Probability and statistics

Daxneeka Ranawat - 59

Answer 1 - (i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Mean $= \frac{\Sigma x_i}{N} = \frac{134}{10} = 13.4$ here $N = 10$

Median $\rightarrow \frac{n+1}{2}$ th term $= 5.5^{th}$ term $= \frac{11+15}{2} = 13$

$SD = \sqrt{\frac{(x_i - \bar{x})^2}{n}}$

$= \sqrt{41.648} = 6.4535$

Mode : most frequently occurring value $= 18$

IQR : $Q_2 = \text{Median} = 13$

$Q_1 = \cancel{\frac{n+1}{4}} = \frac{n}{4}^{th}$ term $= 2.5^{th}$ term $= 8$

$Q_3 = \frac{3n}{4}^{th}$ term $= 7.5^{th}$ term $= 18$

IQR $= Q_3 - Q_1 = 18 - 8 = 10$

(ii)

No. of household	0	1	2	3	4	(x_i)
No. of People	5	11	26	15	3	(f_i)

$\Sigma f_i = 60$

Mean $(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{120}{60} = 2$

Median

x_i	f_i	c.f.
0	5	5
1	11	16
2	26	42
3	15	57
4	3	60

$\frac{n}{2}^{th} \rightarrow \frac{60}{2}^{th}$ term $\rightarrow 30^{th}$

Median $= 2$ $\because$ corresponds to c.f. where 30 lies

Mode $= 2$ since most frequently occurring

IQR $\Rightarrow$ $Q_2 = 2$

$Q_1 = 1$

$Q_3 = 3$

IQR $= Q_3 - Q_1 = 2$

S.D $\Rightarrow$

x_i	f_i	$\bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
0	5	2	4	20
1	11	2	1	11
2	26	2	0	0
3	15	2	1	15
4	3	2	4	12

$S.D = \sqrt{\frac{\Sigma f(x_i - \bar{x})^2}{n-1}}$

$= \sqrt{\frac{58}{59}} = 0.99149$

Question 2 - $X \sim P(0.5)$

(i) $P(X=0) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.5}\lambda^0}{0!} = e^{-0.5} = 0.6065$

(ii) X_1 = no. of years with a claim

$X_1 \sim$ Binomial $(3, 0.3935)$

$P(X_1 = 1) = {}^3c_1 (0.3935)^1 (0.6065)^2 = 0.434$

(iii) $X_2 \sim Exp(0.5)$

$P(X_2 > 2) = \int_2^\infty \lambda e^{-\lambda x} dx = 0.5 \int_2^\infty e^{-0.5x} dx = \left[\frac{e^{-0.5x}}{-0.5}\right]_2^\infty 0.5$

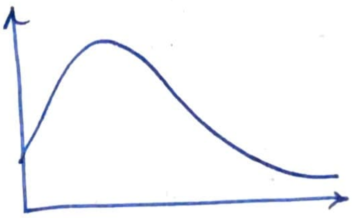
$= -\left[e^{-\infty} - e^{-1}\right] = e^{-1} = 0.368$

Answer 3 - $X \sim$ gamma $(2, 1/2)$

$E(X) = \frac{\alpha}{\lambda} = 4$

$V(X) = \frac{\alpha}{\lambda^2} = 2 \times \frac{1}{1/4} = 8$

(ii) PDF $\rightarrow \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\Gamma\alpha}$



Increases at first then decreases

(iii) CDF $\rightarrow \int_0^x \frac{\lambda^\alpha x^{\alpha-1}}{\Gamma\alpha} e^{-\lambda x}$ when $x > 0$

0 $\quad x < 0$

Answer 5 - ~~P(A)~~ $P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

A : Package arrives late

$P(A/L_1) = 0.2$ $\quad P(A/L_2) = 0.4$ $\quad P(A/L_3) = 0.1$

$$P(L_2/A) = \frac{P(L_2) \cdot P(A/L_2)}{P(L_1) \cdot P\left(\frac{A}{L_1}\right) + P(L_2) \cdot P\left(\frac{A}{L_2}\right) + P(L_3) \cdot P\left(\frac{A}{L_3}\right)}$$

$$= \frac{0.5 \times 0.4}{0.4 \times 0.2 + 0.5 \times 0.4 + 0.1 \times 0.1} = 0.6896$$

