Assignment Pricing and Reserving

1) (i) Constant force of mortality is an assumption to model fractional ages. force of mortality between age $x$ and $x+1$ is constant

(ii) CFM

$$p_x = e^{-\mu_x}$$

$$\mu = -\ln(p_x)$$

$$\mu_{55} = -\ln(1-q_{55})$$

$$= 0.0047010$$

2. (i) A select mortality is a death rate of individual who have recently undergone some medical test.
It has lower mortality rates as individual has cleared some medical test.

(ii)
$$_{20}p_{[25]:[30]} = {}_{20}p_{[25]} \times {}_{20}p_{[30]}$$

$$= \frac{l_{45}}{l_{[25]}} \times \frac{l_{50}}{l_{[30]}}$$

$$= \frac{9801.3123}{9951.9913} \times \frac{9712.0728}{9923.7497}$$

$$= 0.963852$$

Page No.:
Date:

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Page No.:
Date:

3) v) ii)

$${}_{1.4}q_{54.5} = 1 - {}_{1.4}p_{54.5}$$

$$= 1 - {}_{0.5}p_{54.5} \times {}_{0.9}p_{55}$$

$$1 - \left(1 - {}_{0.5}q_{54.5}\right)\left(1 - {}_{0.9}q_{55}\right)$$

$$1 - \left[\left(1 - \frac{0.5 \times q_{54}}{1 - 0.5q_{54}}\right)\left(1 - 0.9q_{55}\right)\right]$$

$$1 - \left(\left(\frac{1 - 0.5 \times 0.00714}{1 - 0.5 \times 0.00714}\right)\left(1 - 0.9 \times 0.00797\right)\right)$$

$$= 0.0107300921$$

4) $$P\ddot{a}_{35:\overline{30|}} = 5600000\, A^{\ \ 1}_{35:\boxed{30|}} + 30\, PA^{\ \ 1}_{35:\overline{30|}}$$

$$\ddot{a}_{35:\overline{30|}} = 14.35$$

$$A^{1}_{35:\overline{30|}} = A_{35} - v^{30}\, {}_{30}P_{35}\, A_{65}$$

$$= 0.04460 -$$

$$A_{35:\overline{30|}}^{\ \ \ 1} = v^{30}\, {}_{30}P_{35}$$

$$(1.06)^{-30}\left(\frac{8121.2612}{9894.4294}\right)$$

$$0.15522581$$

5) (i) $\bar{a}_x$

(ii) $PV = \bar{a}_{\overline{T_x}|}$ $\quad T_x > 0$

$EPV = \bar{a}_x = \int_0^\infty v^s \, {}_sp_x \, ds$

6) (i) $P = 5000000 \, A_{45:\overline{20}|}$

$A^{\;\;1}_{45:\overline{20}|} = A_{45:\overline{20}|} - A_{45:\overline{20}|}^{\;\;\;\;\;\;\;1}$

$= 0.46998 - (1.04)^{-20} \left(\frac{8821.2612}{9801.3123}\right)$

$= 0.069228$

$P = 296140.086$

Joint single premium $= 1000 \times P$

$= 296140086$

Page No
Date

ii) $V = 5000000\, A_{48:\overline{19}|} - 296140.0863$

$5000000\left[0.34599 - (1.06)^{-19}\left(\frac{8821.2612}{9716.953}\right)\right] -$

$= -55694.26419$

$DSAR = 5000000 - (-55694.26417)$
$= 5055694.26417$

$EDS = 10000\, q_{45} \times DSAR$

$= 74065920.97$

$ADS = 10 \times DSAR$
$= 50556942.6$

mortality profit $= EDS - ADS$
$= 23508978.3$

Page No
Date

2) $EPV = 100000\, A^{1}_{40:\overline{20}|} + 200000\, v^{20}\, {}_{20}P_{40}\, A_{60}$

$100000\left(A_{40} - v^{20}\, {}_{20}P_{40}\, A_{60}\right) + 200000\, v^{20}\, {}_{20}P_{40}\, A_{60}$

$A_{40} = 0.12313$ $\quad A_{60} = 0.32692$

$v^{20}\, {}_{20}P_{40} = (1.06)^{-20}\left(\frac{9284.2164}{9856.2863}\right)$

$= 0.293802$

$EPV = \cancel{21917}\ 21917.97945$

$E(PV^2) = 100000^2\left({}^2A^{1}_{40:\overline{20}|}\right) + 200000^2\ {}^2A_{60}\ v^{20}\ {}_{20}P_{40}$

${}^2A^{1}_{40:\overline{20}|} = {}^2A_{40} - (v^{20})^2\, {}_{20}P_{40}\ {}^2A_{60}$

