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Assignment 1

Ans 1) $X \sim Poi(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \overset{\cdot}{\sim} N(0,1)$$

$X = 200$

$$\lambda X = \bar{X} \pm Z_{\alpha/2} \frac{\hat{\lambda}}{n}$$

$$= \bar{X} \pm 1.96 \sqrt{\frac{\bar{X}}{n}} \quad \left[\bar{X} \to N\left(\theta, \frac{\theta}{n}\right) \right]$$

$$\theta = \bar{X} \pm 1.96 \sqrt{\frac{\theta}{n}}$$

Ans 2) i) $X_i : i = 1, 2, \dots n$

i) $E(\bar{X}) = \frac{X_1 + X_2 + X_3 \dots X_n}{n} = \frac{n\mu}{n} = \mu$

$V(\bar{X}) = V(nX) = n\sigma^2 \Rightarrow \frac{n\sigma^2}{n^2} = \sigma^2 \quad [V(ax) = a^2 V(X)]$

$\because$ Variance $= V(X_1 + X_2 + X_3)$
$= V(X_1) + V(X_2) + V(X_3)$
$= n\sigma^2$

$\because V(X+Y) = V(X) + V(Y)$ [In case of i.i.d r.v]

ii) ~~$E(s^2)$~~ $S^2 = \frac{1}{n-1} \Sigma (X_i - \bar{X})^2$

$E(\Sigma (x_i - \bar{x})^2) = E(\Sigma x_i^2 + n\bar{x}^2 - 2\Sigma x_i . \bar{x})$
$= \Sigma E(x_i^2) - E(n\bar{x}^2)$
$\Sigma E(x_i^2) - nE(\bar{x}^2)$
$= n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2$
$= (n-1)\sigma^2$ ——— 1

From ① we get

$$E(S^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2$$

iii) We know,

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

To prove:- $V(S^2) = \frac{2\sigma^4}{n-1}$

$$V(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{(n-1)}.$$

iv) $(50 < x < 150) = \left(\frac{9\times 50}{100} < \chi^2_9 < \frac{150\times 9}{100}\right)$

$$= (4.5, 13.5)$$

Ans 3) ~~n=4~~ $X \sim Bin(4, p)$

i) $P(X = x) = {}^nC_x\, p^x q^{n-x}$

$$L \Rightarrow \left({}^4C_0\, p^0 (1-p)^{4-0}\right)^{86} \cdot \left({}^4C_1\, p^1 \cdot (1-p)^{4-1}\right)^{75} \cdot \left({}^4C_2\, p^2 (1-p)^{4-2}\right)^{16} \cdot \left({}^4C_3\, p^3 (1-p)^{4-3}\right)^2 \cdot \left({}^4C_4\, p^4 (1-p)^{4-4}\right)$$

$$\log L \Rightarrow \left((1-p)^4\right)^{86} \cdot \left(4p(1-p)^3\right)^{75} \cdot \left(6p^2(1-p)^2\right)^{16} \cdot \left(4p^3(1-p)\right)^2 \cdot p^4$$

$$\Rightarrow (1-p)^{344} \cdot 4p^{75}(1-p)^{225} \cdot 6p^{32}(1-p)^{32} \cdot 4p^6(1-p)^2 \cdot p^4$$

$$86\log(1-p)^4 + 75\log(4p(1-p)^3) + 16\log(6p^2(1-p)^2) + 2\log(4p^3(1-$$

$$+ 4\log p$$

$$\Rightarrow \cancel{344\log(1-p)} \; 603\log(1-p) + 117\log 96p$$

$$\frac{d\ln L}{dL} \Rightarrow \frac{603}{1-p} + \frac{117}{96p} = 0$$

$$57888p + 117 - 117p = 0$$

$$57771p = -117$$

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$p = 0.002.$

ii) Goodness of fit test

Ans4) $f(x) = \dfrac{c\beta^2}{(x+\beta)^4}$

Ans 5) i) $X \sim U(a, b)$

Mean $= \frac{a+b}{2}$ — ①

Variance $= \frac{(b-a)^2}{12}$ — ②

$a = 2\bar{x} - b$ (from ①)

$a = b - 2\sqrt{3}s$ (from ②)

