Assignment - 2

A1). Let X be claim sizes on home insurance policy.
& Y be claim sizes on car insurance policy.

$$X \sim N(800, 100^2) \quad , \quad Y \sim N(1200, 300^2)$$

$$P((Y_1 + Y_2 + Y_3) > (X_1 + X_2 + X_3 + X_4) + 800)$$
$$= P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 800)$$

$$Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) \sim N(3(1200) - 4(800), 3(300)^2 + 4(100)^2)$$
$$\Rightarrow \quad \sim N(400, 310000).$$

$$\Rightarrow P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$$

$$= P(Z > 0.718)$$
$$= 1 - P(Z < 0.718)$$

Req. prob. $= 0.23638$

Ans

A2) N follows -ve binomial distribution (type-2) with $k=3$, $p=0.9$.

$$M_N(t) = \left(\frac{0.9}{1-0.1e^t}\right)^3$$

Also, $X \sim Gamma(6,2) \Rightarrow M_X(t) = \left(1-\frac{t}{2}\right)^{-6}$

$$\Rightarrow M_Y(t) = \left(\frac{0.9}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right)^3$$ Ans

ii) $E(X) = \frac{6}{2} = 3$ $\qquad V(X) = \frac{6}{4} = 1.5$

$E(N) = \frac{3 \times 0.1}{0.9} = \frac{1}{3}$ $\qquad V(N) = \frac{3 \times 0.1}{0.81} = \frac{10}{27}$

$$V(Y) = E(N)\,V(X) + [E^2(X)]^2\,V(N)$$

$$= \frac{1}{3}(1.5) + 3^2 \times \frac{10}{27}$$

$$= 0.5 + 3.333.$$

$$= 3.8333.$$

$$\text{Std dev.}(Y) = \sqrt{3.8333}$$

$$= 1.9578$$ Ans

A3). $X \sim Exp.(\lambda)$.

$$f(x) = \lambda e^{-\lambda(x-\alpha)}, \quad x \geq \alpha \; \lambda\alpha, \lambda > 0$$

$$M_X(t) = \int_0^\infty e^{tx} \cdot f_X(x)\, dx.$$

$$= \int_0^\infty e^{tx} \cdot \lambda e^{-\lambda(x-\alpha)}\, dx$$

$$= \frac{\lambda e^{\lambda\alpha}}{\lambda - t} = \left(\frac{\lambda - t}{\lambda e^{\lambda\alpha}}\right)^{-1} \cancel{= \frac{\lambda e}{e}}$$

$$\cancel{= e^{\lambda\alpha} - \frac{t}{\lambda}} = \left(\frac{\lambda}{\lambda e^{\lambda\alpha}} - \frac{t}{\lambda e^{\lambda\alpha}}\right)^{-1}$$

$$= \left(\frac{1}{e^{\lambda\alpha}}\right)^{-1} \left(1 - \frac{t}{\lambda}\right)^{-1}$$

$$M_X(t) = e^{\lambda\alpha}\left(1 - \frac{t}{\lambda}\right)^{-1}$$

$$E(X) = M_X'(t) \Rightarrow \cancel{\frac{d}{dt}\left(e^{\lambda\alpha}\left(1-t\right.\right.}$$

$$= e^{\lambda\alpha} \times \frac{1}{\lambda} \cancel{= e}$$

$$= \frac{e^{\lambda\alpha}}{\lambda}$$

$$V(X) = M''_X(t) - [M'_X(t)]^2$$

$$= \frac{2e^{\alpha\lambda}}{\lambda^2} - \frac{e^{2\lambda\alpha}}{\lambda^2}$$

$$\boxed{= \frac{-e^{2\alpha\lambda} + 2e^{\alpha\lambda}}{\lambda^2} \quad \text{or} \quad \frac{e^{\alpha\lambda}(2 - e^{\alpha\lambda})}{\lambda^2}}$$

~~A4)~~ A3)

$$G_V(t) = p^k(1-qt)^{-k}$$

$$\cancel{G_V(t) = \sum_{\text{all } v} t^v \cdot P(V=4)}.$$

$$\Rightarrow \cancel{p^k(1-qt)^{-k} = \sum_{\text{all } v} t^v \cdot {}^{n-1}C_{k-1}\, p^k (1-p)^{n-k}}.$$

Using binomial expansion from page 2 of tables,

$$G_V(t) = p^k(1-qt)^{-k}$$

$$\cancel{= p^k \quad 1 + (-k)(-qt)}$$

$$= p^k\left(1 + (-k)(-qt) + \frac{(-k)(-k-1)}{2!}(-qt)^2 + \frac{(-k)(-k-1)(-k-2)(-qt)^3}{3!} + (-qt)^4\frac{(-k)(-k-1)(-k-2)(-k-3)}{4!}\right.$$

$$= p^k\left(1 + kqt + \frac{k(k+1)}{2!}(qt)^2 + \frac{k(k+1)(k+2)}{3!}(qt)^3 + \frac{k(k+1)(k+2)(k+3)}{4!}(qt)^4\right.$$

