STATS AND PROBABLITY

Q1] 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

(i) Mean $= \bar{x} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$

$= 13.4$

(ii)

no. of households (x_i)	no. of people (f_i)	cf	$f_i x_i$
0	5	5	0
1	11	16	11
2	26	42	52
3	15	57	45
4	3	60	12
	$\Sigma f_i = 60 = N$		$\Sigma f_i x_i = 120$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{120}{60} = 2$$

$$\text{Median} = l + \frac{(N/2 - cf)}{f} \times h$$

$$= 1.5 + \left(\frac{30-16}{26}\right) \times 1$$

$$= 1.5 + \frac{14}{26}$$

$$= 2.0384$$

Mode = 3 Median - 2 mean
= 3 (2.0384) - 2(2)
= 2.1163.

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term $= \frac{11}{4}$ term $= 2.75$ term

= 2nd term + 0.75 (3rd term - 2nd term)
= 7 + 0.75 (9 - 7)
= 8.5

$Q_3 = 3\left(\frac{n+1}{4}\right)^{th}$ term $= 3 \times \frac{11}{4}$

= 3 x 2.75
= 8.25
= 8th term + 0.25 (9th term - 8th term)
= 18 + 0.25 (18-18)
= 18 + 0.25 (0)
= 18

Inter Quartile range = $Q_3 - Q_1$ = 18 - 8.5
= 9.5

Q2] (i) 0 claim in 1 year $X \sim P(0.5)$

$$P(X=0) = \frac{e^{-x} x^x}{0!} = \frac{e^{-0.5}(0.5)^0}{1}$$

$$= e^{-0.5} = 0.60653$$

(ii) $P(X \geq 1) = 1 - P(X=0)$ $1 - 0.6065.3$

$= 0.039347$

Binomial distribution =

$$P(X<1) = {}^nC_2 \, p^x (1-p)^{n-x}$$

$$= {}^3C_1 \, p^1 (1-p)^{3-1}$$

$$= 6. (0.30347) * (0.60653)^3$$

$$= 0.43424$$

(iii) $P(X>2)$.

Q3] $x = 2, \quad \lambda = 1/2 \quad 0 < x < \infty$

(x^2) = variance

(x) = $\sqrt{\text{variance}}$

$$= \sqrt{\frac{\alpha}{\lambda^2}}$$

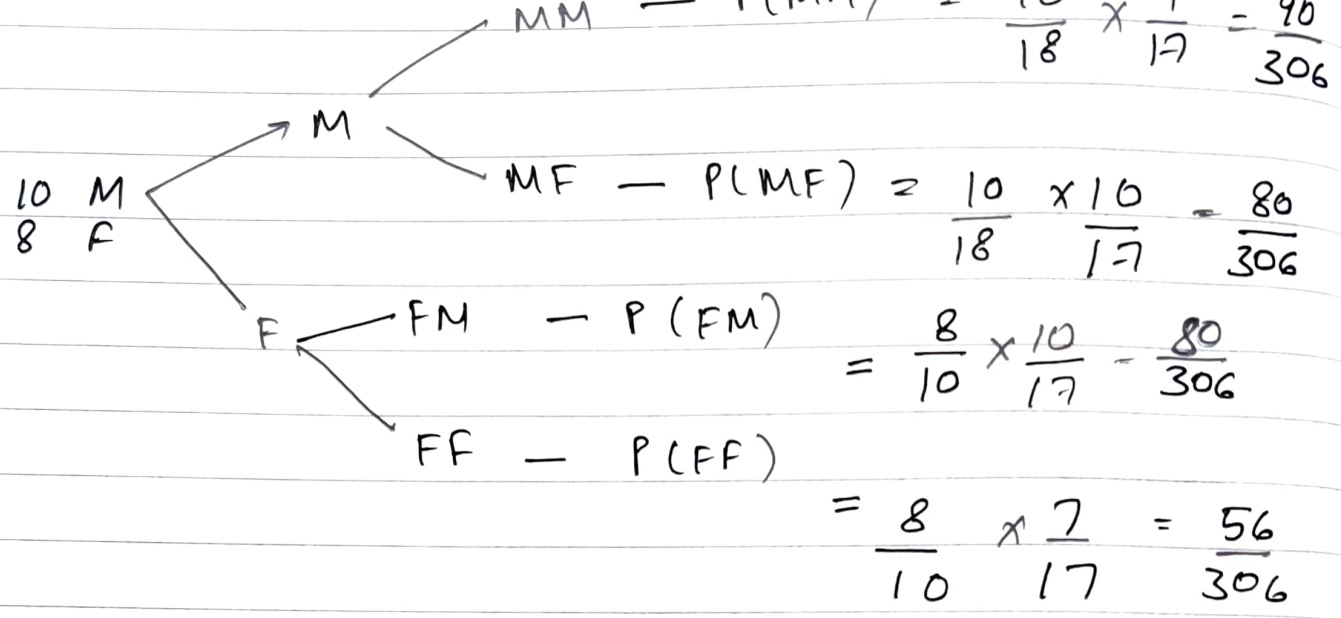
$$= \sqrt{\frac{2}{(1/2)^2}}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