$b = \bar{x} + \sqrt{3}S$

$a = \bar{x} - \sqrt{3}S$

ii) $b = a - 2\bar{x}$

$b = \bar{x} + \sqrt{3}S$

iv) $U(a, b)$

$f(x) = \frac{1}{b}$

$L = \frac{1}{b} \times \frac{1}{b} \cdots$ n times

$= \frac{1}{b^n}$

$\text{Log } L = -(n \log b)$

$\frac{d \ln L}{dL} = -\frac{n}{b}$

Ans 6)

Ans 7) Public (mean) $= \frac{24 + 45 + 29 + 33 + 20 \cdots + 27.5}{9}$

$= 30$

$S^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$

$= \frac{1}{8} \Sigma (x_i - \bar{x})^2 \Rightarrow 64.31$

Private

$$\text{Mean} = \frac{30 + 54.5 + 30 \ldots\ldots\ldots 35.5}{9}$$

$$= 35.056$$

$$s^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$$

$$= \frac{1}{8} \cancel{6}\Sigma (x_i - \bar{x})^2 = 67.4$$

iii) $H_0 \Rightarrow \text{Mean}_{Public} = \text{Mean}_{Private}$

$H_1 \Rightarrow \text{Mean}_{public} \neq \text{Mean}_{Private}$

$$\Rightarrow \frac{30 - 35.056}{\sqrt{8.12}\sqrt{2/9}} = -1.40$$

∴ They don't result in higher claim size.

Ans8) i) Unbiased estimator is when expected value of estimator is equivalent to true value of parameter.

ii) Bias is the difference between value of estimator and true parameter value.

iii) MSE measures the average of squared differences between estimated & actual value.

iv) $\hat{\theta}$ is more efficient since MSE is lower than $\tilde{\theta}$

Ans9) i) $t_k = \frac{N(0,1)}{\sqrt{t_k/k}}$; $\frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

ii) Mean = 0

Variance $= \frac{k}{k-2}$.

ii) a) $n = 10$;
$\bar{x} = 50$
$s^2 = 48.667$ } given.

$\mu = \bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$

$\mu = 50 \pm t_{0.5, 9} \sqrt{\frac{48.667}{10}}$

$= \cancel{57.17}\ (42.83, 57.17)$

b) $\mu = \bar{x} + Z_{\alpha/2} \sqrt{\frac{s}{n}}$

$= 50 \pm Z_{0.5} \sqrt{\frac{98.667}{10}}$

$= (44.32, 55.6)$.

Ans 10) $n = 1000$
$x \sim Poi(3)$
$\Rightarrow x \sim Poi(N\mu)$
$x \sim Poi(3000)$ — (1)
Condition $P(X < 3100)$ then $x \sim Poi(3)$
$\Rightarrow P(x > 3100, \lambda = 3)$ (from (1))
$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right)$
$= P[Z > 1.826)$
$= 3.4\%$.

Ans 11) $\Sigma x = 213,200$
$\Sigma x^2 = 4,919,860,000$
$\bar{x} = 17,767$
$s^2 = 10,144$. } given.

$\Sigma x = 213,200$

$\Sigma x = T + 11,000 + 48,000$

$213,200 - 59,000 = T$

$T = 1,54,200$

$\Sigma x = T + 27,500 + 81,500$

$\bar{x} = \frac{\cancel{213200}}{10}$

Ans 12) $x_1, x_2, x_3 \ldots x_n \sim U(0, \theta)$

i) mean $= \frac{\theta}{2}$ Var. $= \frac{\theta^2}{12}$

$L = \frac{1}{\theta} \times \frac{1}{\theta} \ldots$ n times

$L = \frac{1}{\theta^n}$

$\frac{d \ln L}{dL} = -\frac{n}{\theta}$.

$\because$ θ can be ~~un~~ infinite, Cramér - Rao lower bound is not applicable.

iii) $\text{Bias}[\hat{\theta}(c)] = E[\hat{\theta}c - \theta]$

$= E[y - \theta]$

$= cE(y) - \theta$

$= \frac{cn\theta}{n+1} - \theta$

$\theta\left(\frac{cn}{n+1} - 1\right)$.