Prob. & Stats - 2

Assignment -

1.) $\Sigma x = 850$

$\Sigma x^2 = 925000$

$\Sigma y = 1737$

$\Sigma y^2 = 372717$

$\Sigma xy = 182560$

(a) $S_{xx} = 852750$

$S_{yy} = 71000$

$S_{xy} = 34915$

$$\beta = \frac{S_{xy}}{S_{xx}} = 0.040944$$

$$\alpha = \bar{y} - \beta\bar{x} = 170.219$$

$$\therefore \hat{y} = 170.219 + 0.040944x$$

(b) $\beta = 0.040944$

β is very close to 0

$\therefore$ It is ~~very~~ not significant.

2.) $\Sigma x = 385.2$

$\Sigma x^2 = 12666.58$

$\Sigma y = 1162.5$

$\Sigma y^2 = 119026.9$

$\Sigma xy = 38191.41$

$S_{xx} = 301.66$

$S_{yy} = 6409.7125$

$S_{xy} = 875.16$

$\beta = 2.9011$

$\alpha = 3.7496$

FOR EDUCATIONAL USE

$$\therefore \hat{y} = 3.7496 + 2.9011x$$

(ii) $\sigma^2 = \frac{1}{n-1}\left(S_{yy} - \frac{S_{xy}}{S_{xx}}\right)$

$\sigma^2 = 387.0744$

$\hat{\beta} \sim N(\beta, \sigma^2/S_{xx})$

95% CI

$= \beta \pm (-2.5\%), 12 \left(\frac{\sigma}{\sqrt{S_{xx}}}\right)$

$= 2.9011 \pm (2.228)(1.132)$

$= (0.379, \cancel{5}5.423)$

(iii) $Y_0 \sim N\left(\hat{\alpha} \cdot \hat{\beta} x, \frac{1}{n} \left(\frac{(x_0 - \bar{x})^2}{S_{xx}}\right) \sigma^2\right)$

95% CI $= y_0 \pm t_{\alpha/2, n-2} \sqrt{\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right) \sigma^2}$

$= 119.7936 \pm 22.143$

95% CI $= (97.6504, 141.9306)$

6.) $\Sigma x = 39$

$\Sigma x^2 = 207$

$\Sigma y = 562$

$\Sigma y^2 = 40508$

$\Sigma xy = 2853$

$S_{xx} = 54.9$

$S_{xy} = 661.2$

$S_{yy} = 8923.6$

(i) $r = \dfrac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = 0.944662$

(ii) $\beta = 12.0437$

$\alpha = 9.22957$

$\hat{y} = 9.22957 + 12.0437x$

(iii) $S_{tot} = SS_{reg} + SS_{res}$

$\therefore SS_{reg} = \dfrac{S^2_{xy}}{S_{xx}} = 7963.30$

$SS_{res} = S_{yy} - \dfrac{S^2_{xy}}{S_{xx}} = 960.295$

$SS_{tot} = 7963.30 + 960.295$

$= 8923.6 = S_{yy}$

(iv) $R^2 = (r)^2$

$= (0.944662)^2$

$R^2 = 0.8923$

7.) $\Sigma x = 174.13$

$\Sigma y = 197.84$

$\Sigma x^2 = 7545.90$

$\Sigma y^2 = 4747.45$

$S_{xy} = 2176.84$

$S_{xx} = 4789.4221$

$S_{yy} = 1189.2076$

(i) $\beta = \frac{S_{xy}}{S_{xx}} = 2.20017$

$x = -16.8432$

$\hat{y} = -16.8432 + 2.20017\,x$

(ii) $H_0 : \beta = 0$

$H_1 : \beta \neq 0$

$\hat{\sigma}^2 = 199.8121$

$t_{cal} = \frac{\hat{\beta} - \beta}{\hat{\sigma}/\sqrt{S_{xx}}} = 10.77176$

$t_{tab} = 2.776$

$\therefore t_{cal} > t_{tab}$

$\therefore$ Reject H_0

There is linear relationship between X and Y.

(iv) $r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = 0.912128$

Sundaram®

a.) $\Sigma x = 45$

$\Sigma x^2 = 277.5$

$\Sigma y = 644$

$\Sigma y^2 = 43956$

$\Sigma xy = 2602$

$S_{xx} = 75$

$S_{yy} = 2482.4$

$S_{xy} = -296$

(i) Looking at the plot, I see a negative correlation between x & y.

(ii) $r = \frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} = -0.686$

$\therefore$ There is a strong negative correlation.

(iii) $\tanh^{-1} r \sim N(\tanh^{-1} \rho;\ 1/n-3)$

$H_0 : \rho = 0$

$H_1 : \rho < 0$

$$\frac{\tanh^{-1} r - \tanh^{-1} \rho}{\sqrt{\frac{1}{n-3}}} = Z_{cal}$$

$$Z_{cal} = \frac{-0.84036 - 0}{\sqrt{\frac{1}{7}}} = -2.22338$$

$Z_{tab} = \pm 1.96 = Z_{2.5\%}$

$Z_{0.5} = \pm 2.5758$

at 5% L.O.S. Reject H_0

at 1% L.O.S. do not reject H_0.

There is ~~a~~ negative ~~z~~ correlation at 5% L.O.S.//

There is no correlation at 1% L.O.S.//

— x —