$= 0.02202 - (1.06)^{-40}(0.44698)\left(\frac{9287.2164}{9856.2813}\right)$

$= 0.01415$

$E(PV^2) = 1798358788$

$SD = \sqrt{E(PV)^2 - (EPV)^2}$

$= 36303.73$

Page No.:

Date:

youva

a) $A_{65} = 0.5412$

$A_{66} = 0.5591$

$A_{65} = q_{65} + (1 - q_{65}) A_{66} v$

$A_{65} = q_{65} + A_{66} v - A_{66} v q_{65}$

$A_{65} - A_{66} v = q_{65} (1 - A_{66} v)$

$$\frac{0.5412 - 0.5591 (1+0.08)^{-1}}{1 - 0.5591 (1.06)^{-1}} = q_{65}$$

$= 0.048754$

9) (i) $P = 120000\ a_{20}$

$= 120000\ (\ddot{a}_{20} - 1)$

$= 12000\ (10.562)$

$= 1267440$

(ii) $P(L_0 > 0) = 1$

$P(L_0 \leq 0 \mid G = G_n)$

10) (i) $PV = \bar{a}_{\overline{T_x}|}$

$EPV = \int_0^\infty a_{\overline{t}|} \cdot {}_tP_x\ \mu_{x+t}\ dt = \bar{a}_x$

(ii) $\int_0^\infty e^{-(\mu+\delta)t}\ dt = 16$

$\frac{1}{-(\mu+\delta)} \left[ e^{-(\mu+\delta)t} \right]_0^\infty = 16$

$\frac{1}{-(\mu+0.04)} \left[ 0 - 1 \right] = 16$

$$\mu = \frac{1-0.64}{16}$$

$$= 0.0225$$

$$\text{Var } L = \frac{1}{\delta^2}\left({}^2\bar{A}_x - \left(\bar{A}_x\right)^2\right)$$

$$\bar{A}_x = 1 - \delta\,\bar{a}_x$$

$$\bar{a}_x = 1 - 0.64$$

$$= 0.36$$

$${}^2\bar{A}_x = \int_0^{\infty} e^{-2\delta t}\, e^{-\mu t}\, x_\mu \, dt$$

$$= 0.0225\left[\frac{e^{-(2\delta+\mu)t}}{-(2\delta+\mu)}\right]_0^{\infty}$$

$$= \frac{9}{34}$$

$$SD = \sqrt{\frac{1}{0.04^2}\left(\frac{9}{34} - (0.36)^2\right)}$$

$$= 9.18918702$$

(i) In term assurance, the premium are paid annually in advance

(ii) $P\ddot{a}_{40:\overline{25|}} = 2500000\, A^{1}_{40:\overline{25|}}$

$A^{1}_{40:\overline{25|}} = A_{40:\overline{25|}} - A_{40:\overline{25|}}^{\ \ \ 1}$

$= 0.38907 - (1.04)^{-25}\left(\dfrac{8821.2612}{9856.2863}\right)$

$= 0.05354485019$

$P = \dfrac{2500000\,(0.05354485019)}{15.814}$

$= 8396.602296$

${}_{10}V = 2500000\, A^{1}_{50:\overline{15|}} - P\ddot{a}_{50:\overline{15|}}$

$A^{1}_{50:\overline{15|}} = A_{50:\overline{15|}} - A_{50:\overline{15|}}^{\ \ \ 1}$

$= 0.56719 - (1.04)^{-15}\left(\dfrac{8821.2612}{9712.0723}\right)$

$= 0.062865$

${}_{10}V = 87334.42964$

$= V \quad 250000\, \ddot{a}_{85}$

$= 2900000$

Reserve $= 3500\,(87334.4246) + \ldots$

$= 3943134265.0$

(iii)

(b) (i) $S_{55:\overline{10|}} = a_{55:\overline{10|}}\,(1+i)^{10}\;{}_{10}P_{55}$

$7.61 \times (1.06)^{+10} \left(\dfrac{8821.2612}{9557.8174}\right)$

$= 12.57810568$

(ii) $\ddot{a}^{(4)}_{[40]:\overline{20|}} = \ddot{a}^{(4)}_{[40]} - v^{20}\;{}_{20}P_{[40]}\;\ddot{a}^{(4)}_{60}$

$= \ddot{a}_{[40]} - \dfrac{3}{8} - v^{20}\;{}_{20}P_{[40]}\left(\ddot{a}_{60} - \dfrac{3}{8}\right)$

