Probability and Statistics , Roll - 35

1) X = claim size on a home insurance

$X \sim N(800, 100^2)$

Y = claim size on a car insurance

$Y \sim N(1200, 300^2)$

$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$

$P(y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 > 800)$

$y_1 + y_2 + y_3 - x_1 - x_2 - x_3 - x_4 \sim N(3(1200) - 4(800),$
$3 \times 300^2 + 4 \times 200^2)$

$\therefore \sim N(400, 370000)$

$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$

$= 0.23576$

Q4) $G_V(t) = \left(\frac{p}{1-qt}\right)^K$, $p + q = 1$

$\therefore$ Distribution is -ve binomial

$P(V=4) = \frac{\sqrt{K+4}}{\sqrt{5}\sqrt{K}} p^4 q^4$

Q3) $f_X(x) = \lambda e^{-\lambda(x-\alpha)}$

$M_X(t) = E(e^{tx})$

$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$

$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)} dx$

$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(t-\lambda)x}\right]_0^{\infty} = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(\lambda-t)x}\right]_0^{\infty}$

$$m_x(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda}\left(0 - e^{-\lambda\alpha} e^{t\alpha}\right)$$

$$= \frac{\lambda e^{t\alpha}}{\lambda - t}$$

ii) $m'_x(t) = \lambda \cdot (e^{t\alpha})(\lambda - t)^{-2}(-1)(-1) + \lambda(\lambda - t)^{-1}(e^{t\alpha})(\alpha)$

$$= \frac{\lambda e^{t\alpha}}{(\lambda - t)^{+2}} + \frac{\lambda e^{t\alpha}\alpha}{\lambda - t}$$

$$E(x) = m'_x(0) = \frac{\lambda(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda} = \frac{1 + \lambda\alpha}{\lambda}$$

$m''_x(t) = \lambda e^{tx}(-2)(\lambda - t)^{-3}(-1) + \lambda(\lambda - t)^{-2} e^{t\alpha}(\alpha) +$
$\lambda\alpha e^{t\alpha}(\lambda - t)^{-2}(-1)(-1) + \lambda\alpha(\lambda - \cancel{t})^{-1}(e^{\alpha})(\alpha)$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda - t)^3} + \frac{\lambda\alpha e^{tx}}{(\lambda - t)^2} + \frac{\lambda\alpha e^{tx}}{(\lambda - t)^2} + \frac{\alpha^2 \lambda e^{t\alpha}}{\lambda - t}$$

$$E(x^2) = M''_x(0) = \frac{2\lambda(1)}{\lambda^2} + \frac{2\lambda\alpha}{\lambda^2} + \frac{\cancel{\lambda}\alpha^2}{\cancel{\lambda}}$$

$$= \frac{2}{\lambda} + \frac{2\alpha}{\lambda} + \alpha^2$$

$$V(x) = E(x^2) - E(x)^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha^2 - \frac{2\alpha}{\lambda}$$

$$V(x) = \frac{1}{\lambda^2}$$

2) N = Number of claims

N ~ negative binomial dff (3, 0.9)

X = claim Amount

X ~ gamma (6, 2)

Y = total claim Amount

$$M_Y(t) = M_N \log(M_X(t))$$

$$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

$$M_Y(t) = M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$M_N(t) = \left(\left(\frac{pe^t}{1-qe^t}\right)^k\right)$$

$$M_Y(t) = \left(\frac{p\, e^{\log(1-t/2)^6}}{1-q\, 6\log(1-t/2)6}\right)^3$$

$$M_Y(t) = \left(\frac{(0.9)(1-t/2)^6}{1-(0.1)\left(1-\frac{t}{2}\right)^6}\right)^3$$

$$E(N) = \frac{0.3}{0.9} = \frac{10}{3} = 3.33$$

$$V(N) = \frac{K(0.1)}{(0.9)^2} = 3.33 \times \frac{1}{9} = 0.37$$

$$E(X) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3 \qquad V(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = 1.5$$

$$V(Y) = E(N)\,V(X) + E(X)^2\,V(N)$$

$$= 5 + \frac{10}{3} = \frac{25}{3} = 8.333$$

$$S.D(Y) = \sqrt{\frac{25}{3}} = 2.88$$

Q5) $f(x,y) = ax^2 + axy \qquad 0 < x < 5$

$\qquad 10 < y < 20$

$$1 = \int_{10}^{20}\int_{0}^{5} ax^2 + axy \, dx\, dy$$

$$I = \int_{10}^{20} \frac{125a}{3} + \frac{25}{2} ay \, dy$$

$$I = \frac{125a}{3}(y)_{10}^{20} + \frac{25}{2} a \left(\frac{y^2}{2}\right)_{10}^{20}$$

$$a = \frac{3}{6875}$$

$$f_X(x) = a\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}(10x^2 + 150x)$$