~~P(~~ $P(V=4)$ is coeff. of t^4 in this expansion,

$$\Rightarrow \boxed{P(V=4) = \frac{k(k+1)(k+2)(k+3)}{4!}p^k q^4}$$

Ans

A5) $f(x,y) = ax(x+y)$ $0 < x < 5$, $10 < y < 20$

$P\left(Y > \frac{1}{3}X + 10\right)$

$= P(3Y - X > 30)$.

$$f_{3Y}(y) = \int_{3x} f_{X,Y}(x,y)\,dx$$

$$= \int_0^{15} ax(x+y)\,dx = a\int_0^{15} x(x+y)\,dx$$

$$= a\left(\frac{2250y + 2250}{2}\right) = \frac{2250ay + 2250a}{2}.$$

$$f_X(x) = \int_y f_{X,Y}(x,y)\,dy$$

$$= \int_{10}^{20} ax(x+y)\,dy = a\int_{10}^{20} x(x+y)\,dy$$

$$= a(10x(x+15)). = 10ax^2 + 150ax.$$

$$E(Y/X) = \int x\, f_{X/Y}(x,y)\,dx$$

$$= \int_0^5 x\, \frac{f_{X,Y}(x,y)\,dx}{f_X(x)} = \int_0^5 x\, \frac{\cancel{a}\cancel{x}(x+y)}{\cancel{a}(10\cancel{x}(x+15))}\,dx$$

$$= \frac{1}{10}\int_0^5 \frac{x(x+y)}{10(x+15)}\,dx$$

$$= \cancel{22}\ \cancel{10}\ \frac{1}{10}\left(225 - 15y\right) \cancel{\log} \ln(x+15) + \frac{x^2}{2} \cancel{+(y-15)x}$$

Ans

Q6) $X \sim Poi(\lambda)$ $\qquad$ $Y \sim Poi(\mu)$.

$M_X(t) = \cancel{e^{\lambda(e^t-1)}}\, e^{\lambda(e^t-1)}$ $\qquad$ $M_Y(t) = e^{\mu(e^t-1)}$

Let $Z = X+Y$

$\therefore$

$M_Z(t) = E(e^{tZ}) = E[e^{t(X+Y)}]$.

$\Rightarrow$ $\cancel{M_{X-Y}(t) = e^{(\lambda-\mu)(e^t-1)} = e^{\lambda(e^t-1)-\mu(e^t-1)}}$

$\cancel{M_{X-Y}(t) = e^{(\lambda-\mu)(e^t-1)}}$

$M_Z(t) = \cancel{E(e^{tz}}\, M_X(t) \cdot M_Y(t)$.

$M_Z(t) = e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$

$M_Z(t) = e^{(\lambda+\mu)(e^t-1)}$

$\Rightarrow Z \sim Poi(\lambda+\mu)$.

$\Rightarrow$ $\underline{X+Y \sim Poi(\lambda+\mu)}$.

Let $Q = X-Y$.

$M_Q(t) = E(e^{tQ}) = E[e^{t(X-Y)}]$

$= \cancel{M_X(t) \cdot M_{(Y)}(t)}\ \dfrac{M_X(t)}{M_Y(t)}$

$= \dfrac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$.

$M_Q(t) = e^{(\lambda-\mu)(e^t-1)}$

$\Rightarrow Q \sim Poi(\lambda-\mu)$.

$\Rightarrow$ $X-Y \sim Poi(\lambda-\mu)$

$\Rightarrow$ $\therefore$ Both the statements are true.

A7). (ii) $E(X/Y=2) = \sum_{all\ x} x \cdot P(X=x/Y=2).$

$= \sum_{all\ x} x \dfrac{P(X=x, Y=2)}{P(Y=2)}$

$\cancel{= 1(0.2)+1(0.} \quad \cancel{= 0.2+2(0.3)+3(0.05+0.1)}$

~~P value 1 2~~

~~P(X=x) 0.2 0.3~~

~~P(Y=y) 0.6~~

$= \dfrac{0.2+2(0.3)+3(0.05+0.1)+4(0.15+0.2)}{0.3+0.1+0.2}$

$= \dfrac{2.65}{0.6}$

$E(X/Y=2) = 4.4166. \Rightarrow [E(X/Y)]^2 = 19.5069.$

$E(X^2/Y=2) = \sum_{all\ x} x^2 \dfrac{P[X=x, Y=2]}{P(Y=2)}$

$\cancel{= \dfrac{(0.2)^2+2(0.3)^2+3(0.15)^2+4(0.35)^2}{0.3+0.1+0.1}}.$

$= \dfrac{0.2+4(0.3)+9(0.15)+16(0.35)}{0.6}.$

$E(X^2/Y=2) = 8.35.$

$V(X/Y=2) = E(X^2/Y=2) - [E(X/Y=2)]^2$

$= 19.5069 - 8.35$

$Var(X/Y=2) = -11.1569$

Ans

(ii) $f_{U,V}(u,v) = \frac{48}{67}(2uv - u^2)$; $0 < u < 1$
$\frac{u}{2} < v < 2$

