

## prob and stats -2

(1)  $\Sigma x^2 = 92500$ ,  $\bar{x} = 85$   
 $\Sigma y^2 = 372717$ ;  $\bar{y} = 173.7$   
 $\Sigma xy = 182560$

$$S_{xx} = \Sigma x_i^2 - n(\bar{x})^2 \Rightarrow 20250$$

$$S_{yy} = \Sigma y^2 - n(\bar{y})^2 \Rightarrow 21000.1$$

$$S_{xy} = \Sigma xy - n(\bar{x}\bar{y}) \Rightarrow 34915$$

$$(a) \quad \beta = \frac{S_{xy}}{S_{xx}} = 1.7242$$

$$\alpha = \bar{y} - \beta\bar{x} = 27.143$$

$$\text{So, } \hat{y} = 27.143 + 1.7242x$$

(b) Gradient is no of hours he spend in upon amount withdrawn.

(2)

$$\sum x = 385.2$$

$$\sum x^2 = 12666.58$$

$$(i) \sum y = 1162.5$$

$$\sum y^2 = 119026.9$$

$$\sum xy = 38191.41$$

$$S_{xx} = 301.66$$

$$S_{yy} = 6409.7125$$

$$S_{xy} = 875.16$$

$$\hat{\beta} = 2.901147$$

$$\hat{\alpha} = 3.74818$$

$$\hat{y} = 3.74818 + 2.901147x$$

$$(ii) \hat{\sigma}^2 = \frac{1}{10} \left[ S_{yy} - \frac{S_{xy}^2}{S_{xx}} \right]$$

$$= 387.0745$$

$$-2.228 \leq \frac{\hat{\beta} - \beta}{\hat{\sigma} \sqrt{S_{xx}}} \leq 2.228$$

$$\hat{\beta} - 2.228 \hat{\sigma} \sqrt{S_{xx}} \leq \beta \leq \hat{\beta} + 2.228 \hat{\sigma} \sqrt{S_{xx}}$$

$$(0.377, 5.425)$$

$$(III) \hat{\mu}_0 = 119.794$$

$$s.e.(\hat{\mu}_0) = 10.5989$$

95% confidence interval of  $\mu_0$

$$\Rightarrow 119.794 \pm 2.228(10.598)$$

$$\Rightarrow 119.794 \pm 23.6144$$

$$\Rightarrow (96.17956, 143.4084)$$

(3)

(i) comments on plot:

- The centres of distribution differ for all four cities. Thus there's a Prima Facie case for suggesting that underlying means are different.
- The difference between mean time taken to communicate to office in Peak hours & non peak hours are in order city A, city D [highest, lowest].

(ii) Assumptions

- (\*) The population must be normal
- (\*) The population must have a common variance.
- (\*) The observations are independent



(iii)  $H_0$  : The mean difference is same for each city.

$H_0$  : The mean difference are not same

$$SS_T = 1495 - \frac{103^2}{28} = 1116.11$$

$$SS_B = \frac{1}{7} (64^2 + 44^2 + 8^2 + (1-13)^2) - \frac{103^2}{28} = 51.611$$

$$SS_R = SS_T - SS_B = 600.00$$

### ANNOVA

| Source     | df | SS      | MS     | F    |
|------------|----|---------|--------|------|
| treatments | 3  | 516.11  | 172.04 | 6.88 |
| Residual   | 24 | 600.00  | 25.00  |      |
| Total      | 27 | 1116.11 |        |      |

$$\text{Under } H_0 = F = \frac{172.04}{25} = 6.88 \text{ Using } F_{3,24} \text{ distribution}$$

The 5% critical value is 3.009, so we have sufficient to reject  $H_0$ .

$$\bar{y}_1 > \bar{y}_2 > \bar{y}_3 > \bar{y}_4 ; \quad \hat{\sigma}^2 = \frac{SS_R}{n-k} = 2$$

$$C_{\alpha} = 0.025 * 6 * \sqrt{\left(\frac{1}{7} + \frac{1}{7}\right)} = 2.064 * \sqrt{2}$$

$$\bar{y}_1 - \bar{y}_2 = 2.85, \quad \bar{y}_2 - \bar{y}_3 = 5.15, \quad \bar{y}_3 - \bar{y}_4 = 8$$

observing that all 3 differences less than 5.52, we can conclude that they have no significant difference.

Examining  $\bar{y}_1 - \bar{y}_3 = 8$

hence there's a significant difference between means 1 & 3.

