :-

1) Grouping of ~~test to~~ Sign test for run or clumping of deviation with same sign for overgraduation.

$$P(\text{one positive run}) = \frac{3}{28} = 0.10714$$

greater than 0.05 (one tailed test).

We failed to reject the Null hypothesis that graduated rates are true underlying the crude rates.