$= \left(15.494 - \dfrac{3}{8}\right) - (1.06)^{-10}\left(11.891 - \dfrac{3}{8}\right)\left(\dfrac{9287.2164}{9650.3.36}\right)$

$= 14.6785063$

$_{10}V = 2500000 \ddot{a}_{55}$

$= 3908000$

Reserve $= 8500(87334.42964) + 9900 \times 3908000$
$= 39431542650$

(iii)

12) (i) $S_{55:\overline{10|}} = a_{55:\overline{10|}} (1+i)^{10} \ {}_{10}P_{55}$

$7.61 \times (1.06)^{+10} \left(\frac{8821.2612}{9557.8779}\right)$

$= 12.57810568$

(ii) $\ddot{a}^{(4)}_{[40]:\overline{20|}} = \ddot{a}^{(4)}_{[40]} - v^{20} \ {}_{20}P_{[40]} \ \ddot{a}^{(4)}_{60}$

$= \ddot{a}_{[40]} \cdot \frac{3}{8} - v^{20} \ {}_{20}P_{[40]} \left(\ddot{a}_{60} - 3/8\right)$

$= \left(15.494 - 3/8\right) - (1.06)^{-10} \left(11.891 - 3/8\right)\left(\frac{9207.2164}{9654.3313}\right)$

$= 14.6785063$

Page No.:

Date:

13) $P\ddot{a}_{45:\overline{15}|} = 100000\, A_{45:\overline{20}|} + 100000\, A_{45:\overline{20}|}^{\;\;1}$

$\ddot{a}_{45:\overline{15}|} = 11.881$

$A_{45:\overline{20}|} = 0.46941$

$A_{45:\overline{20}|}^{\;\;1} = (1.04)^{-20}\ {}_{20}P_{45}$

$= 0.41095198$

$P = 7734.608$

${}_{13}V = 100000\, A_{58:\overline{7}|} + 100000\, A_{58:\overline{7}|}^{\;\;1} - P\ddot{a}_{58:\overline{7}|}$

$A_{58:\overline{7}|} = 0.76516$

$A_{58:\overline{7}|}^{\;\;1} = (1.04)^{-7}\ {}_{7}P_{58}$

$= 0.712615$

$\ddot{a}_{58:\overline{7}|} = 1.955$

${}_{13}V = 132603.42$

$DSAR = 100000 - 13V$

$= -32603.42895$

$EDS = 199 \times 0.00560 \times DSAR$

$= -36657.66534$

$ADS = 4 \times DSAR$

$= -130413.7158$

Mortality profit = $EDS - ADS$

$= 93756.05046$

14) (i)

$$\begin{cases} 0 & T_{66} \le 2 \\ 5000\,\bar{a}_{\overline{T_{63}-2}|}\,v^2 & T_{66} > 2 \end{cases}$$

(ii) $5000 \times 100 \; y \; {}_2P_{63} \times v^2\,\bar{a}_{65}$

$50000\,(14.871 - 0.5)\,(1.04)^{-2}\left(\dfrac{9703.703}{9775.818}\right)$

$= 6594343.317$

Page No.:
Date:

(iii) $Var = 5000^2 \times 100 \times \frac{1}{\delta^2}\left(v^{2n}\, {}_nq_x + \delta^4 v^{2n}\, {}^2\bar{A}_{x+n} - \right.$

$\left. v^n\, {}_nq_x + v^n\, {}_nP_x \left(\bar{A}_{x+n}\right)^2\right)\Big]$

$5000^2 \times 100 \times \frac{1}{\delta^2}\left(v^4\, {}_2P_{63} + v^4\, {}_2P_{63}\, {}^2\bar{A}_{65} - \right.$

$\left. v^2\, {}_2q_{63} + v^2\, {}_2P_{63}\left(\bar{A}_{65}\right)^2\right)$

$\bar{A}_{65} = 1 - \delta\, \bar{a}_{65}$

$\bar{A}_{65} = 0.436359$

${}^2\bar{A}_{65} = (1.04)^{0.5}\,(0.20847)$

$= 0.212598$

${}_2P_{63} = \frac{9703.708}{9778.880}$

$= 0.9926$

${}_2q_{63} = 1 - 0.9926$

$= 0.007313$

$Var = 5000^2 \times 100 \times 13.53$

$= (18393.66)^2$

$$({}_{t}V + P)(1+i) - x(1+i)^{1/2} - {}_{5}q_{x+t} = \cancel{t+t}$$

$$ {}_{t+1}V \, P_{x+t}$$

$$({}_{t}V + P)(1+i) - {}_{5}q_{x+t} \cdot R \; {}_{0.5}P_{x+t} = {}_{t+1}V \, P_{x+t}$$