$$f_V(v) = \int_u f_{U,V}(u,v)\,du$$

$$= \int_0^1 \frac{48}{67}(2uv - u^2)\,du$$

$$= \frac{48}{67}\int_0^1 (2uv - u^2)\,du$$

$$= \cancel{\frac{48}{67}} \cancel{2uv - u^2}\ \frac{48}{67}\left(\frac{3v-1}{3}\right)$$

$$= \frac{48v - 16}{67}$$

$$E(U/V=v) = \int_0^1 u \frac{f_{U,V}(u,v)}{f_V(v)}\,du$$

$$= \int_0^1 u \frac{\frac{48}{\cancel{67}}(2uv - u^2)}{\frac{48v-16}{\cancel{67}}}\,du$$

$$= \int_0^1 u \frac{\cancel{16}[3(2uv - u^2)]}{\cancel{16}(3v-1)}\,du$$

$$= \int_0^1 u \frac{6uv - 3u^2}{3v-1}\,du$$

$$= \frac{8v-3}{12v-4}$$ Ans

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Date :

A8) $X \sim \text{Gamma}(3, 2)$.

$E(Y/X=x) = 3x+1$ $\qquad$ $V(Y/X=x) = 2x^2+5$.

Unconditional mean of Y

$= E(E(Y/X=x))$

$= \int_x E(Y/X=x) \cdot \cancel{P(X=x)\, dx}\ f_X(x)\, dx$

$= \int_x (3x+1) \cdot \left(\frac{2^3 e^{-2x} \cdot x^2}{2!}\right) dx$.

$= \int_x (3x+1)\ \cancel{4e^{-2x}} \left(4x^2 e^{-2x}\right) dx$.

$= \frac{-\left(12x^3 + 22x^2 + 22x + 11\right) e^{-2x}}{2}$.

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A9) $E(X) = 3$. $E(Y) = 4$

$V(X) = 2^2 = 4$. $V(Y) = 1^2 = 1$.

$$Corr(X,Y) = -0.3$$

$Z = (X+Y)$. $\Rightarrow Y = Z - X$.

$$Corr(X,Y) = \frac{cov.(X,Y)}{\sqrt{var(X)\ var.(Y)}}$$

$$-0.3 = \frac{cov.(X,Y)}{\sqrt{4 \times 1}}$$

$$(-0.3)(2) = cov.(X,Y)$$

$$\underline{cov(X,Y) = -0.6}$$

~~Cov (X, Z) = cov.(X, X+Y)~~.

ii) $var(Z) = var(X+Y) = V(X) + V(Y) + 2\,cov(X,Y)$

$= 4 + 1 + 2(-0.6)$

$= 5 - 1.2$

$\underline{var(Z) = 3.8}$. Ans

i) $Cov(X,Z)$

$= Cov(X, X+Y) = cov(X,X) + cov(X,Y)$.

$= V(X) + cov.(X,Y)$.

$= 4 + (-0.6)$

$\underline{cov.(X,Z) = 3.4}$ Ans

A10) $f_{X,Y}(x,y) = K x^{-\alpha} e^{-y/\beta}$ $\quad 1 < x < \infty, \ \alpha > 1$
$1 < y < \infty, \ \beta > 0$

$$\int_y \int_x f_{X,Y}(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_1^\infty \int_1^\infty K x^{-\alpha} e^{-y/\beta}\,dx\,dy = 1.$$

$$\Rightarrow \cancel{K\left(\frac{-\beta e^{-y/\beta}}{(-\alpha+1)}\right) = 1}$$

$$\Rightarrow \cancel{K = \frac{\alpha+1}{e^{-y/\beta}}} \quad \cancel{K = \frac{\alpha-1}{e^{-y/\beta}}}$$

$$\Rightarrow \frac{K}{(\alpha-1)} \beta e^{-1/\beta} = 1.$$

$$K\beta e^{-1/\beta} = \alpha - 1$$

$$\cancel{K = \frac{1-\alpha}{\beta e^{-1/\beta}}}$$

$$\Rightarrow \boxed{K = \frac{\alpha-1}{\beta e^{-1/\beta}} \text{ or } \frac{(\alpha-1)e^{1/\beta}}{\beta}}$$

Ans