Q4] $P(M) = \frac{10}{18}$, $P(F) = \frac{8}{18}$, $P(M,0) = \frac{6}{18}$, $P(F,0) = \frac{7}{8}$

Calculate.



$$P(X) = P(FF) + P(MF) + P(FM)$$

$$= \frac{80}{306} + \frac{80}{306} + \frac{56}{306} = \frac{216}{306}$$

$$P(0) = \frac{216}{306} \times \frac{7}{18} = 0.2745.$$

05] $E_1 \rightarrow$ arrives through 1
$E_2 \rightarrow$ arrives through 2
$E_3 \rightarrow$ arrives through 3

$A \rightarrow \neq$ event that the package arrived

$$P(E_2|A) = \frac{P(A|E_2) \cdot P(E)}{P(A|E_1) + P(A|E_2) + P(E_2) + P(A|E_3)P(E_3)}$$

$$= \frac{\frac{50}{100} \times \frac{40}{100}}{\frac{40}{100} \times \frac{20}{100} + \frac{50}{100} \times \frac{40}{100} + \frac{10}{100} + \frac{10}{100}}$$

$$= \frac{\frac{20}{100}}{\frac{29}{100}} = \frac{20}{29}$$

Q6] $X \sim U(1,2)$

$a = 1, \; b = 2$

$$f(x) = \frac{1}{2-1} = \frac{1}{1} = 1$$

$$\therefore f_Y(y) = f_X(w^{-1}(y)) \left| \frac{d}{dy} w^{-1}(y) \right|$$

$$= f_X\left(6 - \frac{3}{Y}\right) \cdot \frac{3}{y^2}$$

$$= 1 \cdot \frac{3}{y^2} = \boxed{\frac{3}{y^2}}.$$

$$Y = \frac{3}{6-X}$$

$$6 - X = \frac{3}{y}$$

$$X = 6 - \frac{3}{Y} = w^{-1}(y)$$

$$X = 6 - 3\left(\frac{1}{y}\right)$$

$$\frac{dx}{dy} = \frac{3}{y^2}$$

07]

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

(i) $F(3) = 0.35$

$= \Sigma P(x=x) \cdot x\,dx$

$= 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.4 + 5 \times 0.5$

$= 3.25$

(iii) $V(X) = E(x)^2 - [E(x)]^2$

$= 1 \times 0.05 + 4 \times 0.15 + 9 \times 0.35 + 16 \times 0.4 + 25 \times 0.05 - (3.25)^2$

$= 0.8875$

08] $P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 \times 0.02^0 \times 0.8815 + {}^{15}C_1 \times 0.2^1 \times 0.8^{14} + {}^{15}C_2 \times 0.2^2 \times 0.813$

$= 0.035184372 + 0.131941395 + 0.230897442$

$= 0.398023209$

09] $n = 7$

$p = 0.01 \Rightarrow p+q = 1$.

$q = 1 - 0.01 = 0.99$

$X \sim$ -ve Bin $(7, 0.01)$

$r = 3$

$P(X=3) = {}^{7-1}C_{3-1} (0.01)^3 (0.99)^{7-3}$

$= {}^6C_2 (0.01)^3 \cdot (0.99)^4 = 0.000014409$

Q11] $P(M) = P(F) = \frac{1}{2}$

woman has 2 girls and 5 boys

$P(\text{net 3 children are boys}) = \frac{1}{2} \times \left(\frac{1}{2}\right)^{3-1}$

$= \frac{1}{8}$

Q12] $\lambda = 10 \qquad X = 0.1$

$F(X)(0.1) = \lambda e^{-\lambda x} = 10\, e^{-10(0.1)} = 10e^{-1}$

$= 3.678794412.$

Q13) $X \sim V(75, 120) \qquad a = 75, \; b = 120$

$\therefore P(80 < x < 100) = f(x)\ 100 - f(x)\ 80$

$= \frac{100}{45} - \frac{80}{45}$

$= \frac{20}{45} = \frac{4}{9} = 0.4444$

Q14] $X \sim \text{Gamma}(5, 0.4)$ $\lambda = 0.4$ $\alpha = 5$

$P(X < 22.89) = ?$

$2\lambda X \sim \chi^2_{10}$

$0.8 X \sim \chi^2_{10}$

$\therefore P(X < 22.89) = P(0.8 X < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$= 0.94992892$

$= 0.9499296$