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R.No - 419

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Financial Engineering
Assignment:

1.i) $S_0 = £65$ $\sigma = 25\%$
$r = 2\%$ $t = 6$ months
$X = £55$

$$C_t = S_0 * \phi(d_1) - X(e^{-rt}) (\phi)(d_2)$$

$$d_1 = \frac{\ln(S_0/X) + (r + 1/2\,\sigma^2)T}{\sigma(\sqrt{T})}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$C_0 = £11.418$$

ii) change in the price of the call option with respect to the change in the price of the underlying is the delta of the call option $= \frac{dC_t}{dS_t}$.

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iii) $\phi(d_1) = 0.862134$

iv) using put - call parity,

$$\Delta P + 1 = \Delta c$$
$$\Delta p = -0.1379$$

2) i) change in the price of the option with respect to the change in the price of the underlying is the delta of it

$$\Delta_c = \frac{dC_t}{dS_t}$$

Vega of an option is defined as the change in the price of the option with respect to the change in the volatility of the underlying.

$$V_c = \frac{dC_t}{d\sigma}$$

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ii) using the put call parity,

$$P_t + S_t = C_t + Ke^{-rt}$$

diff. w.r.t $\sigma$

$$V_p = V_c$$

Hence proved

iii) $S_0 = \$55$ $\quad \sigma = 25\%$

$X = \$50$ $\quad r = 5\%$

$t = 1 \text{ year}$

$$C_t = S_0 \, \phi(d_1) - X e^{-rt} \, \phi(d_2)$$

$$d_1 = \frac{\ln(S_0/X) + (r + 1/2(\sigma^2))T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$d_1 = 0.7062$

$d_2 = 0.4562$

$\therefore C_t = \$9.65$

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using the put-call parity,

$p_t = 2.2140.$

iv) Delta hedged means that change in price of the underlying is not sensitive to the change in ~~pae~~ the value of the derivative.

Vega-hedged means that ~~change~~ change in the volatility of the underlying is not sensitive to its value.

3]. i) Price $= e^{-r(T-t)} E_Q(X_T / F_t)$

ii) European call option.

$$c_t = S_0 \phi(d_1) - X e^{-rt} \phi(d_2)$$

$$d_1 = \frac{\ln(S_0)/(X) + (r + 1/2 \sigma^2) T}{\sigma * T}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

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$d_1 = 0.344$

$d_2 = 0.1673$

$C_t = 4.6604$

iii) Value of call option = 4.6604.

iv) using the put-call parity,

$P_t + S_0 = C_t + Ke^{-rt}$

$P_t = 2.4506$

v) Value of underlying will fall if dividends were payable each time by the amount of dividend payable.

The value of the European call option would decrease as having the option to buy a share which would be less for a fixed price at the expiry date, would be less valuable.

Value of American call will increase compared to European.

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4] i) The CMG ~~theory~~ theorem states that:

Suppose $Z_t$ is a SBM under P. And there exists a measure Q such that P & Q are equivalent measures then,

$$Z\text{bar}(t) = Z_t + Y_t$$

5] i) $\Delta c = \frac{dc_t}{dS_T} = \phi\, d_1$

ii) $S_0 = \$40$

$r = 2\%$ p.a

$X = \$45.91$

$T = 5$ years.

$\Delta = 0.6179$

$$d_1 = \frac{\ln(S_0/X) + (r + 1/2\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$\phi(d_1) = \phi(0.3) = 0.6179$

$d_1 = 0.3$ - from tables.

$\sigma = 32\%$.

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iii) $V_0 = e^{(-rT)} \quad E_Q [X_T / F_0]$

Since, the stock prices are independent,

$V_0 = e^{-rT} * c * Q[S_1/S_0 < k_s]$
$Q[R_1/R_0 < k_e]$

iv) since they are perfectly correlated.

$V_0 = e^{-rT} \cdot c \cdot Q[S_1/S_0 < k_s, S_1/S_0 < k_R]$

$V_0 = e^{-rT} \cdot c \cdot Q[S_1/S_0 < \min(k_s, k_e)]$

$V_0 = e^{-rT} \cdot c \cdot Q[R_1/R_0 < \min(k_s, k_e)]$

v) $1 - \phi(d_2) = \phi(-d_2)$

$= \phi\left( \frac{\ln(X/S_0) - (\mu + 1/2\sigma^2)T}{\sigma\sqrt{T}} \right)$

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$V_0 = e^{-rT}$ $C\ Q[S_1 < S_0 k_S]\ Q[R_1 < R_0 k_R]$