(iii)

(4)

(i) Already in Sol (3), (ii)

$$(ii) H_0 : \mu_1 = \mu_2 = \dots = \mu_5$$

$H_a$  : at least one of the mean is not same

$$\frac{T^2}{N} = \frac{(\sum y_1 + \sum y_2 + \sum y_3 + \sum y_4 + \sum y_5)^2}{N} = \frac{104385.9394}{N}$$

$$SS_{Reg} = \frac{(354)^2}{7} + \frac{(386)^2}{8} + \frac{(87)^2}{6} + \frac{(645)^2}{7} + \frac{(384)^2}{5} - \frac{T^2}{N}$$

$$= 22325.68917$$

$$SS_{TOT} = 69930.0606$$

$$SS_{Res} = SS_{TOT} - SS_{Reg}$$

$$= 47604.37143$$

## ANOVA

| Sources of Variation | df | SS         | MS        | F    |
|----------------------|----|------------|-----------|------|
| Reg                  | 4  | 22325.69   | 5581.4225 | 3.28 |
| Res                  | 28 | 47604.3143 | 1700.154  |      |
| Total                | 33 | 69930.066  |           |      |



So,  $F_{4,28}$  tabulated is : 2.24

and our calculated value is 3.28

So, we have sufficient evidences to reject  $H_0$ .

(C)

$H_0$ : They are independent of each other  
Expected values  $H_a$ : They are dependent

| paper cleaned | 3-5   | 5-8    | 8-10  | 10-12 |
|---------------|-------|--------|-------|-------|
| 0-3           | 27.42 | 23.089 | 14.43 | 11.06 |
| 4-6           | 15.15 | 12.76  | 7.97  | 6.14  |
| 7-9           | 14.43 | 12.15  | 7.59  | 5.82  |

$$\chi^2_{cal} = \frac{(E_i - O_i)^2}{E_i}$$

$$\Rightarrow 11.27 + 0.4133 + 4.925 + 3.3204 + 4.384 \\ + 4.1079 + 0.133 + 0.00319 + 6.162 + \\ 1.4175 + 7.234 + 6.56$$

$$= 39.929$$

Since  $\chi^2_{cal} > \chi^2_{Tab}$  ; hence we have sufficient evidences to reject  $H_0$ .

| ⑤ Source of | SS | MSS    | F     |
|-------------|----|--------|-------|
| Regression  | 3  | 21.153 | 7.051 |
| Residual    | 8  | 1.236  | 0.155 |
| Total       | 11 | 22.38  |       |

$$F_{3,8,51} = 7.951$$

$\therefore$  We reject  $H_0$ .

⑥ (i) correlation coefficient

$$S_{yy} = 8923.6$$

$$S_{xx} = 54.9$$

$$S_{xy} = 661.2$$

$$r = \frac{S_{xy}}{\sqrt{S_{yy} \cdot S_{xx}}} = 0.94466$$

(ii)  $\hat{\beta} = 12.0437$

$$\hat{\alpha} = 9.22957$$

$$y = 9.22957 + 12.0437 x$$



$$(iii) SS_{RES} = \left[ S_{yy} - \frac{S_{xy}^2}{S_{xx}} \right] = 960.2951$$

$$S_{TOT} = 8923.6$$

$$S_{REG} = 7963.305$$

$$(iv) R^2 = \frac{SS_{REG}}{SS_{TOT}} = \frac{7963.305}{8923.6} = 0.89148$$

Hence: 89% of variation is shown by our model

$$\underline{7.} \quad S_{xx} = \sum x^2 - n(\bar{x})^2$$

$$= 4789.4221$$

$$S_{yy} = \sum y^2 - n(\bar{y})^2 = 1189.2077$$

$$S_{xy} = \sum (x - \bar{x})(y - \bar{y}) = 2176.84$$

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = 0.45451$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 10.9056$$

$$\text{so required eqn} = y = 10.79056 + 0.45451x$$

(II)  $H_0 : \beta = 0$

$H_a : \beta \neq 0$

$\hat{\sigma}^2 = 22.20136$

$$Z_c = \frac{\hat{\beta} - \beta}{\hat{\sigma} / \sqrt{S_{xx}}} = \frac{0.45451}{\sqrt{22.20136} / 4789.4221}$$

$Z_c = 6.67567$

$Z_{tab} = 2.262$

since  $Z_{cal} > Z_{tab}$

Hence, we have insufficient evidences to reject  $H_0$

(III)  $r = \frac{s_{xy}}{\sqrt{s_{xx} \cdot s_{yy}}} = \frac{2176.84}{\sqrt{4789.4221 \times 1189.2077}}$