Answer 4 –

10 M → 6 FQ, 4 PQ

8 F → 7 FQ, 3 PQ

$$1F \rightarrow \frac{{}^7C_1 \times {}^6C_1}{{}^{18}C_2}$$

$$2F \rightarrow \frac{{}^7C_2 \times {}^6C_0}{{}^{18}C_2}$$

$$\text{Answer} = \frac{{}^7C_1 \times {}^6C_1}{{}^{18}C_2} + \frac{{}^7C_2 \times {}^6C_0}{{}^{18}C_2}$$

$$= \frac{7 \times 6 + 21}{153} = \frac{63}{153} = 0.411764705$$

Answer 6 – $X \sim U(1,2)$

$$Y = \frac{3}{6-X}$$

$$F_Y(Y) = P(Y \le y) = P\left(\frac{3}{6-X} \le y\right) = \cancel{P(3 \le y(6-x))}$$

$$= P\left(\frac{6-x}{3} \ge y\right)$$

$$= P(x - 6 \le -3y)$$

$$= P(x \le 6 - 3y)$$

$$F_Y(Y) = \begin{cases} 0 & y > 2 \\ 6-3y & 1 \le y \le 2 \end{cases}$$

$$f'_{(Y)}Y = \begin{cases} 0 & y > 2 \\ -3 & 1 \le y \le 2 \end{cases}$$

Answer 7 – (i) $F(3) = F(X \le 3) = 0.05 + 0.15 + 0.35 = 0.55$

(ii) $E(X) = 1(0.05) + 2(0.15) + 3(0.35) + 4(0.4) + 5(0.05) = 3.25$

(iii) $E(X^2) = 1(0.05) + 4(0.15) + 9(0.35) + 16(0.4) + 25(0.05) = 11.45$

$\therefore V(X) = E(X^2) - (E(X))^2 = 11.45 - (3.25)^2 = 0.8875$

(iv) Coefficient of skewness $= E(X - \bar{X})^3$

Answer 8 - $X \sim B(15, 0.2)$

$$P(X<3) = P(X=0) + P(X=1) + P(X=2)$$

$$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)^1 (0.8)^{14} + {}^{15}C_2 (0.2)^2 (0.8)^{13}$$

$$= 0.39802$$

Answer 9 - $P(X=7) = {}^{6}C_3 (0.01)^3 (0.99)^4 = 0.000019409$

Answer 10 - $X \sim P(0.4)$

$$P(X>2) = 1 - P(X \leq 2)$$

$$= 1 - 0.99207$$

$$= 0.0793$$

Answer 11 - $P(\text{Next 3 Boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Answer 12 - $X \sim Exp(10)$

$$P(X<0.1) = \int_0^{0.1} \lambda e^{-\lambda x} dx$$

$$= 10 \int_0^{0.1} e^{-10x} dx = 10 \left[\frac{e^{-10x}}{-10}\right]_0^{0.1} = 1 - e^{-1} = 0.6321$$

Answer 13 - $X \sim U(75, 120)$

PDF $\rightarrow \int_{80}^{100} \frac{1}{b-a} dx$

$$P(X \text{ lies b/w 80 and 100}) = \frac{1}{45}[100-80] = \frac{20}{45} = \frac{4}{9}$$

Answer 14 - $X \sim Gamma(5, 0.4)$

$$2\lambda X \sim \chi^2_{2\alpha = n}$$

$$2 \times 0.4 \times X \sim \chi^2_{10}$$

$$0.8X \sim \chi^2_{10}$$

$P(X < 22.89)$

$P(\chi^2 < 18.312)$

$\Rightarrow 0.949898$

Table Book & Interpolation

$18.0 \to 0.9450$

$18.5 \to 0.9529$

$0.5 \to 0.0079$

$1 \to \frac{0.0079}{0.5}$

$0.31 \to \frac{0.0079}{0.5} \times 0.31 = 0.004898$

<u>Answer 15</u> –

PDF $f(x) = \frac{k}{(x+5)^4}$ $x > 0$

$$\int_0^\infty f(x)\,dx = 1$$

$$k\int_0^\infty (x+5)^{-4}\,dx = 1$$

$$\frac{k}{-3}\left[(x+5)^{-3}\right]_0^\infty = 1$$

$$-\frac{k}{3}\left[0 - (5)^{-3}\right] = 1$$

$$-\frac{1}{125} \times -\frac{k}{3} = 1$$

$$k = 125 \times 3 = 375$$

(ii) $P(X \le 3) = \int_0^{10} f(x)\,dx = 375\int_0^{10} (x+5)^{-4}\,dx = \frac{375}{-3}\left[(x+5)^{-3}\right]_0^{10}$

$= -125\left[\frac{1}{3375} - \frac{1}{125}\right] = 0.962963$

(iii) $E(X) = \int_0^\infty x f(x)\,dx = 375\int_0^\infty x\,(x+5)^{-4}\,dx = 375\int_0^\infty x(x+5)^{-4}\,dx$

$375\int_5^\infty t^{-3} - 5t^{-4} = 375\left[\frac{1}{25}\left(\frac{1}{2} - \frac{1}{3}\right)\right] = \frac{5}{2} = 2.5$

$E(X) = 2.5$