$$E(y/x) = y f(y/x) = y \int_{10}^{20} \frac{f(x,y)}{f_X(x)}$$

$$= \int_{10}^{20} \frac{y\left(\frac{3}{6875}\right)(x^2 + xy)}{\frac{3}{6875}(10x^2 + 150x)} dy$$

$$= \left(\frac{1}{10(x+15)}\right)\left(\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}\right)$$

$$= \frac{1}{10(x+15)}\left(150x + \frac{7000}{3}\right)$$

$$E(y|x) = \frac{45x + 7000}{3(x+15)}$$

$P(y > \frac{1}{3}x + 10)$

$$f_y(y) = \int_0^5 \frac{3}{6875}(x^2 + xy)\,dx$$

$$= \frac{1}{6878}\left(\frac{250 + 75y}{2}\right)$$

$$P(y > \tfrac{1}{3}x + 10) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875}\left(\frac{250 + 75y}{2}\right)dy$$

$$= \frac{1}{6875(2)}\left[10{,}000 - \frac{1250x}{3} - \frac{25x^2}{3}\right]$$

6)

$X \to Poi(\lambda) \quad y \sim Poi(\mu)$

$M_x(t) = e^{\lambda(e^t - 1)}$

$M_y(t) = e^{\mu(e^t - 1)}$

$$M_{x-y}(t) = E(e^{t(x-y)})$$

$$= E(e^{tx - ty})$$

$$= \frac{E(e^{tx})}{E\,e^{t(y-1)}} = \frac{e^{\lambda(e^t - 1)}}{e^{\mu(e^t - 1)}}$$

$$= e^{\lambda - \mu(e^t - 1)}$$

HP

$$M_{x+y}(t) = E(e^{t(x+y)})$$

$$= E(e^{tx})\,E(e^{ty})$$

$$= e^{\lambda(e^t - 1)}\,e^{\mu(e^t - 1)}$$

$$= e^{\lambda + \mu(e^t - 1)}$$

Q7) i) $Var(x/y=2) = E(x^2/y=2) - (E(x/y=2))^2$

$= \frac{\sum n^2 P(X=n, y=y)}{P(y=2)} - \left(\sum n \frac{P(n=n, y=y)}{P(y=y)}\right)^2$

$= \frac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.1)}{0.6} - [1(0) + 2(0.3) + 3(0.1) + 4]$

$= \frac{53}{6} - \frac{289}{30} = \frac{29}{36}$

ii) $f(v, v) = \int_0^1 \frac{48}{67}(2vu - u^2)\,du$

$= \frac{48}{67}\left[\frac{2u^2 v}{2} - \frac{v^3}{3}\right]_0^1$

$= \frac{48}{67}\left(v - \frac{1}{3}\right) = \frac{48}{67}\left(\frac{3v-1}{3}\right) = \frac{16}{67}(3v-1)$

from ① $E(v/v=v) = \frac{\int_0^1 v\left(\frac{48}{67}\right)(2vu - v^2)\,du}{\frac{16}{67}(3v-1)}$

$= \frac{3}{3v-1}\int_0^1 2u^2 v - u^3\,du$

$= \frac{3}{3v-1}\left[\frac{3v}{3} - \frac{1}{v}\right]$

$= \frac{8v-3}{4(3v-1)}$

Q8) $\alpha = 3$, $\lambda = 2$

$X \sim \text{Gamma}(3, 2)$:

$E(Y/X=x) = 3x+1$ Var $(y/x=x) = 2x^2+5$

$E(y) = E(E(y/x=x))$

$= E(3x+1)$

$= 3E(x)+1$

$= 3(3/2)+1 = 11/2$

$V(Y) = E(V(y/x=x)) - V E(y/x=x)$

$= E(2x^2+5) + V(3x+1)$

$= 2(3)+5+9(3/4) = 6+5+6.75$

$= 17.75$

Q10) $f(x,y) = kx^{-\alpha} e^{-y/\beta}$ $1 < x < \infty$

$1 < y < 8$

$\alpha > 1$, $\beta > 0$

$1 = K \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta} \, dx \, dy$

$1 = K \left[\frac{1}{\alpha+1}\right]\left[-\beta e^{-1/\beta}\right]$

$K = \dfrac{-1+\alpha}{\beta e^{-1}\beta}$

Q9) $Z = \cancel{\phi}x + y$

$E(X) = 3 \quad E(y) = 4$

$\sigma_x = \sqrt{V(x)} = 2 \quad \sigma_y = \sqrt{V(y)} = 1$

$Cov(X, Z) = Cov(X, X+y)$

$= Cov(X, X) + Cov(X, y)$

$= V(X) + (-0.6)$

$= 3.4$

$Var(Z) = Var(x+y)$

$= V(X) + V(y) + Cov(x+y)$

$= 4 + 1 + 2(0.6)$

$= 3.8$