$= 0.91213$

(IV) given :  $x^3 = 25 \Rightarrow 2.924$

WOW = 1

for mean response

$\hat{H}_0 = 10.79 + 0.4545(2.924)$

$\hat{H}_0 = 12.11773$

$V(\hat{H}_0) = \hat{\sigma}^2 \left[ \frac{1}{n} + \frac{(x - \bar{x})^2}{S_{xx}} \right]$

$$= \frac{22 \cdot 20136}{11} + \frac{(2.724 - 15.83)^2}{4789.4221}$$

$$\boxed{\text{Val}(\hat{H}_0) = 2.790417}$$

95% CI :

$$\hat{H}_0 \pm 2.262 \sqrt{2.790417}$$

$$\Rightarrow 12.11773 \pm 2.262 \sqrt{2.790417} \Rightarrow (8.3392, 15.8962)$$

for individual response

$$\hat{y}_0 = 12.11773$$

$$v(\hat{y}_0) = 24.991777$$

95% CI :

$$\hat{y}_0 \pm 2.262 \sqrt{24.991777}$$

$$\Rightarrow 12.11773 \pm 2.262 \sqrt{24.991777}$$

$$= (0.8096, 23.42587) \text{ soln }$$

(iv) since there is some pattern hence, we can't say it exhibits normal distribution.



~~(11)~~  
8. 9  
 (1) The main purpose of factor analysis is to reduce many individual item into a fewer number of dimensions. Principle component analysis is a method to decrease dimensions without data loss.

| (II) | PC   | Diagnol Entry | Pci/Sum (PC <sub>i</sub> ) |
|------|------|---------------|----------------------------|
|      | PC 1 | 0.456         | 65%                        |
|      | PC 2 | 0.137         | 18.5%                      |
|      | PC 3 | 0.08          | 11.4%                      |
|      | PC 4 | 0.0165        | 2.4%                       |
|      | PC 5 | <u>0.72</u>   | <u>1.7%</u>                |
|      |      | 0.1015        | 100%                       |

(III) As 1st 3 principle components explain over 95% of total variance, dimensions can be reduced to 3 for this data set.



9. (i) → Appears to be negatively correlated.

(ii)  $S_{xx} = 75$  ,  $S_{xy} = -296$

$$S_{yy} = 2482.4$$

$$r = \frac{-296}{\sqrt{2482.4(75)}} = -0.686$$

This shows ~~the~~ (-ve) ~~correlation~~ coefficient.

(iii)  $H_0 : \rho = 0$        $H_a : \rho < 0$ .

$$w = \tanh^{-1}(r) = -0.84036$$

$$w \sim N\left(0, \frac{1}{7}\right)$$

$$Z_c = \frac{-0.84036 - 0}{\sqrt{\frac{1}{7}}} \Rightarrow -2.2234$$

$$Z_{tab} = -1.96$$

$$\text{Since } Z_c < Z_{tab}$$

hence we reject null hypothesis

10

$$(IV) \quad \hat{\beta} = \frac{S_{xy}}{S_{xx}} = -3.94667$$

$$2 = \bar{y} - \hat{\beta} \bar{x} = 82.16$$

$$y = 82.16 - 3.94667x$$

$$(V) \quad R^2 = 0.470596$$

so, 47% of variation is shown by our model,

so, when  $x = 1$

$$y = 82.16 - 3.94667 \\ \approx 78.21333$$

when  $x = 2$

$$y = 74.26666$$

for every additional 1 hour spent, expected change is

$$\boxed{-3.94667}$$

$$10. S_{xx} = 0.0042$$

$$S_{yy} = 0.00126$$

$$S_{xy} = 0.0022$$

$$SS_{TOT} = 0.00126$$

$$SS_{REG} = \frac{S_{xy}^2}{S_{xx}} = 0.00115$$

$$SS_{RES} = SS_{TOT} - SS_{REG} \\ = 0.0011$$

### ANNOVA TABLE

| Source     | df | SS      | MSS     | F      |
|------------|----|---------|---------|--------|
| Regression | 1  | 0.00115 | 0.00115 | 10.454 |
| Residual   | 1  | 0.00011 | 0.00011 |        |
| Total      | 2  | 0.00126 |         |        |

$$H_0: \beta = 0$$

$$H_1: \beta \neq 0$$

$$F_{1,1} 5\% = 161.42$$

$$F_{cal} < F_{table}$$

$\therefore$  Hence will not reject  $H_0$ .