$V_0 = \$ 1.6075$

6]

i) a) $\Delta_p = \phi(d_1) - 1$

b) $V_0 = \psi - 24830\ (S_0)$

$\Delta_V = -24{,}830.$

we set $100000\ \Delta_p = \Delta V$

$$\Delta_p = \frac{\Delta V}{100000}$$

$= -0.2483.$

ii) Since,

$\Delta_p = \phi(d_1) - 1 = 0.2483$

$\phi d_1 = 0.7517.$

$d_1 = 0.68$

$$d_1 = \frac{\ln(S_0/X) + (r + 1/2\ \sigma^2)\ T}{\sigma\sqrt{T}}$$

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iii) b) $V_0 = \Psi - 24830\ (6.40)$

$\Psi = 165{,}872$

I] i) Delta - change in price of derivative w.r.t change in price of underlying

Gamma - change in the delta of der w.r.t change in price of underlying

Vega - change in price of underlying w.r.t volatility of underlying

ii) $\phi d_1 = 0.8010.$

iii) $V_0 = \psi + \phi(y_0) \rightarrow$ replicating portfolio

$\therefore \Delta v = \phi$
$= 0.8010$

Substituting

$V_0 = \Psi + 0.801\ (60)$

$V_0 = \Psi + 48.06$ $(V_0 = 17.91)$

$\therefore \Psi = -30.15$

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iv) $\frac{dc}{d\sigma} = \frac{\Delta C}{\Delta \sigma}$

$29 = \frac{c_2 - c_1}{2\%}$

$\therefore C_2 = \$18.49$

8] $S_0 = \$100$ $r = 3\%$ $t = 1yr$

$X = \$109.42$

$\Delta c = 0.4207$

$\phi(-0.20) = 0.4207$

$d_1 = -0.2$

$\sigma = 20\%$

9] i) func. g must satisfy:

$$\frac{dg}{dt} + (r-q) S_T \frac{dg}{dS_T} + \frac{1}{2}\sigma^2 S_t^2 \frac{d^2g}{dS_T^2} = rg$$

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boundary condition applies at maturity

$D_T = g(T, S_t) = f(S_T)$

10) $C_t + Ke^{-r(T-t)} - P_t = S_t$

$X = 120$ $t = 1$ yr $r = 2\%$

$C_t = 10.09$

$S_0 = 110$

$d_1 = \frac{\ln(S_0/X) + (r + 1/2\,\sigma^2)T}{\sigma\sqrt{T}}$

$d_2 = d_1 - \sigma(\sqrt{T})$

$\sigma = 30\%$

The payoff from portfolio P,

$S_1 - 121 \leq D \leq S_1 - 120$

It follows that the initial price, V, of the portfolio should satisfy.

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$S_0 - 121e^{-r} \leq V \leq S_0 - 120e^{-r}$

ie: $-8.604 \leq V \leq -7.624$

$17.714 \leq P_0 \leq 18.6914$

The Black - Scholes price is \$18.35

11) $X = \$150$
$r = 2\%$ p.a
$S_0 = 117.98$

$\Delta_{portfolio} = 100000\,\Delta_c - 18673\,\Delta_s$

$\Delta_s = 1$
$\Delta_{portfolio} = 0$.

$\Delta_c = 0.18673$

Under the Black Scholes option pricing model $\Delta_c = \phi(d_1)$.

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$\phi d_1 = 0.18673$
$d_1 = -0.89$

using the black scholes formula,

$\sigma = 22\%$ p a

$$p_t = Ke^{-rt}\,\phi(-d_2) - S_0 \cdot \phi(-d_1)$$

$$d_1 = \frac{\ln(S_0/X) + (r + 1/2\,\sigma^2)\,T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$p_t = \$\,31.45$

taking partial derivative

$\Delta c = \Delta p + \Delta s$
$\gamma_c = \gamma_p + \gamma_s$.

$\Rightarrow$ 100,000 put options.

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delta of portfolio

$= 100000\,\Delta_c - 10000\,\Delta_p + x\,\Delta_s$

$= 0$

$x = -100000$

12) Assumptions :

- complete divisibility
- no taxes
- no transaction costs
- unlimited buying & selling
- underlying asset follows a contin. path
- geometric brownian motion
- risk free rate is constant
- volatility is asset
- investors are ration
- investors are risk averse
- $dS_t = \mu S_t dt + \sigma S_t dB_t$

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$S_0 = £8$

$X = £9$

$r = 2\%$ p.a

$\sigma = 20\%$ p.a

$T = 3$ months $= 3/12$ yrs.

using Black scholes options formula,

$$p_t = Ke^{-rt}\,\phi(-d_2) - S_0\,\phi(-d_1)$$

$$d_1 = \frac{\ln(S_0/X) + (r + 1/2\,\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$p_t = £1.01$